Quadratic Equations and Parabolas

A quadratic equation is usually in the form:

$$y = ax^2 + bx + c$$

But it can also be written like this:

$$y = a(x - h)^2 + k$$ Pg 17

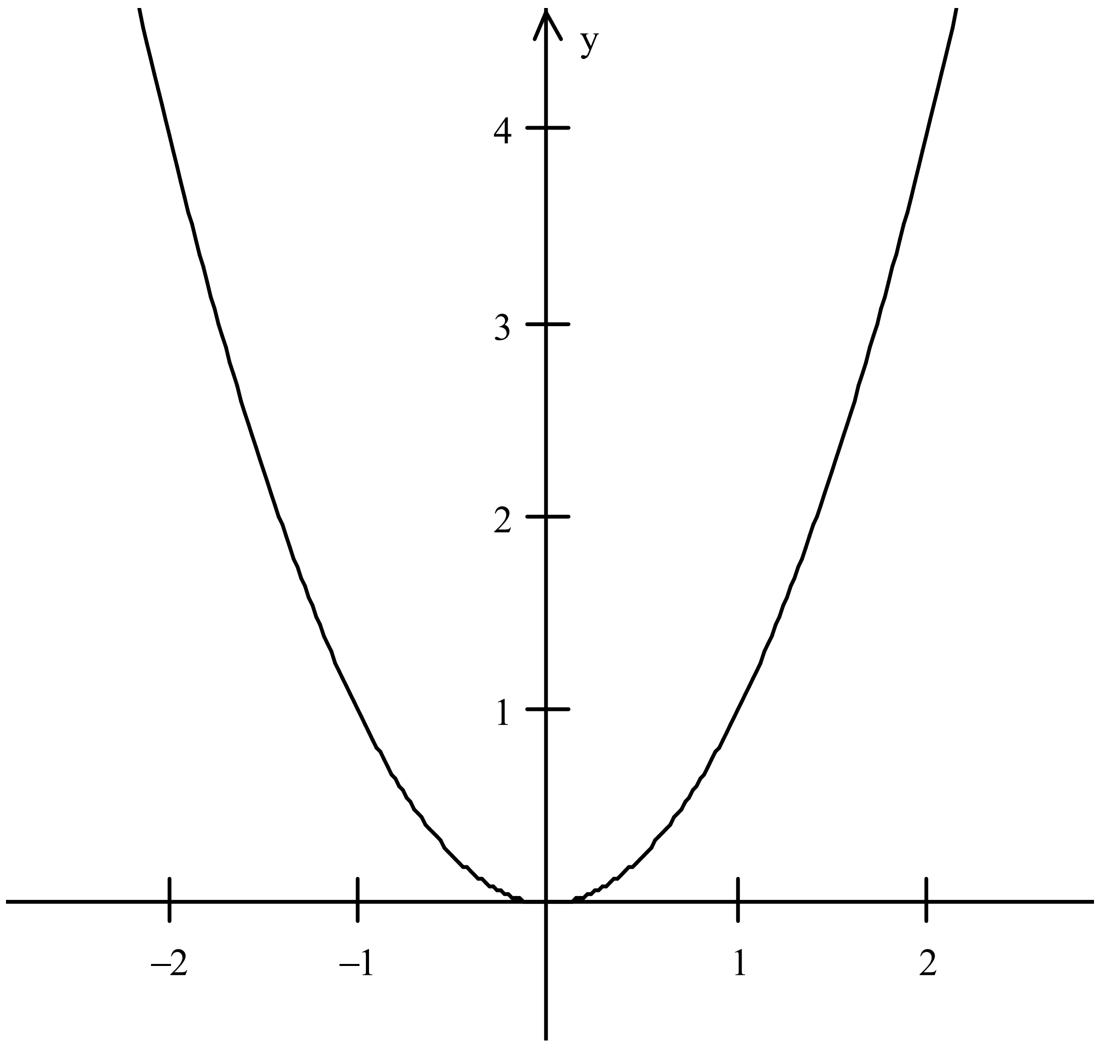
The values of h and k will be important

What does a quadratic equation look like when we graph it?

