AP Statistics Name: Key

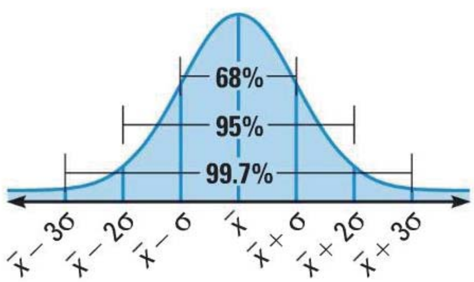
Mr Murphy Date:

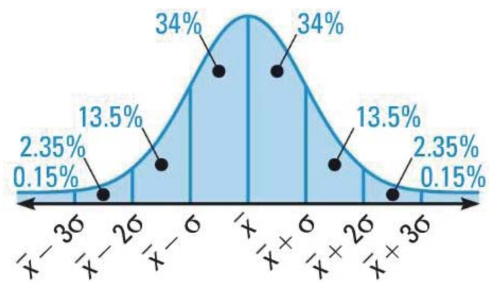
Empirical Rule, Percentiles, and *Z* Scores Worksheet Period:

The Empirical Rule

A lot of large data samples can be referred to as being normally distributed. When data is normally distributed, it has certain characteristics:

1. The mean, median, and the mode are all equivalent.
2. The data fits a bell shaped curve (normal curve).
3. About 68% of the data falls within 1 standard deviation from the mean.
4. About 95% of the data falls within 2 standard deviation from the mean.
5. About 99.7% of the data falls within 3 standard deviation from the mean.

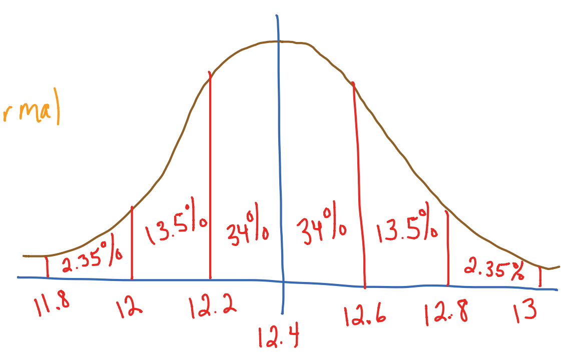




1. A machine fills 12 ounce Potato Chip bags. It places chips in the bags. Not all bags weigh exactly 12 ounces. The weight of the chips placed is normally distributed with a mean of 12.4 ounces and with a standard deviation of 0.2 ounces. The company has asked you to determine the following probabilities to aid in consumer relations concerning the weight of the bags purchased.

   (a) If you purchase a bag filled by this dispenser what is the likelihood it has less than 12 ounces?

   $p = 0.025 \Rightarrow$ 2.5% are to the left of 12 on the normal curve



   (b) If you purchase a bag filled by this dispenser what is the likelihood it has less than 12.6 ounces?

   $p = 0.84 \Rightarrow$ 84% are to the left of 12 on the normal

   (c) If you purchase a bag filled by this dispenser what is the likelihood it has between 12 and 12.6 ounces?

   $p = 0.815 \Rightarrow$ 68% + 13.5% = 81.5% of the normal curve data are between 12 and 12.6

Percentiles - A percentile is a measure that tells us what percent of the total (relative) frequency scored at or below that measure

   (d) What weight of the bag is represented by the 84th percentile?

   84% is to the right of 12.6 so the answer is 12.6 oz.

z–scores

- tell us how many standard deviations a term is above or below the mean
- use *z*–scores to normalize data
- $z = \frac{x - \mu}{\sigma}$

2. If we have a normally distributed data set where $\mu = 8$ and $\sigma = 2$, find the *z*-score for the following data points.

   (a) $x = 12$

   $z_1 = \frac{12-8}{2}$

   $= 2$

   (b) $x = 7$

   $z = \frac{7-8}{2} = -\frac{1}{2}$

Conclusion about *z*-scores…

z-scores have a ___positive___ value if the element lies above the mean.

z-scores have a ___negative___ value if the element lies below the mean.

3. Women's heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. Jodi is 61.1 inches tall.

   (a) What is the difference between Jodi's height and the mean? $61.1 - 63.6 = -2.5$

   (b) How many standard deviations is that? -1 standard deviations

   (c) Convert Jodi's height to a z-score.

   (d) How do your answers to (b) and (c) compare?

   c) $z = \frac{61.1 - 63.6}{2.5} = -1$ d) The same!

4. Heights of men are normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches. Find each of the following *z*-scores and tell if it is unusual.

   (a) Shaquille O'Neal is 7 ft. 1 in. tall.

