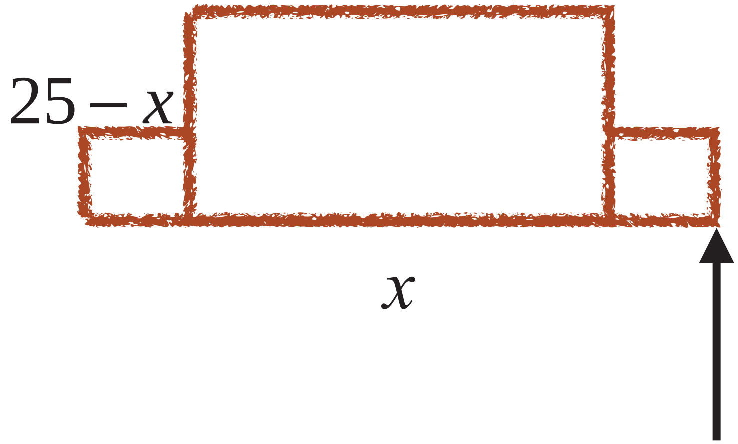
1-5 Optimization

Finding Optimum Values Part 1

You have 50 feet of fence to enclose a rectangular garden. What is the maximum area that you can enclose?

$w = x$



$P = 50 = 2x + 2l$

$2l = 50 - 2x$

$$l = \frac{50 - 2x}{2} = 25 - x$$

Imagine the length being very small, almost 0

$$A(x) = lw = (25 - x)x$$

If the perimeter is 50, what value would *x* approach?

Consider $A(x)$ to be the area formula as a function of *x*.

Because there is an *x* on two sides of the garden each value would max out at 25 feet.

What is the domain?

$0 < x < 25 \;\; feet$ Why?

You have 50 feet of fence to enclose a rectangular garden. What is the maximum area that you can enclose?

$w = x$

$25 - x$

x

$$P = 50 = 2x + 2l$$

$$2l = 50 - 2x$$

$$l = \frac{50 - 2x}{2} = 25 - x$$

How do we find the maximum value of $A(x)$?

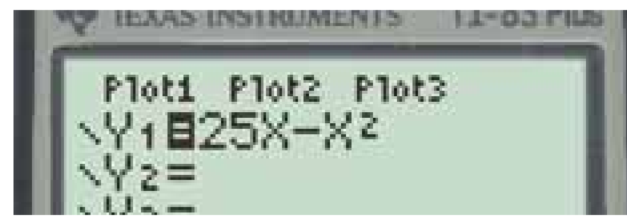
$$A(x) = 25x - x^2 \longleftarrow$$

Is an upside down parabola with the maximum at its vertex

$$A(x) = lw = (25 - x)x$$

Consider $A(x)$ to be the area formula as a function of x.

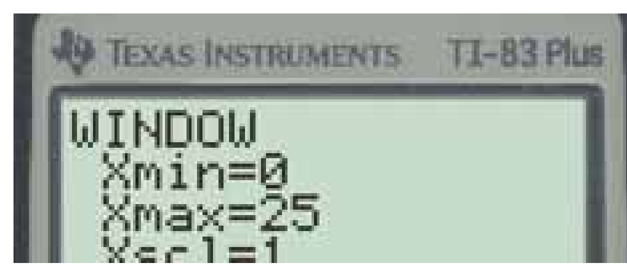
Plug this into the calculator



What is the domain?

$$0 < x < 25 \ \ feet$$

with a window setting of...



