

3-3 Homework: p. 156, #1-14

1. $c(x) = 2000 + 100x - 0.1x^2$

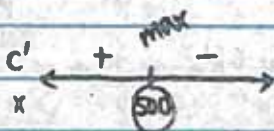
b) $\frac{dc}{dx} = 100 - 0.2x$

$\frac{dc}{dx} = 100 - 0.2(100) = 80$

c) $0 = 100 - 0.2x$

$0.2x = 100$

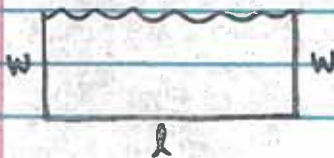
$x = 500$ machines



a) $c(100) = \frac{2000 + 100(100) - 0.1(100)^2}{100} = \frac{11000}{100} = 110$

average $\rightarrow 100$

2.



$3000 = l + 2w = P$

$l = 3000 - 2w \rightarrow l = 3000 - 2(750) = 1500$ yds.

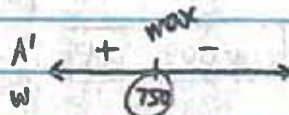
$A = lw = w(3000 - 2w)$

$A = 3000w - 2w^2$

$A' = 3000 - 4w = 0$

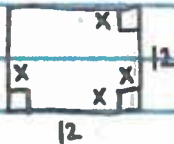
$4w = 3000$

$w = 750$ yds.



$A = 750 \times 1500 = 1,125,000$ yd²

3.



$V = l \cdot w \cdot h = (12 - 2x)(12 - 2x)(x)$

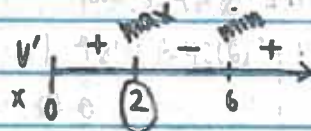
$V = 144x - 48x^2 + 4x^3$

$\frac{dV}{dx} = 144 - 96x + 12x^2$

$12 - 8x + x^2 = 0$

$(x + 6)(x - 2) = 0$

$x = 2, 6 \rightarrow$ squares: 2×2 in



domain: $x \in [0, 6]$

4. $108 = 2\pi r + l$, where $l = h$

$$l = 108 - 2\pi r$$

$$l = 108 - 2\pi \left(\frac{36}{\pi}\right) = 36$$

$$r = \frac{36}{\pi}, l = 36 \text{ in}$$

$$V = \pi \left(\frac{36}{\pi}\right)^2 (36) = \frac{36^3}{\pi} \approx 14851.066$$

$$V = \pi r^2 h \rightarrow V = \pi r^2 (108 - 2\pi r)$$

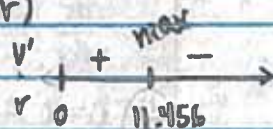
$$= 108\pi r^2 - 2\pi^2 r^3$$

$$\frac{dV}{dr} = 216\pi r - 6\pi^2 r^2$$

$$0 = 6\pi r (36 - \pi r)$$

$$r = 0, \frac{36}{\pi}$$

$$r = 11.456$$



5. Moon: $832t - 2.6t^2 = 0$

$$t(832 - 2.6t) = 0$$

$$t = 0, \boxed{320 \text{ sec}}$$

$$s' = 832 - 5.2t = 0$$

$$t = 160$$

$$s(160) = 832(160) - 2.6(160)^2 = \boxed{66,560 \text{ ft}}$$

Earth: $s = 832t - 16t^2 = 0$

$$t(832 - 16t) = 0$$

$$t = 0, \boxed{52 \text{ sec}}$$

$$s' = 832 - 32t = 0$$

$$t = 26$$

$$s(26) = 832(26) - 16(26)^2 = \boxed{10,816 \text{ ft.}}$$

6. $A = 16 = lw$

$$l = \frac{16}{w}$$

$$\boxed{w = 4, l = 4}$$

$$P = 2l + 2w$$

$$P = 2\left(\frac{16}{w}\right) + 2w$$

$$P = \frac{32}{w} + 2w \rightarrow 32w^{-1} + 2w$$

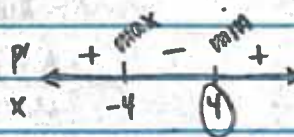
$$P' = -\frac{32}{w^2} + 2 = 0$$

$$w^2 = 32$$

$$2w^2 = 32$$

$$w^2 = 16$$

$$w = \pm 4$$



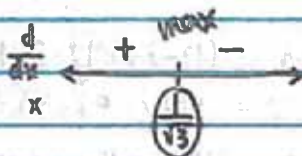
7. $x - x^3 = 0$; $x \in [0, 1]$

$$\frac{d}{dx} (1 - 3x^2) = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \approx 0.577$$



8.

