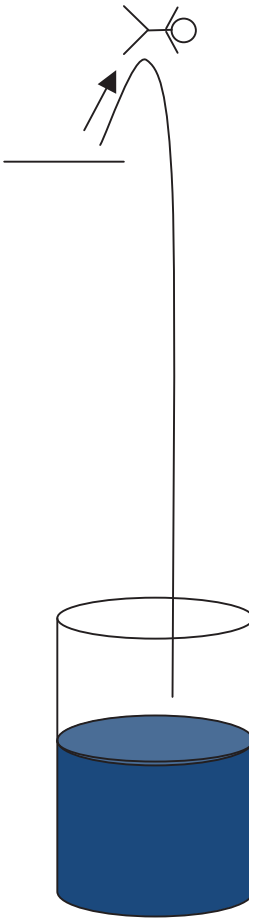
Math Analysis **Name**_____

Average & Instantaneous Velocity

Morgan, Maggie, and Sarah build a nice springy diving board for Mr. Murphy's next dive into the tank. Because of this, instead of just going straight down off the diving board, Mr. Murphy is able to get some upward motion and therefore some extra height before falling into the water (see diagram below). Rocky, Nicole, and Bella attentively study and time Mr. Murphy's path to the water and find the equation for his height to be $h(t) = 192 + 16t - 16t^2$

$192 + 48 - 144$

$=$



1) How long was Mr. Murphy in the air?

$0 = 192 + 16t - 16t^2$

$0 = -16(t^2 - t - 12) = -16(t-4)(t+3) \rightarrow t = 4$ seconds

2) Use your calculator to find Mr. Murphy's maximum height and at what time.

Graph from 0 to 4 and use max function on calc

Answer: $\left(\frac{1}{2}, 196\right) \rightarrow t = \frac{1}{2}$ second height = 196 feet

3) What was Mr. Murphy's average velocity over the period of time that he was in the air?

Started at $t = a$, landed at $t = b$ $\quad h(4) = 0 \quad h(0) = 192$

$a = 0 \quad b = 4$

$$\text{avg vel} = \frac{h(b) - h(a)}{b - a} = \frac{h(4) - h(0)}{4 - 0} = \frac{0 - 192}{4 - 0} = -48 \text{ ft/sec}$$

4) What was Mr. Murphy's average velocity from the time he started falling to the time he hit the water?

$a = \frac{1}{2} \quad b = 4$

$$\frac{h(4) - h\left(\frac{1}{2}\right)}{3\frac{1}{2}} = \frac{0 - 196}{\frac{7}{2}} = -\cancel{196}^{28} \cdot \frac{2}{\cancel{7}} = -56 \text{ ft/sec}$$

5) Find Mr. Murphy's instantaneous velocity at 1, 2, and 3 seconds.

$$\lim_{t\to 1}\frac{h(t)-h(1)}{t-1}=\frac{192+16t-16t^2-192}{t-1}=\lim_{t\to 1}\frac{-16(t-1)(t+1)}{t-1}=-16(1+1)=-32\ ft/sec$$

$$\lim_{t\to 3}\frac{h(t)-h(2)}{t-2}=\lim_{t\to 2}\frac{192+16t-16t^2-160}{t-2}=\lim_{t\to 2}\frac{32+16t-16t^2}{t-2}=\frac{-16(t^2-t-2)}{t-3}=\lim_{t\to 2}\frac{-16(t-2)(t+1)}{t-2}$$

$$=-16(3)=-48\ ft/sec$$

$$\lim_{t\to 3}\frac{h(t)-h(3)}{t-3}=\lim_{t\to 3}\frac{192+16t-16t^2-96}{t-3}=\lim_{t\to 3}\frac{96+16t-16t^2}{t-3}=\frac{-16(t^2-t-6)}{t-3}=\lim_{t\to 3}\frac{-16(t-3)(t+2)}{t-3}$$

$$=-16(5)=-80\ ft/sec$$

6) Find Mr. Murphy's instantaneous velocity when he hit the water.

$$\lim_{t\to}\frac{h(t)-h(4)}{t-4}=\lim_{t\to 4}\frac{192+16t-16t^2}{t-4}=\lim_{t\to 4}\frac{192+16t-16t^2}{t-4}=\frac{-16(t^2-t-12)}{t-4}=\lim_{t\to 3}\frac{-16(t-4)(t+3)}{t-4}$$

$$=-16(6)=-96\ ft/sec$$