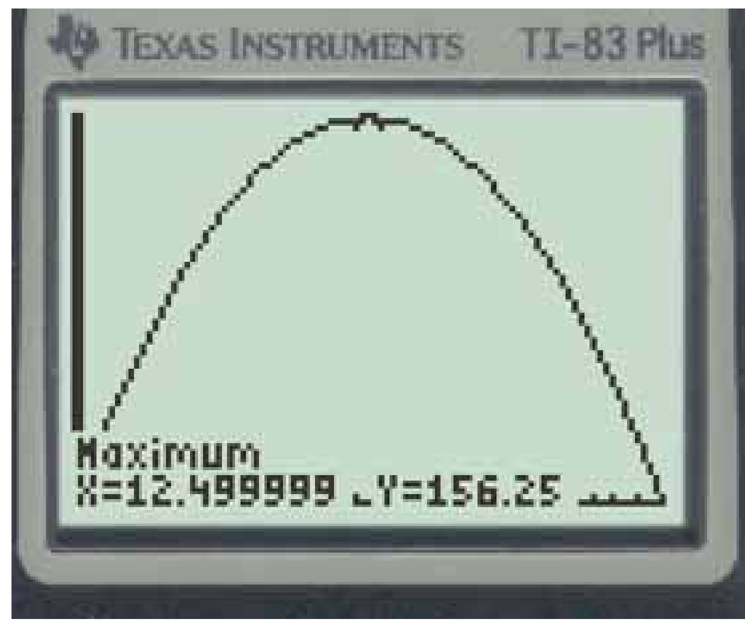
You have 50 feet of fence to enclose a rectangular garden. What is the maximum area that you can enclose?

$$l = 12.5 \ feet$$

$$w = 12.5 \ feet$$

Now just use the ZOOMFIT function (Zoom --> 0)

The maximum here is at x = 12.5 ft with the maximum area being 156.25 ft^2



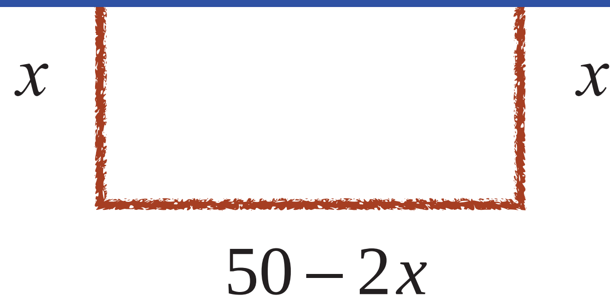
We can also use x = 12.5 to find l

Optimization

1. Draw a picture! This can go a long way towards setting up your solution.
2. Write a function for the value that you are trying to optimize.
3. If your function is in terms of more than one variable, use substitution to reduce it to one.
4. Find the domain of this function in order to set the x-window
5. Graph it on the calculator
6. Determine the maximum/minimum values
7. Check the end points of the domain to determine if either is a max or min.

Your 50 feet of fence must now enclose a rectangular space along the bank of a river. What is the maximum area that you can enclose?

$l = x$



$P = 50 = 2l + w$

$w = 50 - 2x$

$50 = 2x + w$

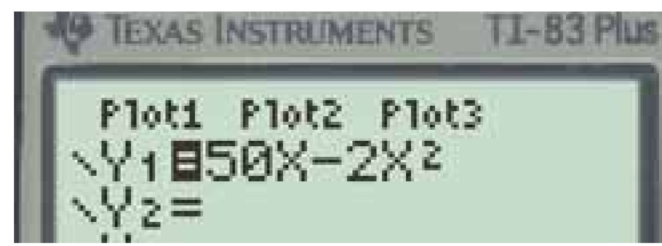
How do we find the maximum value of $A(x)$?

$$A(x) = 50x - 2x^2$$

$$A(x) = lw = x(50 - 2x)$$

Consider $A(x)$ to be the area formula as a function of x.

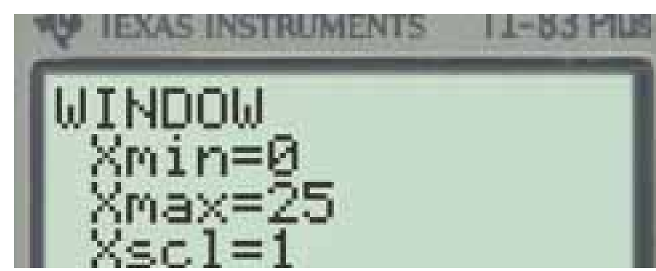
Plug this into the calculator



What is the domain?

