The Quotient Rule **Name** Solutions

Find the critical points of the rational function as well as the equation of the tangent line at a given point when asked.

1) a) Find the critical points indicating a maximum or a minimum for the graph of $y = \dfrac{2x-9}{x^2+3}$

$$\left.\begin{array}{ll} f = 2x-9 & g = x^2+3 \\ f' = 2 & g' = 2x \end{array}\right\} \quad y' = \frac{2(x^2+3) - 2x(2x-9)}{(x^2+3)^2} = \frac{2x^2+6-4x^2+18x}{(x^2+3)^2} = \frac{-2x^2+18x+6}{(x^2+3)^2} = 0$$

but needs quadratic formula and calculator

$(x^2+3)^2$ never $= 0$

$$-2x^2+18x+6 = -2\underbrace{(x^2-9x-3)}$$

$$x = \frac{9 \pm \sqrt{81+12}}{2} = \frac{9 \pm \sqrt{93}}{2}$$

$$\frac{9+\sqrt{93}}{2} \approx 9.322 \qquad \frac{9-\sqrt{93}}{2} \approx -0.322$$

extreme value $(-0.322, -3.1)$ extreme value $(9.322, 0.107)$

$-$ 0 $+$ 0 $-$

-0.322 9.322

b) Find the equation of the tangent line at $x = -1$

$m_T \Rightarrow$ plug $x=-1$ into y'

$$m_T = \frac{-2x^2+18x+6}{(x^2+3)^2} = \frac{-2(1)^2+18(1)+6}{(1^2+3)^2} = \frac{-2-18+6}{16} = \frac{-14}{16} = -\frac{7}{8}$$

at $x=-1$ $\quad y = \dfrac{2(-1)-9}{(-1)^2+3} = \dfrac{-11}{4}$

Point-Slope

$$y = \frac{-7}{8}(x+1) - \frac{11}{4}$$

2) a) Find the critical points indicating a maximum or a minimum for the graph of $y = \dfrac{x^2+5}{x+2}$

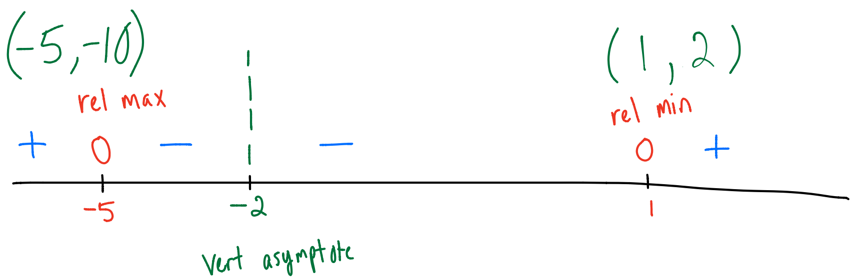
$f = x^2+5 \qquad g = x+2$

$f' = 2x \qquad g' = 1$

$$y' = \frac{2x(x+2) - 1(x^2+5)}{(x+2)^2} = \frac{2x^2+4x-x^2-5}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2} = 0 \text{ or undef}$$

$$= 0 = \underline{x^2+4x-5} = (x+5)(x-1) \quad x = -5, 1$$

$$= \text{undef} \Rightarrow \underline{(x+2)}^2 = 0 \quad x = -2$$



b) Find the equation of the tangent line at $x = 0$

$m_T \Rightarrow$ plug $x=0$ into y'

$$m_T = \frac{x^2+4x-5}{(x+2)^2} = \frac{0^2+4(0)-5}{(0+2)^2} = \frac{-5}{4}$$

$$x = 0 \quad y = \frac{0^2+5}{0+2} = \frac{5}{2}$$

Point-slope

$$y = -\frac{5}{4}(x-0) + \frac{5}{2}$$

or

$$y = -\frac{5}{4}x + \frac{5}{2}$$

3) Given $y = \dfrac{1}{x^4+x^2+1}$ find y'

$f = 1 \qquad g = x^4+x^2+1$

$f' = 0 \qquad g' = 4x^3+2x$

$$y' = \frac{0(x^4+x^2+1) - 1(4x^3+2x)}{(x^4+x^2+1)^2}$$

$$= \frac{-4x^3-2x}{(x^4+x^2+1)^2}$$