Let's look at three different ones:

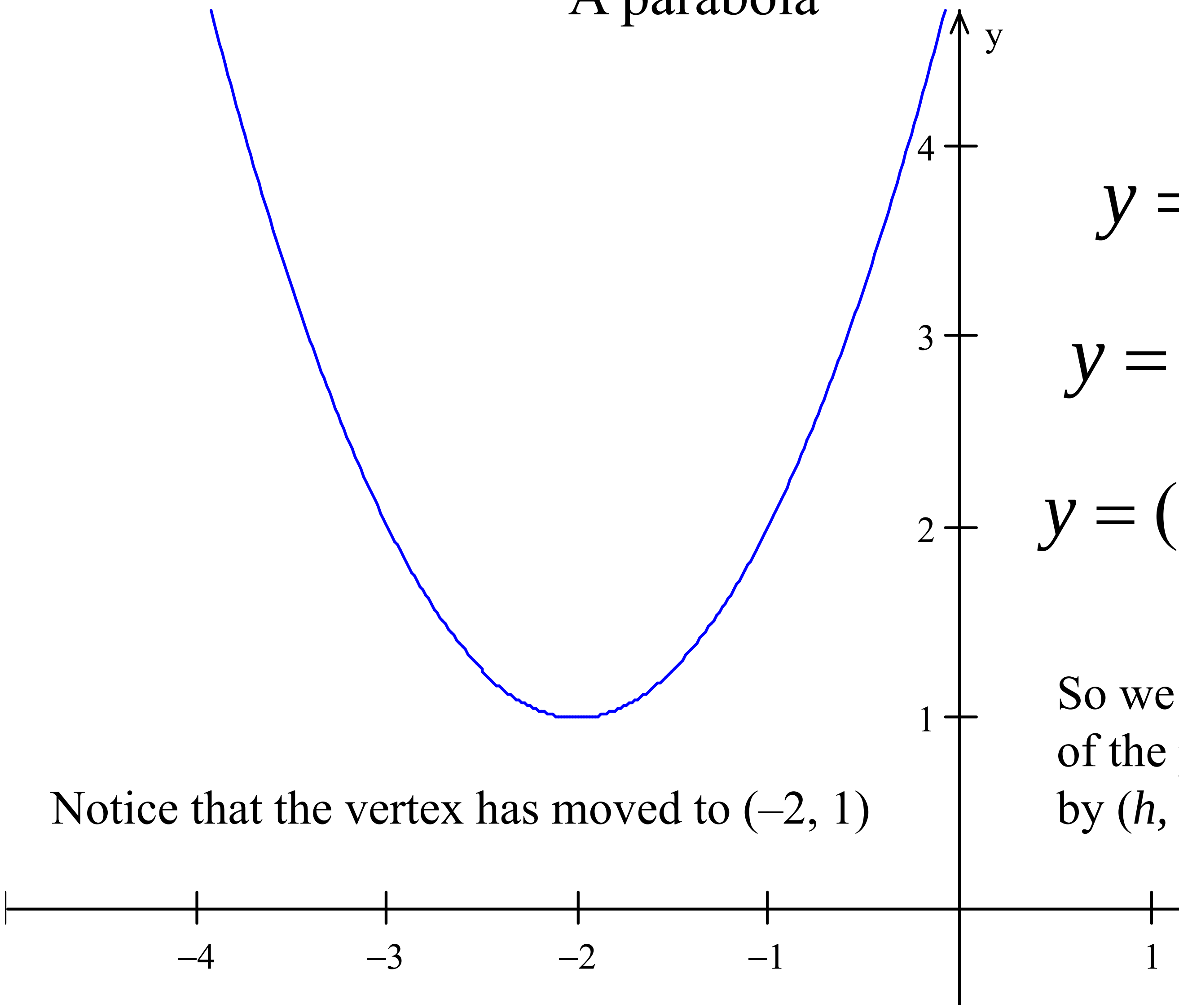
$y = x^2$ $\quad$ $y = (x+2)^2 + 1$ $\quad$ $y = x^2 + 2x + 3$