$0 < x < 25$ *feet*

with a window setting of...



Your 50 feet of fence must now enclose a rectangular space along the bank of a river. What is the maximum area that you can enclose?

x

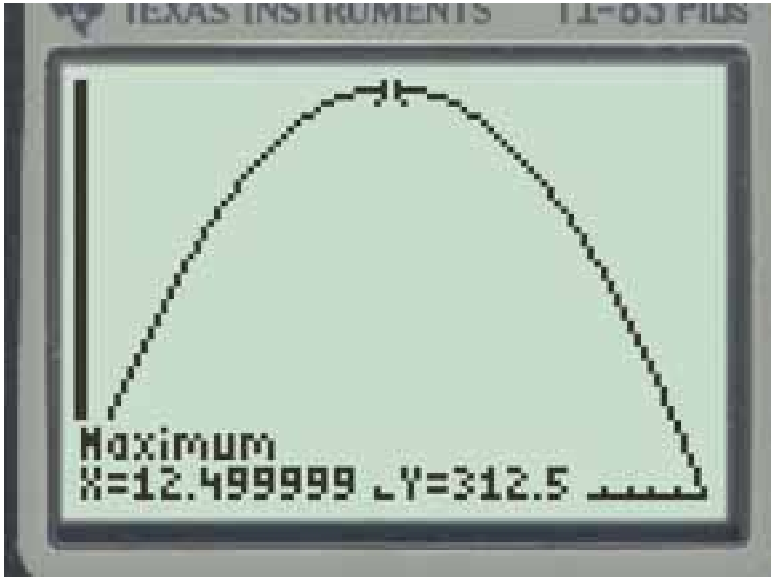
$l = x = 12.5\ feet$

$w = 50 - 2x = 25\ feet$

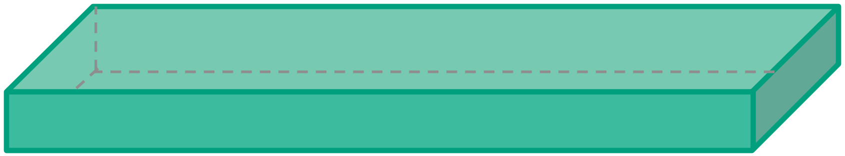
Now just use the ZOOMFIT function (Zoom --> 0)

The maximum here again is at x = 12.5 ft but the results are a little bit different.

$$A_{max} = 312.5\ feet^2$$



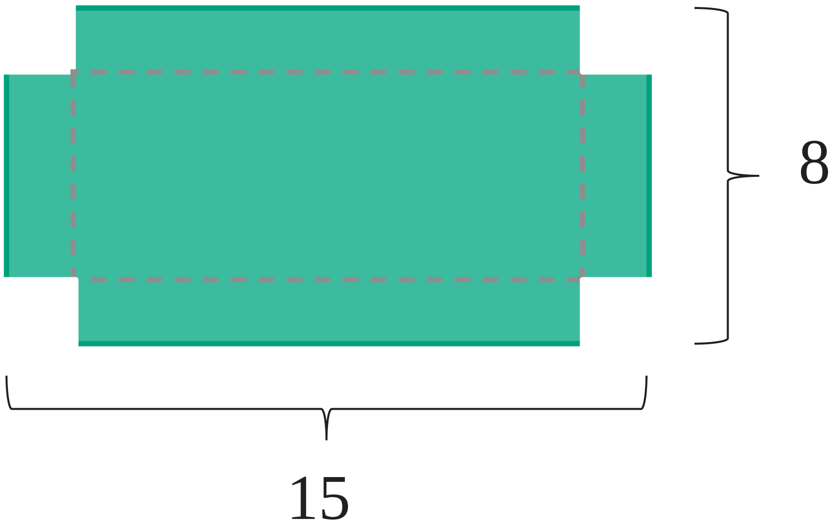
You are making an open top rectangular box



