**Mr. Murphy**
**AP Statistics**
**10.4 Power and Probability of Type II Error**
**HW Pg. 548 #10.64, 10.67, 10.69**

Goals:
1. Calculate the power of a test.
2. Name the factors that affect power.
3. How hungry can a person by at 10:31am!?

- The **power of a test** is the probability of rejecting the null hypothesis, when the null hypothesis is false ← this is a good thing. You would want to reject the null hypothesis if it were false.

This number, **power**, will help us assess how "good" our test procedure is.

A "good" procedure should have a small probability of rejecting $H_0$ when it is true (a Type I error), and a high probability of rejecting $H_0$ when it is false (power).

Ex1 A cigarette manufacturer claims that the mean nicotine content of its cigarettes is 1.5mg. We might investigate this claim by testing

$$H_0 : \mu = 1.5 \text{ versus } H_a : \mu > 1.5$$

where $\mu$ is the true mean nicotine content. A random sample of $n$ = 36 cigarettes is to be selected, and the resulting data will be used to reach a conclusion. Suppose $s$ = 0.20, $\alpha = 0.01$, and by some miracle we found out that $\mu = 1.6$. What is the power of this test?

Suppose $\mu = 1.7$. What is the power of this test? What is the power of the test if $\mu = 2.0$ ?

Now suppose $\alpha = 0.05$. What is the power of the test? What is the power when $\alpha = 0.10$?

Suppose $n$ = 57. What is the power of this test? What is the power when $n$ = 100?

- **Effect of Various Factors on the Power of a Test**
    1. The larger the size of the discrepancy between the hypothesized value and the true value of the population characteristic, the higher the power.
    2. The larger the significance level, $\alpha$, the higher the power of the test.
    3. The larger the sample size, the higher the power of the test.

- When $H_0$ is false, power $= 1 - \beta$.

| | **Fail to reject $H_0$** | **Reject $H_0$ i.e Accept $H_a$** |
|---|---|---|
| **$H_0$ True** | Hooray! | Type I error |
| **$H_a$ True** | Type II error | Power |

Ex2 A package delivery service advertises that at least 90% of all packages brought to its office by 9 a.m. for delivery in the same city are delivered by noon that day. Let $p$ denote the proportion of all such packages actually delivered by noon. The hypothesis of interest are

$$H_0 : p = 0.9 \text{ versus } H_a : p < 0.9$$

where the alternate hypothesis states that the company's claim is untrue. Suppose, again by miracle, we learned that $p = 0.80$. Given that $n$ = 225 and we were testing at a 0.01 level, what is the probability of a Type II error? What is the power?

**Checkpoint**
**Multiple Choice**

1. If the $p$-value is less than alpha in a one tail test, what can conclusion can be drawn?

(a) The null hypothesis should not be rejected.
(b) The null hypothesis should be rejected.
(c) A two tailed test should be used.
(d) Alpha should be changed.
(e) The alternate hypothesis is rejected.

2. The power of a statistical test is the probability of rejecting the null hypothesis when it is _____. When you increase alpha, the power of the test will _____.

(a) true, decrease
(b) false, decrease
(c) true, increase
(d) false, increase
(e) true, neither increase nor decrease

3. The value of $1-\alpha$ is called the
(a) probability of a Type I error.
(b) power of the test.
(c) confidence level.
(d) probability of a Type II error.
(e) point estimate.

4. If a hypothesis test has a Type 1 error probability of 0.01, what does this mean?

(a) If the null hypothesis is true, we do not reject it 1% of the time.
(b) If the null hypothesis is true, we reject it 1% of the time.
(c) If the null hypothesis is false, we do not reject it 1% of the time.
(d) If the null hypothesis is false, we reject it 1% of the time.
(e) The null hypothesis is true.

5. As the sample size increases, how is the power of the study affected?

(a) There is no effect.
(b) The power approaches 0.
(c) The power is decreased.
(d) The power is increased.
(e) The power approaches 2.

6. If the probability of making a Type II error is 0.15, what is the power?

(a) 0.85
(b) 0.225
(c) 1.15
(d) 0.78
(e) 15

7. If the probability of making a Type II error is 0.22, what is the power?

(a) 0.85
(b) 0.225
(c) 1.15
(d) 0.78
(e) 15

8. Which of the following alphas would be associated with the greatest statistical power?

(a) 0.1
(b) 0.05
(c) 0.025
(d) 0.01
(e) 0.001

9. Which of the following alphas would be associated with the greatest likelihood of making a Type II error?

(a) 0.1
(b) 0.05
(c) 0.025
(d) 0.01
(e) 0.001

10. When a researcher decreases the risk of making a Type I error, the risk of making a Type II error is

(a) not affected.
(b) equal to the chance of making a Type I error.
(c) decreased.
(d) increased.
(e) None of the above

11. When alpha is set at 0.1, what is the chance of rejecting the null hypothesis erroneously?

(a) 1
(b) 1 in 80
(c) 1 in 100
(d) 1 in 10
(e) 1 in 99