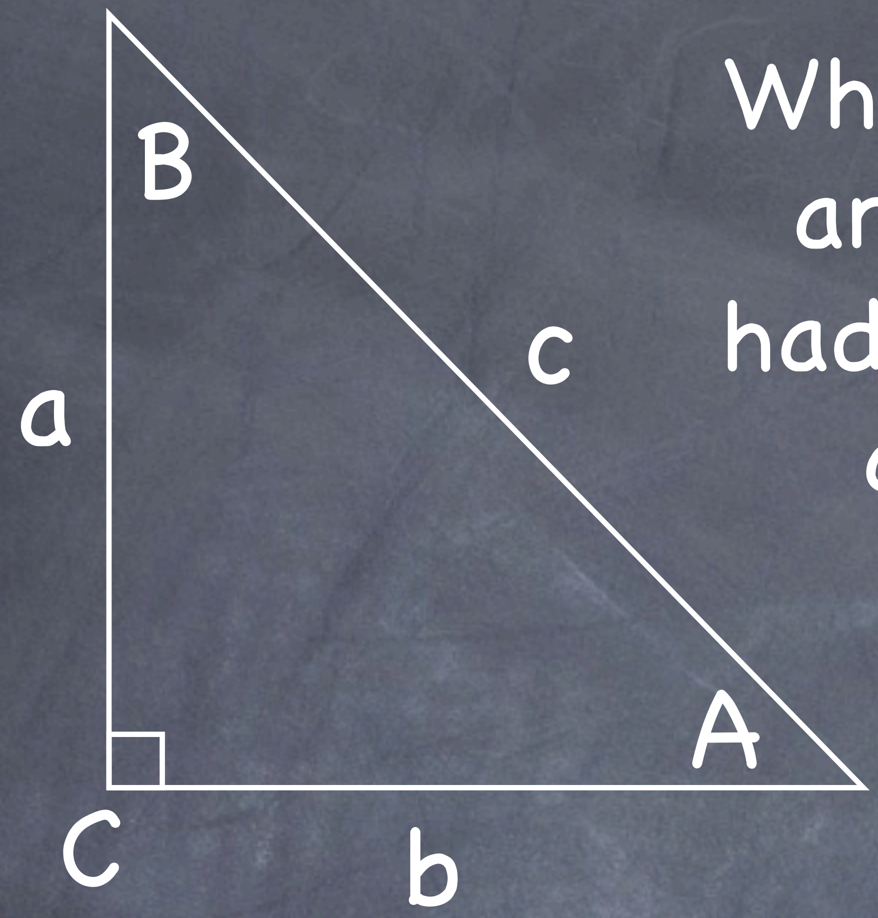
Law of Cosines

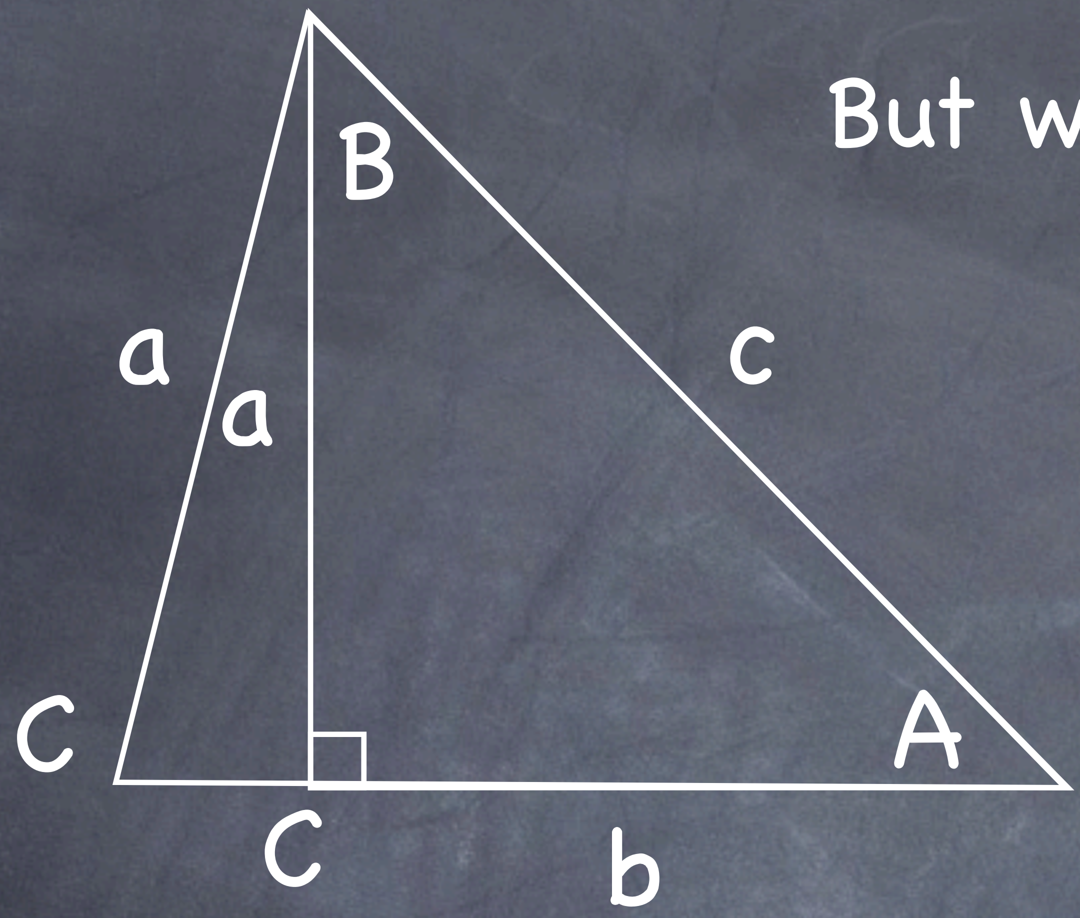


When solving for missing sides and angles with a right triangle, we had the pythagorean theorem plus all the trig functions to use.

$$a^2 + b^2 = c^2$$

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

$$\sin B = \frac{b}{c} \qquad \cos B = \frac{a}{c} \qquad \tan B = \frac{b}{a}$$



But what happens if it is not a right triangle?

With some clever mathematical manipulation we won't discuss here, we have a way of solving these:

The Law of Cosines

$$a^2 + b^2 - \boxed{2ab\cos C} = c^2$$

Notice how this is actually the Pythagorean Thm with the added term

The Law of Cosines

$$a^2 + b^2 - 2ab\cos C = c^2$$

$$b^2 + c^2 - 2bc\cos A = a^2$$

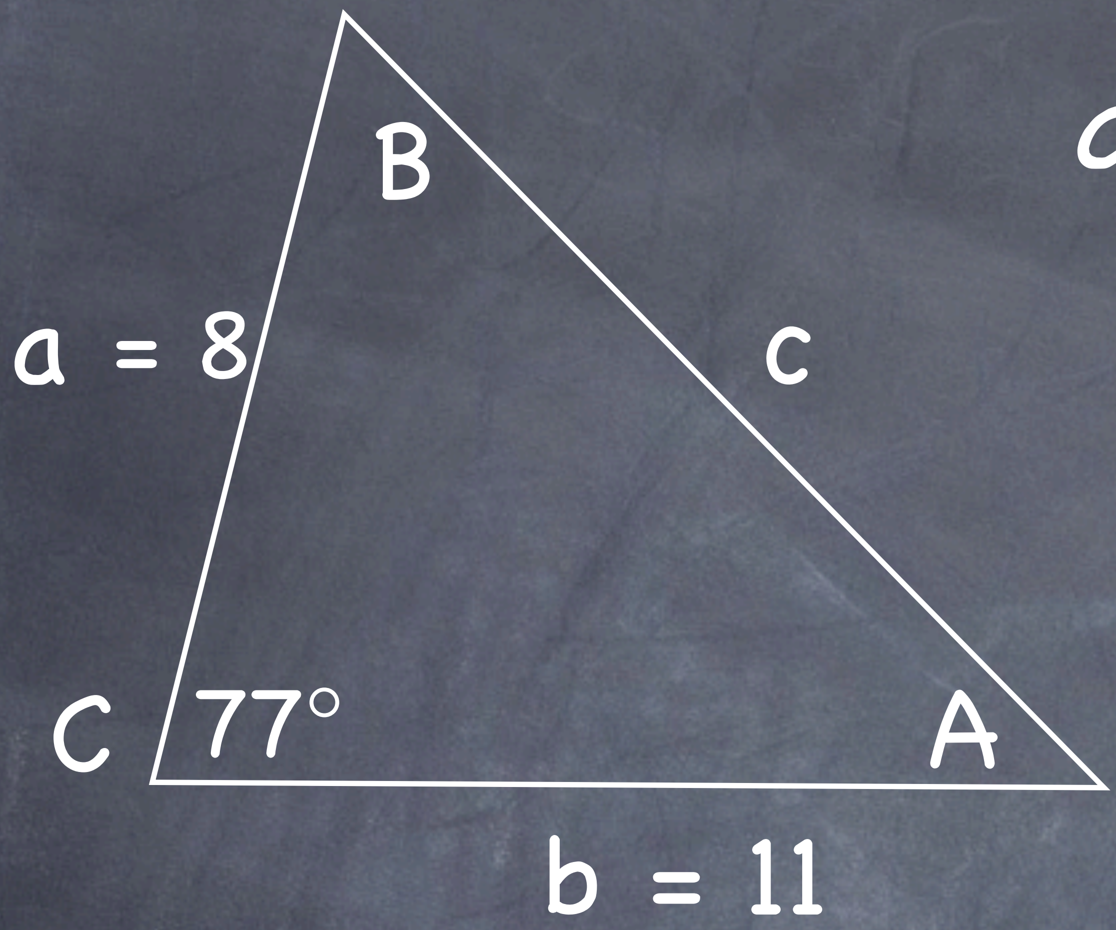
$$a^2 + c^2 - 2ac\cos B = b^2$$

They're all the same. This is just to assure you that the law applies to any labeling you use for a triangle.

And when we need to use it to solve for a missing angle, we have

$$C = \cos^{-1}\frac{a^2 + b^2 - c^2}{2ab}$$

More on this in class...