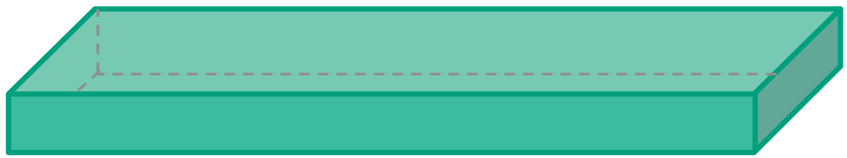
Find the dimensions of such a box with the largest volume.

from an 8 by 15 inch piece of cardboard

by cutting square pieces off of each corner and folding up the sides



You are making an open top rectangular box



Find the dimensions of such a box with the largest volume.

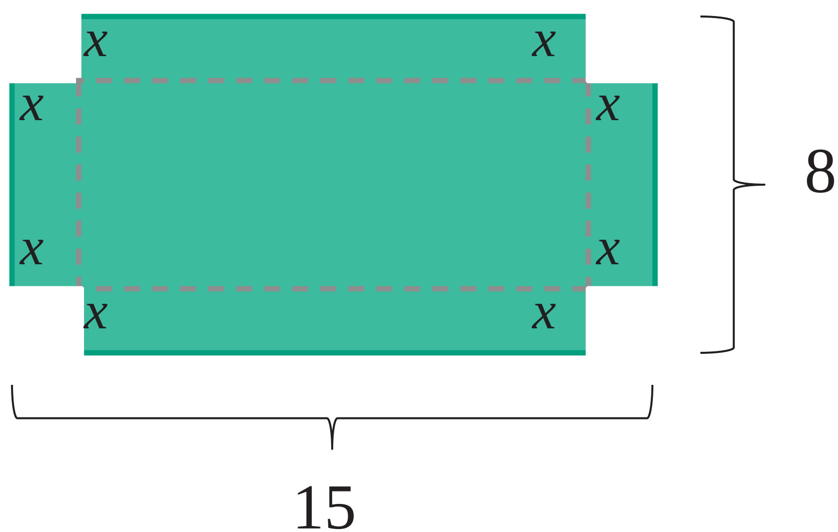
from an 8 by 15 inch piece of cardboard

Each square that we cut off would have sides equal to x

$$0 < x < 4$$

Why?

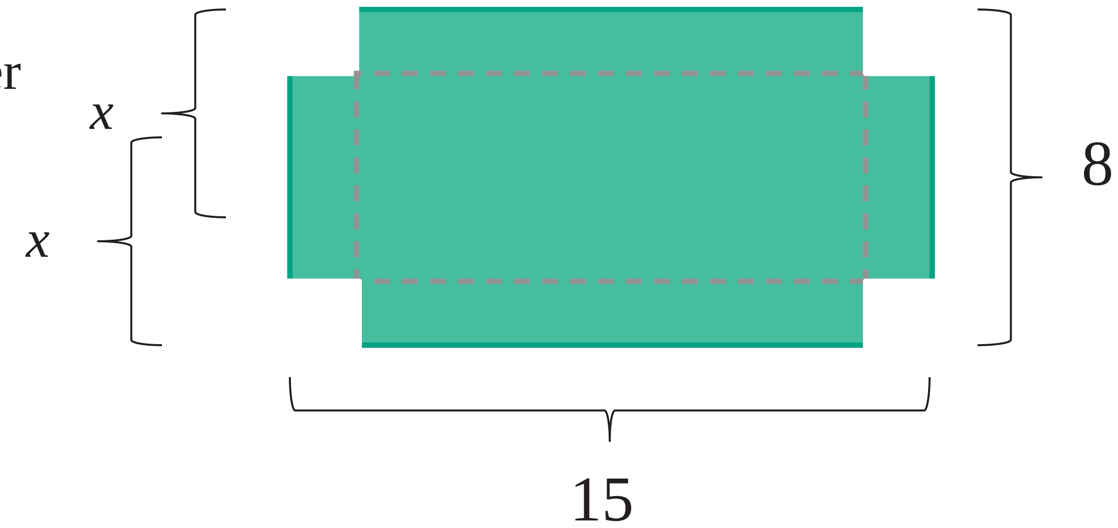
Do we have a limited domain for x values?



