A.M.D.G.

AP Statistics **Name:_____________________**

Single Sample 1 Proportion *z*-Test **Date:___________ Period:____**

Hypothesis testing is a process that is basically the inverse of a confidence interval. In confidence intervals, we are making a statement about how likely the true value is within a given interval. With hypothesis testing, we are trying to gather evidence that a sample that we have collected ***is*** unusual (that is, it is not inside the 90%, 95%, or 99% region; rather, it is outside that region). If we find this value outside the region, it is evidence that the original claim we made is faulty.

The main idea behind rejecting and failing to reject is similar to determining guilt in a trial.

In a trial, the accused individual is presumed innocent (the null hypothesis).
This is equivalent to our null hypothesis – the proportion we are assuming is true.
In hypothesis testing, we are presuming this (null hypothesis, H_0) is true

In that same trial, we are trying to prove guilt (the alternative hypothesis).
This is the equivalent to our alternative hypothesis (p >, p <, or p ≠ the value in the null hypothesis.
In hypothesis testing, we are trying to gather evidence of this (the alternative hypothesis, H_a) in order to reject the null hypothesis.

We do not prove someone innocent – we only say we do not have enough evidence to convict (i.e. we "fail to reject" the null hypothesis)

- If we have evidence to "convict" – we reject the null hypothesis (innocence) in favor of the alternative hypothesis (guilt)
- If we do not have enough evidence to "convict" – we fail to reject the null hypothesis (innocence) in favor of the alternative hypothesis (guilt)

STEPS IN HYPOTHESIS TESTING

1. Define the population characteristic (i.e. parameter) about which hypotheses are to be tested.

2. State the null hypothesis H_0.
    a. This is always p = the proportion you are asking about.

3. State the alternative hypothesis H_a.
    a. This is always what you are trying to evidence to support. (<, >, or ≠ the proportion from step 2).

4. State the significance level α for the test.
    a. This is the level given in the problem (usually 0.10, 0.05, or 0.01)

5. Check all assumptions.
    a. The same ones from proportions: $np_0 \geq 10$, $n(1-p_0) \geq 10$, sample size small relative to population.

6. State the name of the test.
    a. 1-proportion z-test

7. *State degrees of freedom if applicable* **(not applicable with proportions)**.

8. Display the test statistic to be used without any computation at this point. $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$

9. Compute the value of the test statistic, showing specific numbers used.

10. Calculate the P – value (using normalcdf, where $\mu = 0, \sigma = 1$)

11. Sketch a picture of the situation.
    a. Draw the normal curve with the z-score and appropriate shading.

12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0. Always start by stating the P – value compared to the significance level, α, of the test
        - If the P – value is **<u>less than</u>** α, then we **reject the null hypothesis (*H_0*)** at the significance level we tested.
        - If the P – value is **<u>greater than</u>** α, then we **fail to reject the null hypothesis (*H_0*)** at the significance level we tested.

    2. Summarize in context discussing H_a .

        - **If we reject *H_0*** state that "we have evidence that the proportion of _______ is …, therefore, the *(initial claim)* is incorrect."
        - **If we fail to reject *H_0*** state that "we have insufficient evidence that the proportion of _______ is …, therefore, we cannot reject the *(initial claim)*."

Example 1 The article "Credit Cards and College Students: Who Pays, Who Benefits?" described a study of credit card payment practices of college students. According to the authors of the article, the credit card industry asserts that at most 50% of college students carry a balance from month to month. However, the authors of the article report that, in a random sample of 310 college students, 217 carried a balance each month. Does this sample provide sufficient evidence that the percentage is higher, rejecting the industry claim? We will answer this question by carrying out a hypothesis test using a 0.05 significance level. Does this sample provide sufficient evidence to reject the industry claim with a 0.05 significance level?

Answer:

p = proportion of college students who carry a balance on a credit card from month to month

H_0: $p = 0.5$ $\qquad$ H_a: $p > 0.5$ $\qquad$ $\alpha = 0.05$

Assumptions:

1. $np \geq 10$ $\qquad$ $310(0.5) = 155 \geq 10$
2. $n(1-p) \geq 10$ $\qquad$ $310(0.5) = 155 \geq 10$
3. Sample Size Small Relative to Population (310 is less than 10% of college students)

