**AP Statistics**
**Mr. Murphy**
**11.2 Inferences Concerning the Difference Between Means Using Paired Samples**
**HW Pg. 606 #11.33, 11.36, 11.37**

Goals: 1. Run a hypothesis test for the difference in means between two PAIRED SAMPLES.
2. Construct a confidence interval for the difference in means between two PAIRED SAMPLES.

### ▪ Summary of the Paired *t* Test for Comparing Two Population or Treatment Means

**Null hypothesis:** $H_0: \mu_d =$ hypothesized value

**Test statistic:** $t = \dfrac{\bar{x}_d - \text{hypothesized value}}{\dfrac{s_d}{\sqrt{n}}}$

where $n$ is the number of sample differences and $\bar{x}_d$ and $s_d$ are the sample mean and standard deviation of the differences, respectively. This test is based on df $= n - 1$.

| **Alternative Hypothesis** | ***P*-Value** |
|---|---|
| $H_a$: $\mu_d >$ hypothesized value | Area under the appropriate $t$ curve to the right of the calculated $t$ |
| $H_a$: $\mu_d <$ hypothesized value | Area under the appropriate $t$ curve to the left of the calculated $t$ |
| $H_a$: $\mu_d \neq$ hypothesized value | (1) 2(area to the right of $t$) if $t$ is positive, or<br>(2) 2(area to the left of $t$) if $t$ is negative |

**Assumptions:**
1. The samples are *paired.*
2. The $n$ sample differences can be viewed as a *random sample* from a population of differences.
3. The *number of sample differences is large* (in general, at least 30) or the *population distribution of differences is approximately normal.*

### ▪ Paired *t* Confidence Interval for $\mu_d$

When

1. the samples are *paired*,
2. the $n$ sample differences can be viewed as a *random sample* from a population of differences, and
3. the *number of sample differences is large* (in general, at least 30) or the *population distribution of differences is approximately normal*,

the paired $t$ confidence interval for $\mu_d$ is

$$\bar{x}_d \pm (t \text{ critical value})\left(\frac{s_d}{\sqrt{n}}\right)$$

For a specified confidence level, the $(n - 1)$ df row of Appendix Table 3 gives the appropriate $t$ critical value.

Ex1 Can taking chess lessons and playing chess daily improve memory? A study in which sixth-grade students who had not previously played chess participated in a program where they took chess lessons and played chess daily for 9 months. Each student took a memory test before starting the chess program and again at the end of the 9-month period. Data are given in the table.

The author of the study proposed using these data to test the theory that students who participated in the chess program tend to achieve higher memory scores after completion of the program. We can consider the pretest scores as a sample of scores from the population of sixth-grade students who have not participated in the chess program and the posttest scores as a sample of scores from the population of sixth-grade students who have completed the chess training program.

| | Memory Test Score | |
|---|---|---|
| Student | Pretest | Posttest |
| 1 | 510 | 850 |
| 2 | 610 | 790 |
| 3 | 640 | 850 |
| 4 | 675 | 775 |
| 5 | 600 | 700 |
| 6 | 550 | 775 |
| 7 | 610 | 700 |
| 8 | 625 | 850 |
| 9 | 450 | 690 |
| 10 | 720 | 775 |
| 11 | 575 | 540 |
| 12 | 675 | 680 |

Ex2 The effect of exercise on the amount of lactic acid in the blood was examined in a study. Eight males were selected at random from those attending a week-long training camp. Blood lactate levels were measured before and after playing three games of racquetball, as shown in the following table:

| Player | Before | After |
|---|---|---|
| 1 | 13 | 18 |
| 2 | 20 | 37 |
| 3 | 17 | 40 |
| 4 | 13 | 35 |
| 5 | 13 | 30 |
| 6 | 16 | 20 |
| 7 | 15 | 33 |
| 8 | 16 | 19 |

Use the data to construct a 95% confidence interval to estimate the mean change in blood lactate level.

**Checkpoint**
**Multiple Choice**

1. Which of the following is an example of a matched pairs design?

(a) A teacher compares the pre-test and post-test scores of students.
(b) A teacher compares the scores of students using a computer-based method of instruction, with the scores of other students using a traditional method of instruction.
(c) A teacher compares the scores of students in her class on a standardized test with the national average score.
(d) A teacher calculates the average score of students on a pair of tests and wishes to see if this average is larger than 80%.

2. Nine students took the SAT. Their scores are listed below. Later on, they read a book on test preparation and retook the SAT. Their new scores are also listed below. Construct a 95% confidence interval for their difference in scores.

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Scores before reading book | 720 | 860 | 850 | 880 | 860 | 710 | 850 | 1200 | 950 |
| Scores after reading book | 740 | 860 | 840 | 920 | 890 | 720 | 840 | 1240 | 970 |

(a) (–20.341, 4.852) (b) (–163.7, 132.58) (c) (0.615, 30.496)
(d) (–152.5, 121.35) (e) (2.858, 28.254)

3. Seven sets of identical twins are given psychological tests to determine whether the firstborn of the twins tends to be more aggressive than the second born. The results are shown in the following table, where the higher score represents greater aggressiveness.

If we are willing to assume that the distribution of differences is symmetric about the median but not necessarily normal, then the value of the appropriate test statistic is:

(a) 22.5 and we would reject $H_0$ at $\alpha = 0.05$

(b) 40 and we would reject $H_0$ at $\alpha = 0.05$

(c) 1.71 and we would not reject $H_0$ at $\alpha = 0.05$

(d) 22.5 and we would not reject $H_0$ at $\alpha = 0.05$

(e) 1.71 and we would reject $H_0$ at $\alpha = 0.05$

| Set | First Born | Secon d Born | Difference |
|---|---|---|---|
| **1** | 86 | 88 | -2 |
| **2** | 77 | 65 | 12 |
| **3** | 91 | 90 | 1 |
| **4** | 70 | 65 | 5 |
| **5** | 75 | 80 | -5 |
| **6** | 88 | 81 | 7 |
| **7** | 87 | 72 | 15 |