Each square that we cut off would have sides equal to x

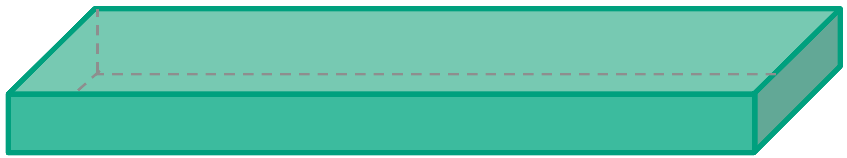
$0 < x < 4$ Why?

Suppose that x measures larger than 4 as shown to the right



It wouldn't make sense for two squares to add up to 8 or more inches.

You are making an open top rectangular box



Find the dimensions of such a box with the largest volume.

from an 8 by 15 inch piece of cardboard

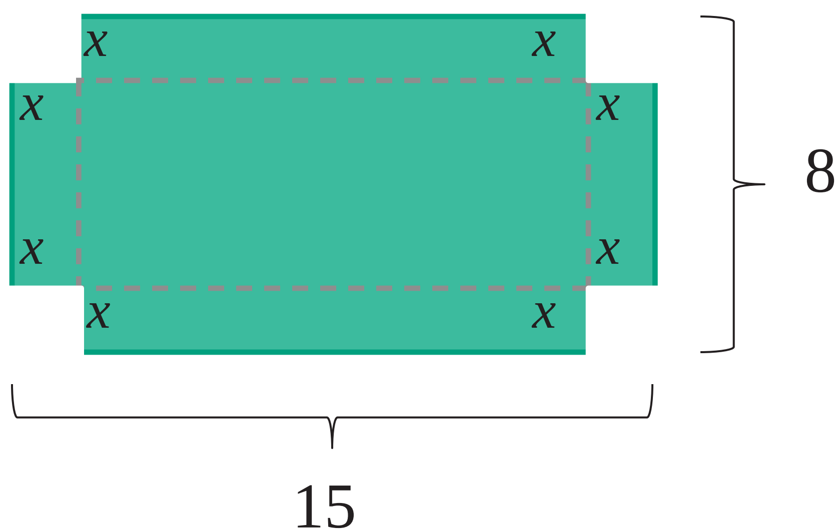
Each square that we cut off would have sides equal to x

$$V = l \cdot w \cdot h$$

length width height

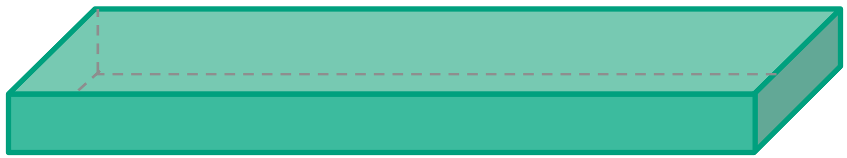
$$0 < x < 4$$

Why?