A parabola



$$y = x^2$$

A parabola



$$y = (x+2)^2 + 1$$

$$y = a(x-h)^2 + k$$

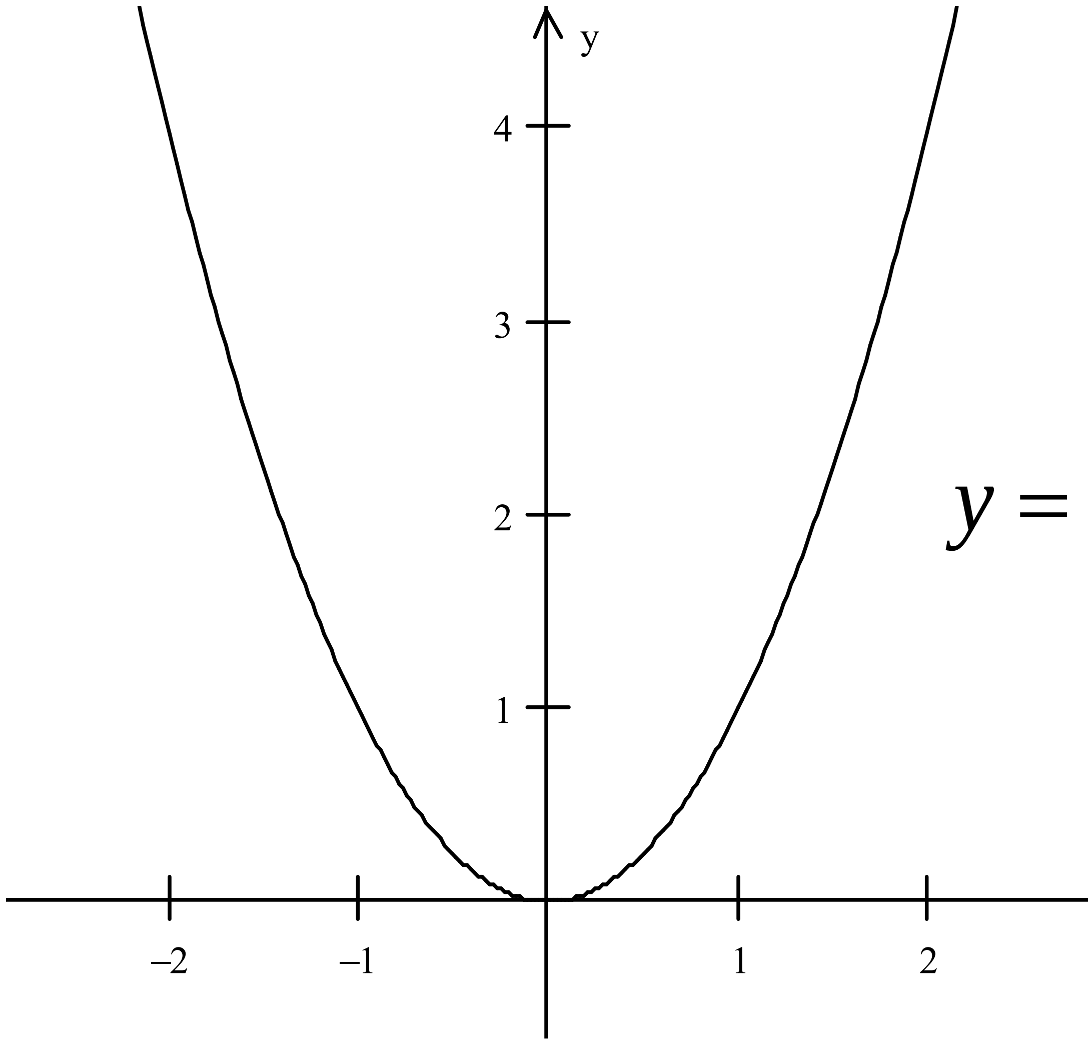
$$y = (x-(-2))^2 + 1$$

Notice that the vertex has moved to (–2, 1)

