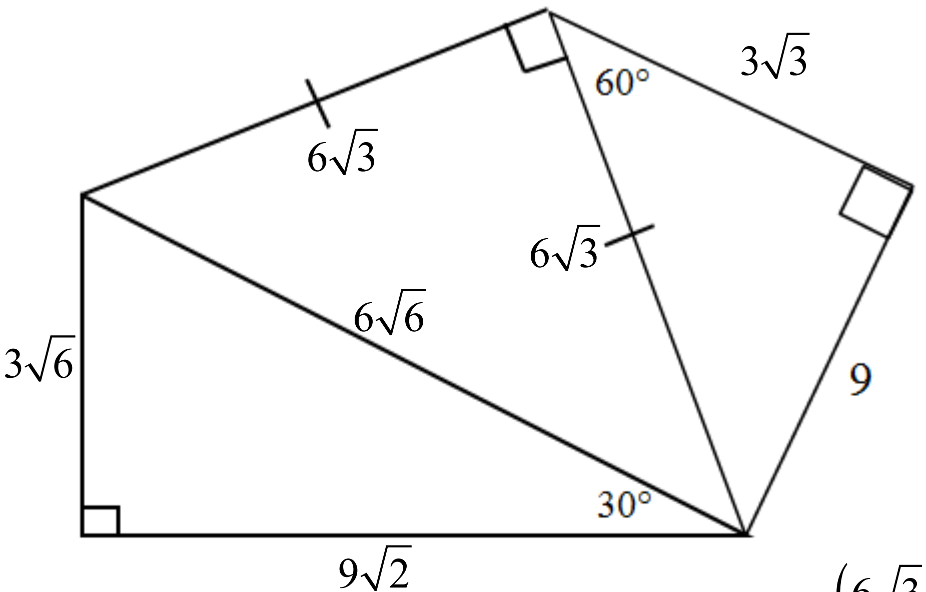
# 12.5

Angle Relationships within Circles




$$9 = x\sqrt{3}$$

$$x = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

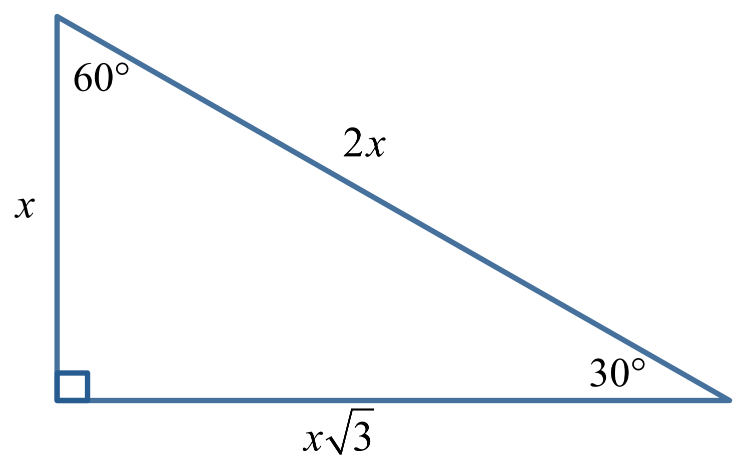
$$2x = \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