Test: 1-proportion z-test

Degrees of Freedom: not applicable

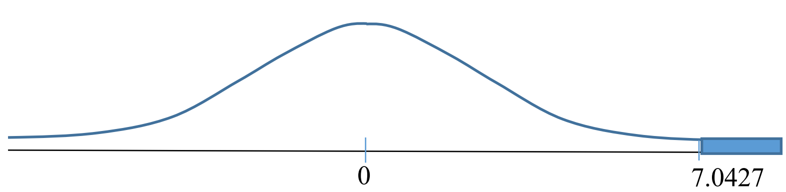
Test Statistic: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$ $\qquad$ z value: $z = \dfrac{\dfrac{217}{310} - 0.5}{\sqrt{\dfrac{0.5(1-0.5)}{310}}} = \mathbf{7.0427}$

(notice that our z-score is above 7, that is **a lot** of standard deviations out – so it should be obvious at this point that we will reject the null hypothesis)

Probability statement and value:

$P(z > 7.0427) = \text{normalcdf(lower} = 7.0427, \text{ upper} = \infty, \mu = 0, \sigma = 1) = 9.48 \times 10^{-13} = \mathbf{0}$

Sketch:



Conclusion:

a. Since $P(z > 7.0427) = 0 < 0.05$, then we **reject the null hypothesis (H_0)** at the 0.05 significance level.

b. We have evidence that the proportion of college students who carry a credit card balance month to month is greater than 0.5, therefore, the claim that the proportion of students is 0.5 is incorrect."

1) Mr. Maychrowitz is convinced that Hoslow's Petal Whip is the best weapon in the video game, Elden Ring. He decides to survey the community of players (approximately 20 million people are in the player community) by taking a simple random sample of 2705 people and 623 people agree that it is, indeed, the best weapon. Does this sample provide convincing evidence that less than ¼ of the Elden Ring community agrees with Mr. M? Perform the test at the 0.05 significance level.

1. p = _The proportion of Elden Ring players who think Hoslow's Petal Whip is the best weapon____

2. H_0: __________ 3. H_a: ______________ 4. α =___________

5. Assumptions:
    1. $np \geq 10$,
    2. $n(1-p) \geq 10$,
    3. 2705 is/is not less than 10% of 20 million (sample size small relative to population)

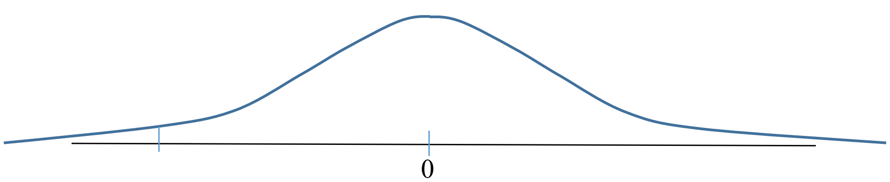
6. Test:_1-proportion *z*-test________________________

7. Degrees of Freedom:_*Not Applicable*________________

8. Test Statistic: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$ 9. z value:

11. Probability statement and value:

12. Sketch:



13. Conclusion:

2) You poll 30 SI seniors (out of the approximately 360 seniors) to find out how anxious they are about being accepted into a college. 18 out of 30 of the randomly selected students surveyed said that they had a high level of anxiety about being accepted into a college. Does this provide convincing evidence that more than half of all SI Seniors are anxious about being accepted into a college at the 0.10 significance level?

1. $p =$ ____

2. H_0: ________ 3. H_a: ________ 4. $\alpha =$ ________

5. Assumptions:
    1. ____________________
    2. ____________________
    3. ____
6. Test: ____________________________________
7. Degrees of Freedom: __________________________________
8. Test Statistic:
9. z value:
10. Probability statement and value: ____
11. Sketch:
12. Conclusion:
    a.

    b.

3. According to a report from the Institute for Higher Education titled, "Average Won't Do: Performance Trends in California Higher Education as a Foundation for Action" (January 2014), 53% of students graduating from California high schools go on to attend a 2 or 4-year college the year after graduation. A representative (random) sample of 1500 Bay Area students estimated the college going rate to be 50.6%. Use a hypothesis test with significance level $\alpha = 0.05$ to determine if there is sufficient evidence to suggest that the Bay Area proportion is different from that of the state.

   1. $p =$ ____

   2. H_0: ________ 3. H_a: ________ 4. $\alpha =$ ________

   5. Assumptions:
      1. ______________________
      2. ______________________
      3. ____

   6. Test: ________________________________

   7. Degrees of Freedom: ________________________________

   8. Test Statistic: 9. z value:

   10. Probability statement and value: ________________________________

   11. Sketch:

   12. Conclusion:
       a.

       b.