Now we just need a volume formula in terms of x

You are making an open top rectangular box

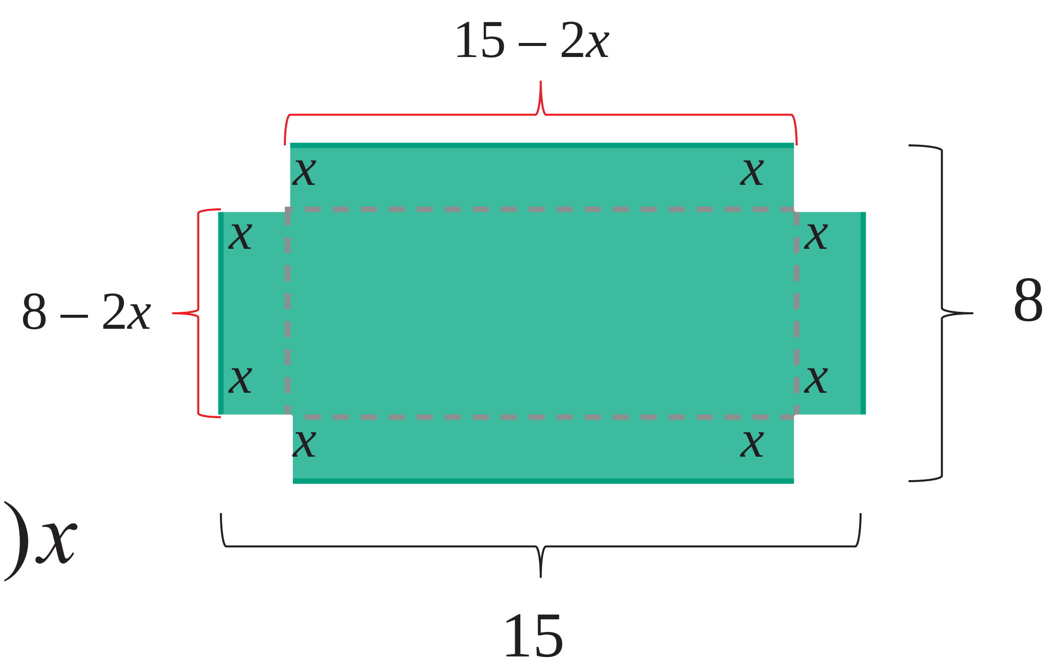


Find the dimensions of such a box with the largest volume.

from an 8 by 15 inch piece of cardboard

$$V = l \cdot w \cdot h$$

$l \to 15 - 2x$, $w \to 8 - 2x$, $h \to x$

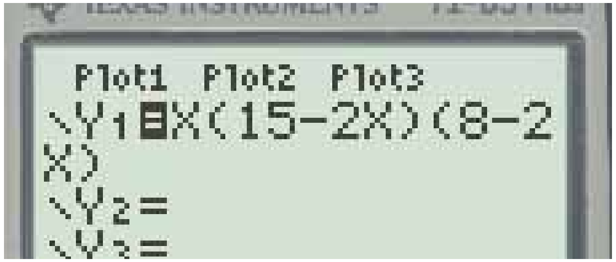


$$V = (15 - 2x)(8 - 2x)x$$

$$V = (15 - 2x)(8 - 2x)x$$

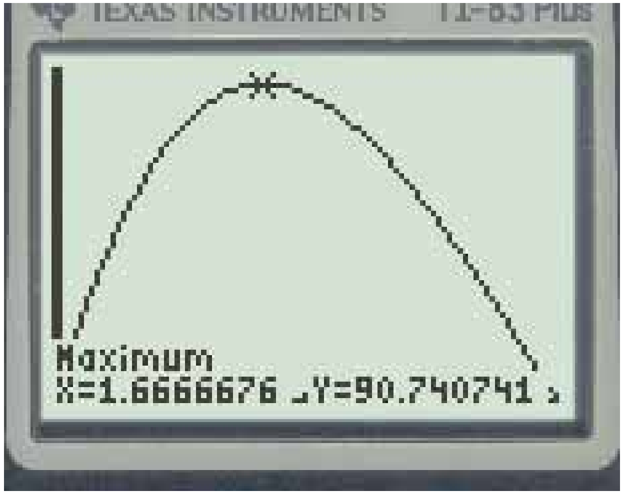
$$0 < x < 4$$

Enter the function for volume



Set the window according to the domain

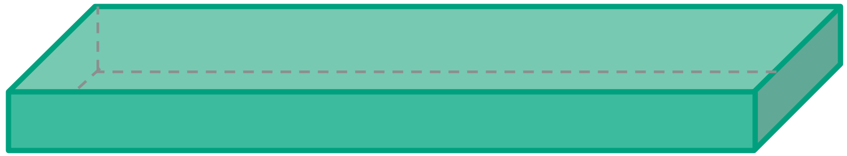
Find the maximum value



The *x* coordinate 1.6666...

is actually 5/3

You are making an open top rectangular box



Find the dimensions of such a box with the largest volume.