$$V = \pi r^2 h$$

$$\pi r^2 h = 32\pi$$

$$h = \frac{32\pi}{\pi r^2}$$

$$h = \frac{32}{r^2}$$

$$\text{cost: top} = \pi r^2$$

$$\text{bottom} = \pi r^2$$

$$\text{sides} = 2\pi r h$$

$$2(\text{top} + \text{bottom}) = \text{sides}$$

$$2(\pi r^2 + \pi r^2) = 2\pi r h$$

$$C = 4\pi r^2 + 2\pi r h$$

$$4\pi r^2 + 2\pi r \left(\frac{32}{r^2} \right)$$

$$C = 4\pi r^2 + \frac{64\pi}{r} ; r \in [0, 2.99]$$

$$= 4\pi r^2 + 64\pi r^{-1}$$

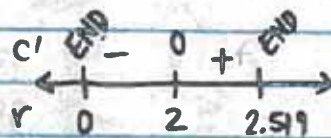
$$C' = 8\pi r - 64\pi r^{-2} = 0$$

$$8\pi r = \frac{64\pi}{r^2}$$

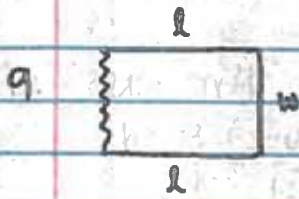
$$8\pi r^3 = 64\pi$$

$$r^3 = 8$$

$$\boxed{r = 2 \text{ in.}}$$



$$C|_{r=2} = \$150.80$$



$$P = 160 = 2l + w$$

$$w = 160 - 2l$$

$$w = 160 - 2(40)$$

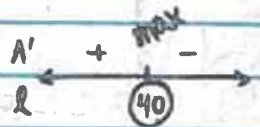
$$\boxed{w = 80'}$$

$$A = lw = l(160 - 2l)$$

$$A = 160l - 2l^2$$

$$A' = 160 - 4l = 0$$

$$\boxed{l = 40'}$$

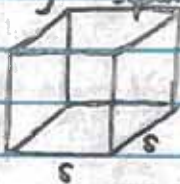


$$10. \quad l + g = 108$$

$$\text{given: } g = P_{\text{square}} = 4s \quad \therefore l + 4s = 108$$

$$l = 108 - 4s$$

$$l = 108 - 4(18) = 36$$



$$V = Bh = s^2 l$$

$$V = s^2(108 - 4s)$$

$$V = 108s^2 - 4s^3$$

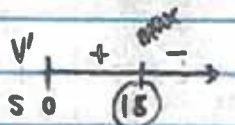
$$V' = 216s - 12s^2$$

$$V = 18^2 [108 - 4(18)]$$

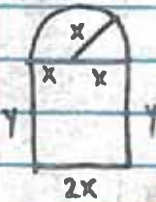
$$\boxed{V = 11,664 \text{ in}^3}$$

$$12s(18 - s) = 0$$

$$s = 0, 18$$



11.



$$P = 2x + 2y$$

$$C = \frac{2\pi x}{2} = \pi x$$

$$20 = 2x + 2y + \pi x$$

$$2y = 20 - 2x - \pi x$$

$$y = 10 - x - \frac{\pi x}{2} \longrightarrow y = 10 - 2.8 - \frac{\pi(2.8)}{2}$$

$$A = 2xy + \frac{\pi x^2}{2}$$

$$y = 10 - x - \frac{\pi x}{2}$$

$$= 2x \left(10 - x - \frac{\pi x}{2} \right) + \frac{\pi x^2}{2}$$

$$A = 20x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2 \longrightarrow A = 20(2.8) - 2(2.8)^2 - \pi(2.8)^2 + \frac{1}{2}\pi(2.8)^2$$

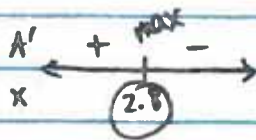
$$A' = 20 - 4x - 2\pi x + \pi x$$

$$= 20 - 4x - \pi x = 0$$

$$4x + \pi x = 20$$

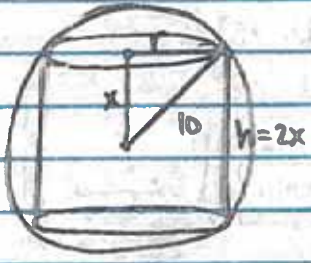
$$x(4 + \pi) = 20$$

$$\boxed{x = 2.8 \text{ ft}}$$



$$\boxed{A = 28.005 \text{ ft}^2}$$

12.



$$x = r$$

$$x^2 + x^2 = 10^2$$

$$2x^2 = 100$$

$$x^2 = 50$$

$$r = x = 5\sqrt{2}$$

$$h = 2x = 10\sqrt{2}$$

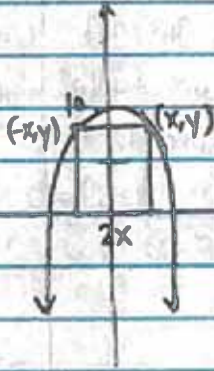
$$V = \pi r^2 h =$$

$$= \pi (5\sqrt{2})^2 (10\sqrt{2})$$

$$V = \pi (50) (10\sqrt{2})$$

$$V = 500\pi\sqrt{2} \approx \boxed{2221.441 \text{ m}^3}$$

13.



$$A = lw = 2xy; \quad y = 16 - x^2$$

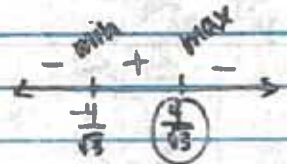
$$A = 2x(16 - x^2) \longrightarrow A = 2\left(\frac{4}{\sqrt{3}}\right)\left[16 - \left(\frac{4}{\sqrt{3}}\right)^2\right] = \boxed{49.267}$$

$$A = 32x - 2x^3$$

$$A' = 32 - 6x^2 = 0$$

$$6x^2 = 32$$

$$x^2 = \frac{16}{3}; \quad x = \pm \frac{4}{\sqrt{3}}$$



$$14. \quad R = H^2 \left(\frac{C-M}{2} \right) = \frac{C}{2} H^2 - \frac{1}{3} M^3$$

$$R' = CH - M^2 = 0$$

$$H(C-M) = 0$$

$$H = 0, C$$

