Testing Series

Divergence

Test for Divergence

$\lim_{n\to\infty} a_n \neq 0$ series diverges

Integral Test

$a_n = f(n)$ where f is a positive, decreasing function for all positive n

$\int_c^\infty f(x)\,dx = \infty$ series diverges

Comparison Test

$a_n \geq b_n$ where $\sum b_n$ diverges

Limit Comparison Test

$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ (including = ∞) and $\sum b_n$ diverges

Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$$

Root Test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$$

Alternating Series Test

$\sum (-1)^n a_n$ where $\lim_{n\to\infty} a_n \neq 0$

Convergence

Test for Divergence

$\lim_{n\to\infty} a_n = 0$ series **may** converge

Integral Test

$a_n = f(n)$ where f is a positive, decreasing function for all positive n

$\int_c^\infty f(x)\,dx = L$ where L is a unique number, series converges

Comparison Test

$a_n \leq b_n$ where $\sum b_n$ converges

Limit Comparison Test

$0 \leq \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$ and $\sum b_n$ converges

Reminder: limit does *not* include ∞

Ratio Test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$ Note: if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, inconclusive

Root Test

$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$ Note: if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, inconclusive

Alternating Series Test

$\sum (-1)^n a_n$ where

a) $a_{n+1} \leq a_n$

b) $\lim_{n\to\infty} a_n = 0$

Absolute Convergence: If $\sum |a_n|$ converges then $\sum a_n$ also converges.
Conditional Convergence: If $\sum a_n$ converges but $\sum |a_n|$ does not.

Geometric Series

$\sum_{n=1}^{\infty} ar^{n-1}$ converges to $\frac{a}{1-r}$ if $|r| < 1$ and diverges otherwise

P-series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise

Convergence Theorem for Power Series

If $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$ converges for $x = c$, then it converges absolutely for all $|x| < |c|$

If $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$ diverges for $x = d$, then it diverges for all $|x| > |d|$

Taylor Series of *f* at *a*

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = \frac{f(a)}{0!}(x-a)^0 + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \ldots \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

where $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ where c is a number between x and a. $R_n(x)$ is called the **Lagrange error bound.**

Maclaurin Series

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ or a Taylor series with $a = 0$.

Interval of Convergence: What values of x for which a series converges.
Radius of Convergence: One half the actual length of the interval of convergence.