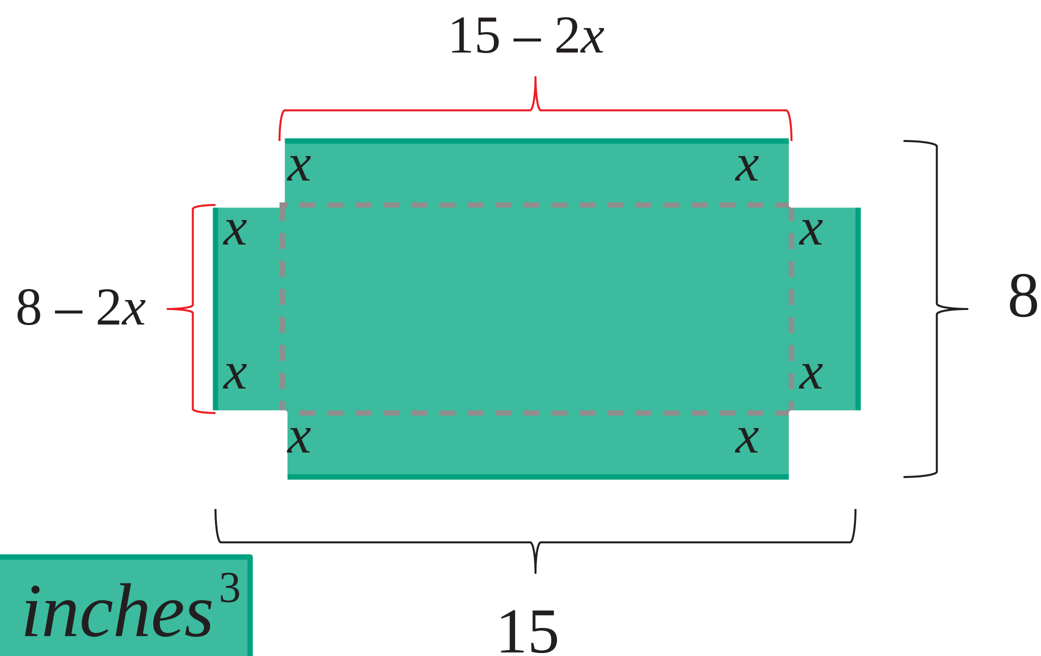
from an 8 by 15 inch piece of cardboard

So our dimensions are:

$$l = 15 - 2\left(\frac{5}{3}\right) = \frac{35}{3}$$

$$w = 8 - 2\left(\frac{5}{3}\right) = \frac{14}{3}$$

$$h = \frac{5}{3} \qquad V\left(\frac{5}{3}\right) \approx 90.74 \ inches^3$$

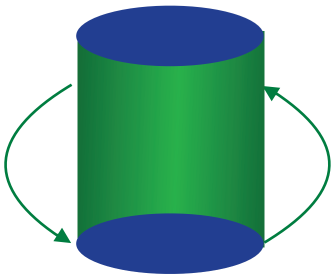


The KQ cola company wants to use as little aluminum per can of cola as possible for a 355 cm^3 cylindrical can.

What this problem is really asking for is the minimum surface area for the can.

So we are trying to minimize this function:

$$A = \underbrace{2\pi r^2}_{\text{area of circular ends}} + \underbrace{2\pi r h}_{\text{lateral area}}$$