   $z_S = \frac{85-69}{2.8} = 5.714$

   (b) Bob Jenkins 5 ft. 4 in.

   $z_B = \frac{64-69}{2.8} = -1.79$

   (c) Textbook's author 69.72 inches tall.

   $z_T = \frac{69.72-69}{2.8} = 0.26$

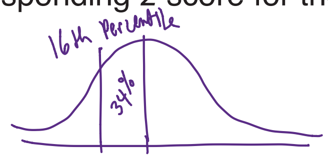
   not at all unusual

Why do we find the *z*-score? .... A way to compare apples and oranges!

5. The average apple has a diameter of 3.25 inches with a standard deviation of 0.5 inch. The average orange has a diameter of 4.5 inches and has a standard deviation of 1 inch. If I have an apple with a diameter of 4 inches and an orange with a diameter of 5.5 inches, which fruit is largest compared to others of its kind?

$z_A = \frac{4-3.25}{0.5} = 1.5$ $\qquad z_o = \frac{5.5-4.5}{1} = 1$ $\qquad$ Apple wins!

6. Inside what interval does the corresponding *z*-score for the 30th percentile fall?
   (a) $-2 < z < -1$
   (b) $-1 < z < 0$
   (c) $0 < z < 1$
   (d) $1 < z < 2$
   (e) $2 < x < 3$



Since it falls in the range within 1 S.D. to the left of the mean, the z score will fall between −1 and 0

Checkpoint

1. A study of elite distance runners found a mean body weight of 63.1 kilograms (kg), with a standard deviation of 4.8 kg. Assume a normal distribution.

   (a) Use the Empirical Rule to find intervals centered at the mean that will include 68%, 95%, and 99.7% of the weights of the runners. Draw the normal curve to the right.

68%: $63.1 \pm 4.8 \Rightarrow 58.3$ to 67.9 kg

95%: $63.1 \pm 2(4.8) \Rightarrow 53.5$ to 72.7 kg

99.7%: $63.1 \pm 3(4.8) = 48.7$ to 77.5 kg

   (b) What weight would represent the 84th percentile?

67.9 kg (50% + 34% = 84%)

   (c) A weight in what range would represent the bottom 16% of the weights?

58.3 kg (1 S.D. to the left of the mean)

   (d) What percent of weights are higher than 77.5kg?

0.15% (3 S.D. to the left of the mean)

   (e) The 80th percentile would be between what two weights?

Between 63.1 and 67.9 kg (but closer to 67.9 kg)

2. Batteries of 2 brands are compared. Brand A has a mean life of 48 months and a standard deviation of 2 months. Brand B has a mean of 48 months and a standard deviation of 6 months. Which brand would you say is the better choice? Why?

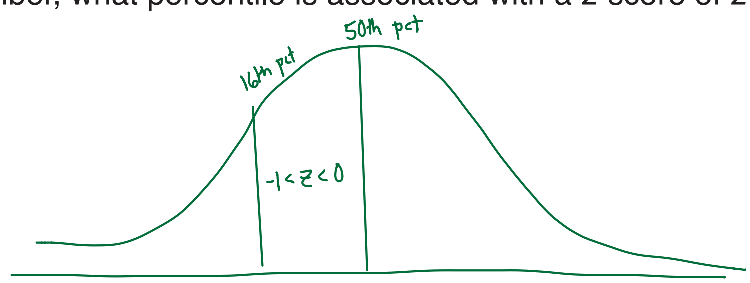
Brand A — smaller range

3. Test 1 has a mean of 128 and s = 34. Test 2 has a mean of 86 and s = 18. Test 3 has a mean of 15 and s = 5. Which of these scores is the highest relative score? Test 1 score of 144 or Test 2 score of 90 or Test 3 score of 18.

$z_1 = \frac{144-128}{34} = 0.471$ $\quad z_2 = \frac{90-86}{18} = 0.222$ $\quad z_3 = \frac{18-15}{5} = 0.6$

4. To the nearest whole number, what percentile is associated with a *z*-score of *z* = -0.68?

(a) 10th percentile
(b) 40th percentile
(c) 50th percentile
(d) 25th percentile
(e) 75th percentile



5. If the range of a normally distributed data set is 25, what's a reasonable estimate for the standard deviation?

(a) 1
(b) 2.5
(c) 5
(d) 10
(e) 15

The normal curve covers six S.D. overall so...

6. SAT scores of females have a normal distribution with a mean of 998 and s = 202. A college has a minimum of 900 as one of its requirements for admission. About what percentage of females do NOT satisfy this requirement?

$\approx 32\% \Longrightarrow$