So we see that the vertex of the parabola is given by (*h*, *k*)

We'll deal with a later. For now, we'll just have problems in which $a = 1$

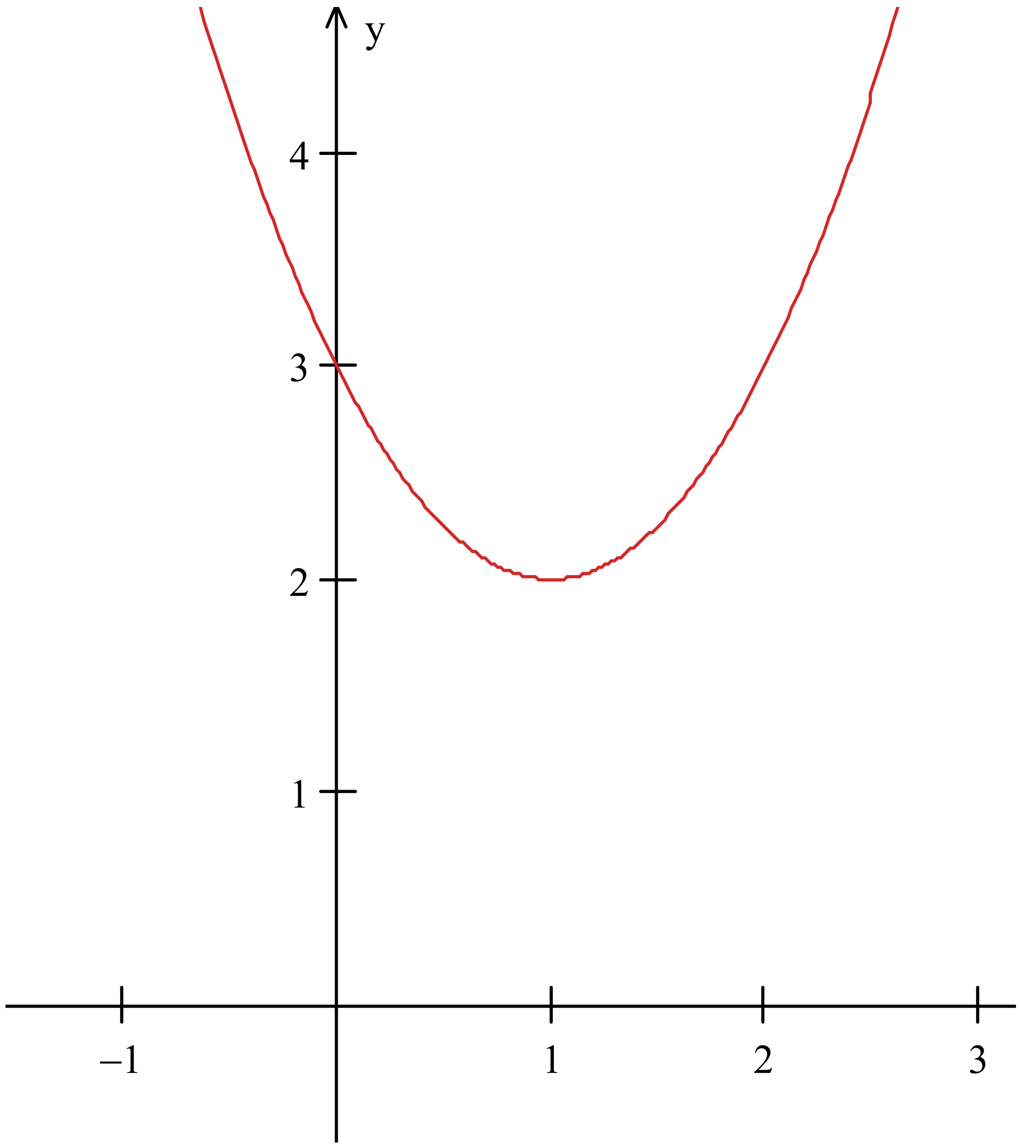
By the way…



$$y = x^2$$

Notice

$$y = (x - 0)^2 + 0$$