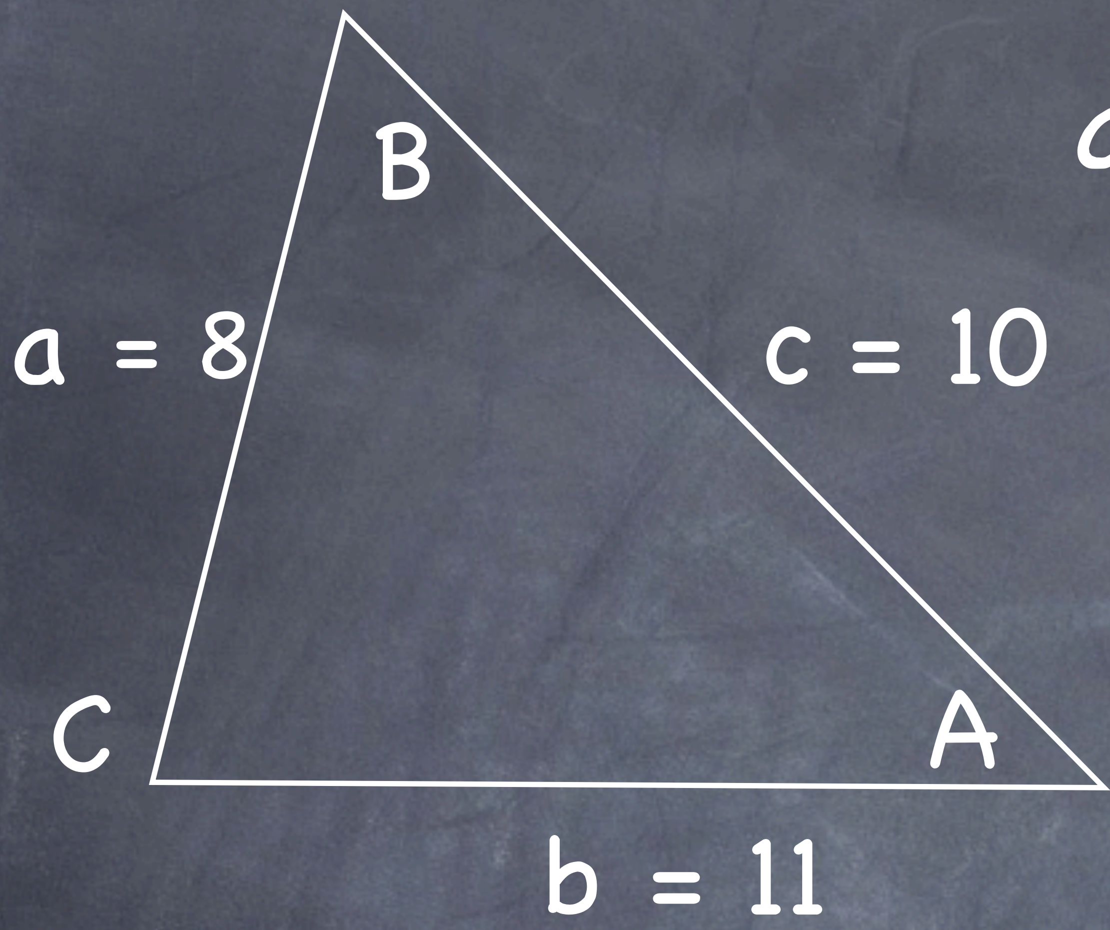
$$a^2 + b^2 - 2ab\cos C = c^2$$

Find the missing side

$$8^2 + 11^2 - 2(8)(11)\cos 77 = c^2$$

$$c^2 \approx 145.409$$

$$c \approx 12.059$$



$$a^2 + b^2 - 2ab\cos C = c^2$$

Find the missing angle

$$8^2 + 11^2 - 2(8)(11)\cos C = 10^2$$

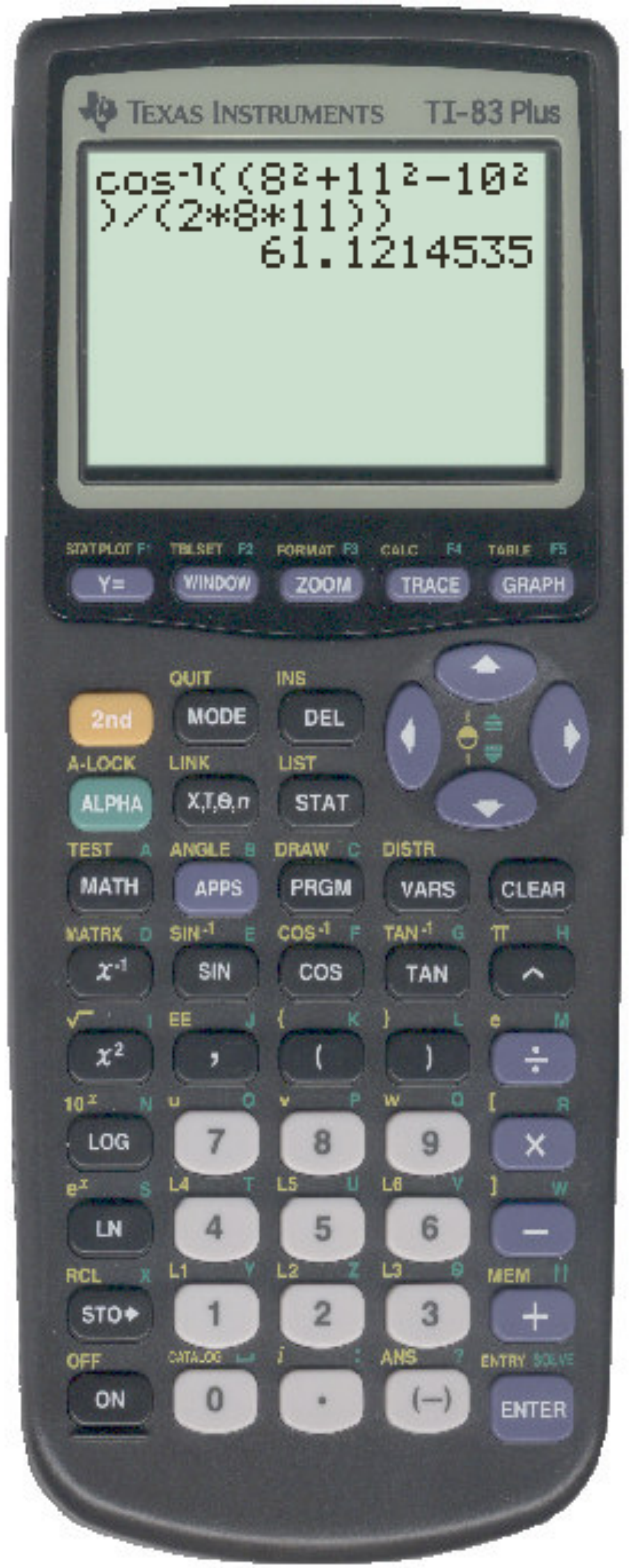
$$-2(8)(11)\cos C = 10^2 - 8^2 - 11^2$$

$$\cos C = \frac{10^2 - 8^2 - 11^2}{-2(8)(11)}$$

Be careful entering this into the calculator

$$\cos C = \frac{8^2 + 11^2 - 10^2}{2(8)(11)}$$

$$C = \cos^{-1}\left(\frac{8^2 + 11^2 - 10^2}{2(8)(11)}\right) \approx 61.121^\circ$$



Both the numerator and the denominator need parentheses. The calculator needs to see the terms like this:

$$C = \cos^{-1}\left(\frac{(8^2 + 11^2 - 10^2)}{(2(8)(11))}\right)$$

Without them the calculator will read it like this:

$$C = \cos^{-1}\left(8^2 + 11^2 - \frac{10^2}{2} * 8 * 11\right)$$