$$A = 2\pi r^2 + 2\pi r \frac{355}{\pi r^2}$$

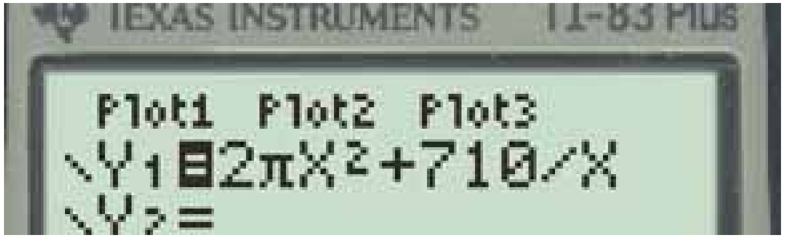
$$A = 2\pi r^2 + \frac{710}{r}$$

We need to eliminate one of these variables through substitution

Since we also know that

$$V = 355 cm^3 = \pi r^2 h$$

$$\frac{355}{\pi r^2} = h$$

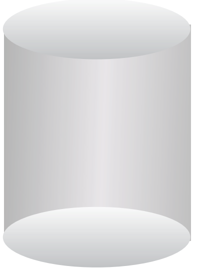


Since we're trying to minimize the area the only domain restriction here is that $r > 0$

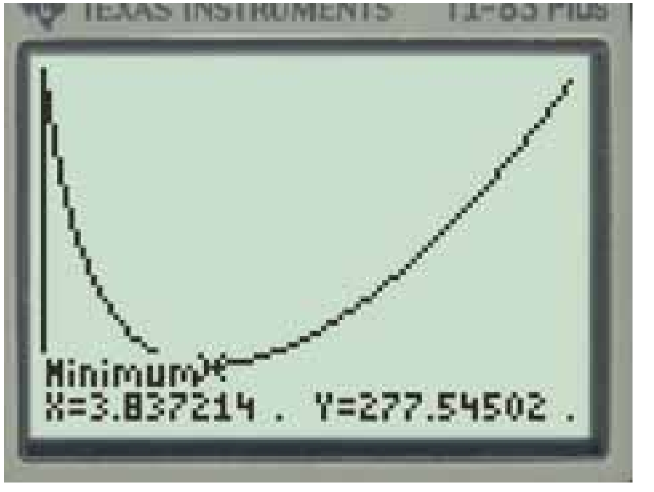
The KQ cola company wants to use as little aluminum per can of cola as possible for a 355 cm^3 cylindrical can.

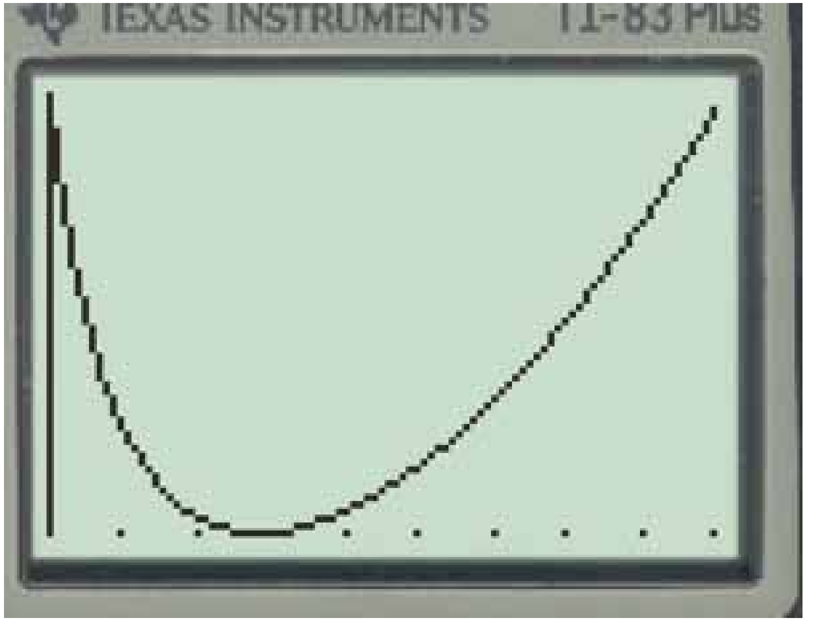
What this problem is really asking for is the minimum surface area for the can.

So we are trying to minimize this function:



Now let's graph it and find the minimum





$$r \approx 3.837\,cm$$

$$h \approx 7.674\,cm$$

And the minimum area possible is:

$$A \approx 277.545\,cm^2$$

Remember these examples when working on Assignment 1-5