$$y = x^2 - 2x + 3$$

But how did we do this?

$$y = x^2 - 2x + 3$$

We want to make look like

$$y = ax^2 + bx + c \qquad y = a(x-h)^2 + k$$

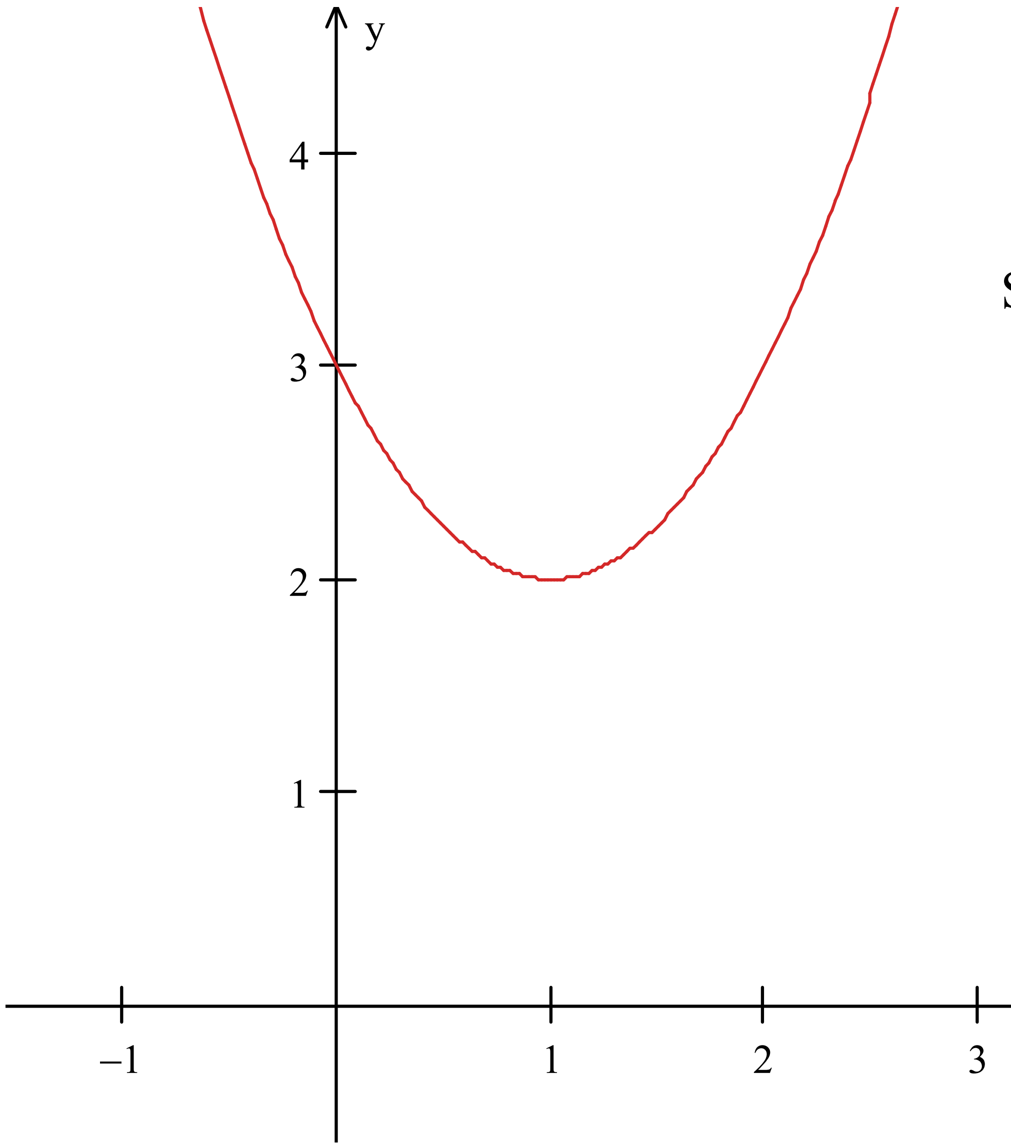
Remember a technique called completing the square?

