Math Analysis Name ______________________________

Midterm Exam Prep Worksheet

1) Rocky, having checked out completely from swimming, is floating in an inner tube in a wave pool. She is 7 feet from the bottom of the pool when she is at the trough of a wave. A stopwatch starts timing at this point. After 2 s, she is on the crest of the wave, 11 feet from the bottom of the pool.

a) Determine the equation of the function that expresses Rocky's distance from the bottom of the pool in terms of time.

$a = (\text{max} - \text{min})/2$
$= (11 - 7)/2 = 2$

Vertical shift = max − amp
= 11 − 2 = 9

period = 4 $\quad b = \frac{2\pi}{4} = \frac{\pi}{2}$

horizontal shift = 2 (first max occurred at 2 seconds)

b) What is the amplitude of the function, and what does it represent in this situation?

a = 2

equation

$y = 9 + 2\cos\left[\frac{\pi}{2}(t-2)\right]$

c) How far above the bottom of the pool is Rocky at *t* = 4 s?

$y = 9 + 2\cos\left[\frac{\pi}{2}(4-2)\right] = 7$ feet

d) If data are collected for only 40 s, how many complete cycles of the sinusoidal function will there be?

Ten 4 second periods

e) If the period of the function changes to 3 s, what is the equation of this new function?

$y = 9 + 2\cos\left[\frac{2\pi}{3}(t-2)\right]$

2) Given $\csc A = -\frac{5}{4}$ in Q III, and $\left(-2, \sqrt{5}\right)$ is on the terminal side of B, find the **exact** values of: (Do background info here)

$\sin A = -\frac{4}{5}$

$(-2)^2 + (\sqrt{5})^2 = 9$

$\sin B = \frac{\sqrt{5}}{9}$

$r = 3$

$\cos A = -\frac{3}{5}$

$\cos B = -\frac{2}{9}$

a. sin (A + B) $= \sin A \cos B + \cos A \sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{2}{9}\right) + \left(-\frac{3}{5}\right)\left(\frac{\sqrt{5}}{9}\right) = \frac{8}{45} - \frac{3\sqrt{5}}{45} = \frac{8-3\sqrt{5}}{45}$

b. cos (A – B) $= \cos A \cos B + \sin A \sin B$

$= \left(-\frac{3}{5}\right)\left(-\frac{2}{9}\right) + \left(-\frac{4}{5}\right)\left(\frac{\sqrt{5}}{9}\right)$

$= \frac{6}{45} - \frac{4\sqrt{5}}{45} = \frac{6-4\sqrt{5}}{45}$

3) Find **the general solutions** in degrees to the following equations:

a. $\underbrace{\cos 30° \cos 2x - \sin 30° \sin 2x}_{\cos(30+2x) = \sqrt{3}/2} = \frac{\sqrt{3}}{2}$ *for* $x \in \{\text{Re}\,als\}$

$30 + 2x = \pm 30° \pm 360n$

$2x = 0° \pm 360n$
$-60° \pm 360n$

$0° \pm 180n$
$-30° \pm 180n$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

b. $\underbrace{\cos x\sin 70^\circ - \sin x\cos 70^\circ}_{\sin(x-70) = -\frac{1}{2}} = -\frac{1}{2}$ *for* $x \in \{0^\circ, 360^\circ\}$

$x - 70 = \begin{matrix} 30^\circ \pm 360n \\ 150^\circ \pm 360n \end{matrix} \rightarrow x = \begin{matrix} 100^\circ \pm 360n \\ 220^\circ \pm 360n \end{matrix}$

Prove the given identities

4) $\sin x\cos x\tan x = 1 - \cos^2 x$

$\sin x \cancel{\cos x} \frac{\sin x}{\cancel{\cos x}} = 1 - \cos^2 x$

$\sin^2 x = 1 - \cos^2 x$ ✓

5) $\sin x\left(\cot x + \cos x\tan x\right) = \cos x + \sin^2 x$

$\cancel{\sin} x \frac{\cos x}{\cancel{\sin} x} + \sin x \cancel{\cos x} \frac{\sin x}{\cancel{\cos x}} = \cos x + \sin^2 x$

$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{1}{\cos^2\theta} \frac{1}{\sin^2\theta}$

$\frac{1}{\cos^2\theta}\frac{\sin\theta}{\sin\theta} + \frac{1}{\sin^2\theta}\frac{\cos\theta}{\cos\theta} = \frac{1}{\cos^2\theta}\frac{1}{\sin^2\theta} \Rightarrow \frac{(\sin^2\theta + \cos^2\theta)}{\cos^2\theta\sin^2\theta} = \frac{(1)}{\cos^2\theta\sin^2\theta}$

6) $\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$

$\frac{1}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta\sin^2\theta}$ ✓

7) $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \frac{0}{0}$

$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$

$\lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x}+2)} = \lim_{x\to 4}\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim_{x\to 4}\frac{1}{(\sqrt{4}+2)} = \frac{1}{4}$

8) $\lim_{x\to 7}\frac{x^2-49}{x^2+2x-63} = \frac{0}{0} = \lim_{x\to 7}\frac{(x-7)(x+7)}{(x-7)(x+9)} = \lim_{x\to 7}\frac{\cancel{(x-7)}(x+7)}{\cancel{(x-7)}(x+9)} = \lim_{x\to 7}\frac{(7+7)}{(7+9)} = \frac{14}{16} = \frac{7}{8}$