$$y = x^2 - 2x + 3 = \underbrace{(x^2 - 2x + 1)}_{} + 2$$

Add $\dfrac{b^2}{4}$ or $\dfrac{(-2)^2}{4}$

$$y = (x-1)^2 + 2$$

Now we have a vertex of (1, 2)



$$y = x^2 - 2x + 3$$

So this is the same as writing

$$y = (x-1)^2 + 2$$

A quadratic equation is usually in the form:

$$y = ax^2 + bx + c$$

But it can also be written like this:

$$\boxed{y = a(x-h)^2 + k}$$

The values of h and k will be important

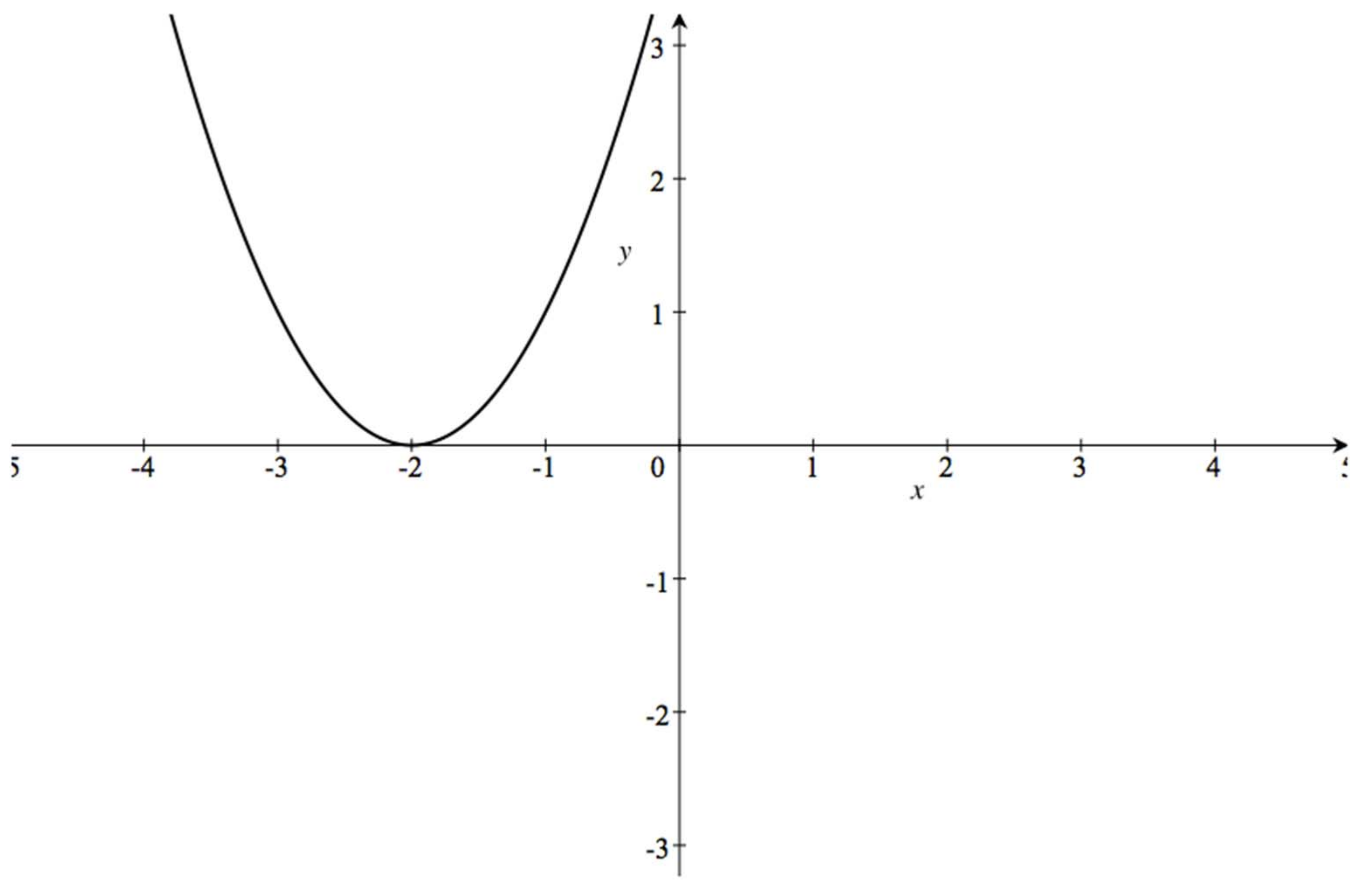
Now let's apply that second equation to this one:

$$y = x^2 + 4x + 4 \longrightarrow y = (x+2)^2 + 0$$

$$y = a(x-h)^2 + k$$

$$y = (x+2)^2$$

And the graph would look like this:



Find the equation of the parabola that passes through the points
$(-1, 0)$ $(2, 0)$ $(0, -4)$

Since -1 and 2 are both zeros then we can write the quadratic equation like this:

$$y = a\underbrace{(x+1)(x-2)}$$

See the first two points

$$-4 = a(0+1)(0-2)$$

$$-4 = -2a$$

$$a = 2$$

Now plug this answer in to the first equation

$$y = 2(x+1)(x-2)$$

$$\boxed{y = 2x^2 - 2x - 4}$$

Next we’ll talk about what this all has to do with…

PHYSICS

Now let’s try looking at a projectile…

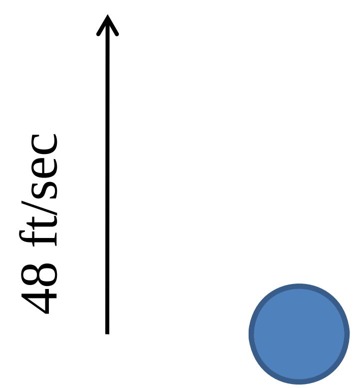
Initial velocity = 48 ft/sec

But what does this look like?

$$h = -16t^2 + 48t$$

This is what earth’s gravity does to a projectile. More on that in a minute.

This is determined by the projectile’s initial velocity



Now let's try launching it from a platform that is 20 feet off the ground

Initial velocity = 48 ft/sec

$$h = -16t^2 + 48t + 20$$

This is what earth's gravity does to a projectile

This is determined by the projectile's initial velocity

The object's initial height

$$h = -\frac{1}{2}gt^2 + v_0 t + h_0$$

In meters it's 9.81 m/s^2

In feet it's 32 ft/s^2

Now let’s try launching it from a platform that is 40 feet off the ground

Initial velocity = 32 ft/sec

Let’s find the projectile’s highest point…

$$h = -16t^2 + 32t + 40$$

…or the vertex of this parabola

$$h = -16(t - 2t + \quad)^2 + 40 -$$

What do we add here to complete the square?

What do we subtract here?

$$h = -16(t-1)^2 + 56$$

$$h = -16(t - 2t + 1)^2 + 40 - 1(-16)$$

Add 1 inside the parentheses

Subtract 1 times -16 because of the -16 in front of the parentheses

So the maximum height is 56 feet which occurs at 1 second into its flight.