$$\left(6\sqrt{3}\right)\sqrt{2} = 6\sqrt{6}$$

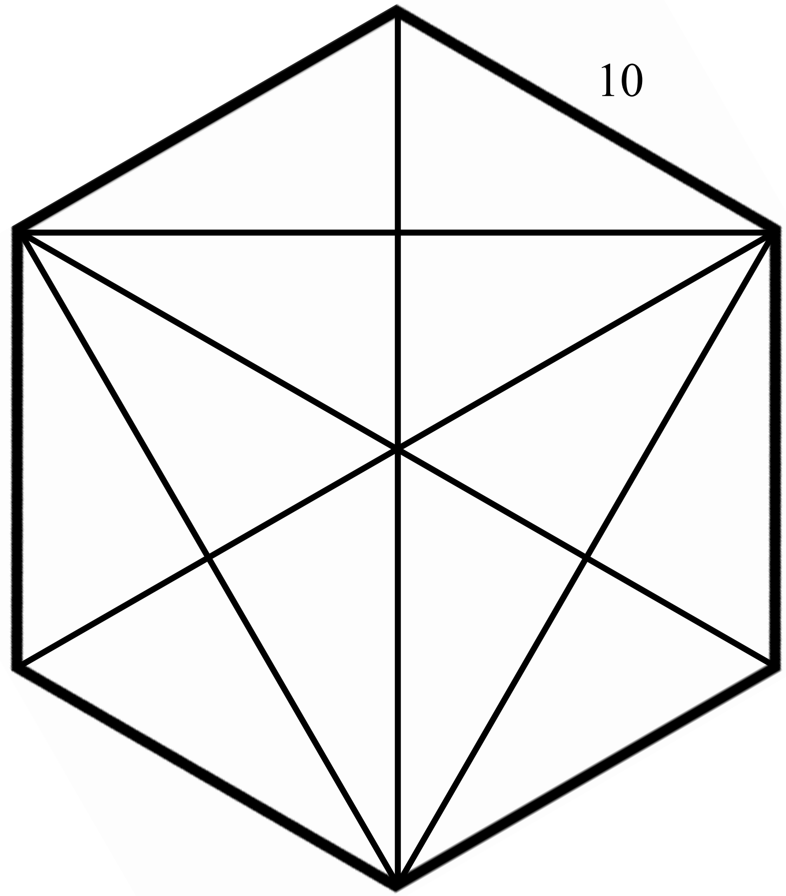
$$2y = 6\sqrt{6}$$

$$y = 3\sqrt{6}$$

$$y\sqrt{3} = 3\sqrt{6}\sqrt{3} = 9\sqrt{2}$$



Given the regular hexagon of side length 10, find the lengths of all the interior diagonals



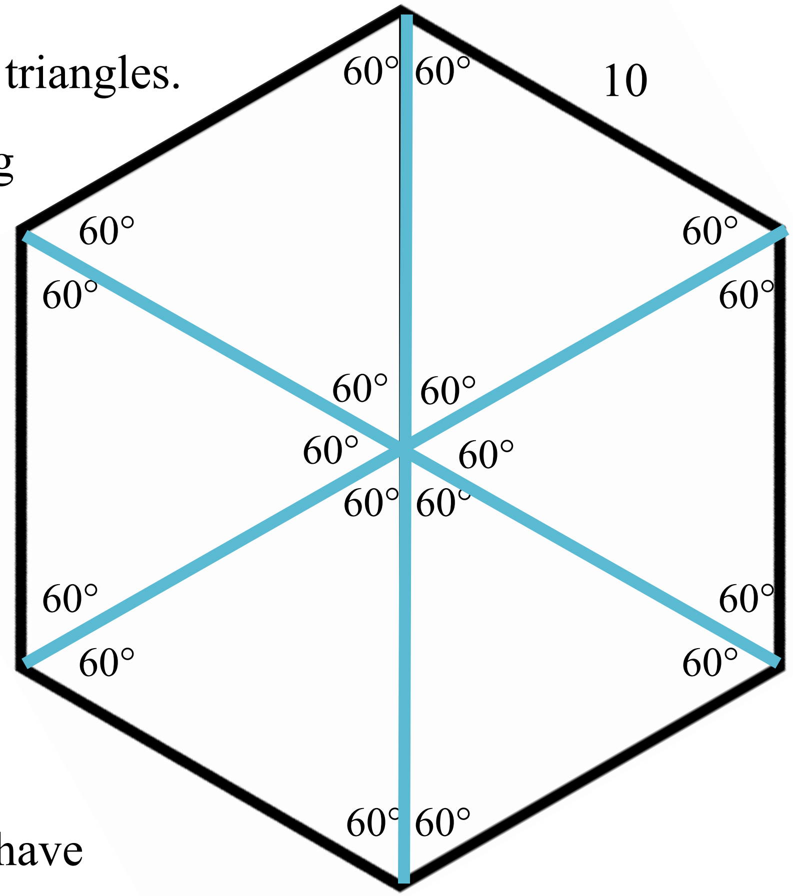

Now before anyone panics…

Given the regular hexagon of side length 10, find the lengths of all the interior diagonals

These are all congruent triangles.

They are also something else…

equilateral



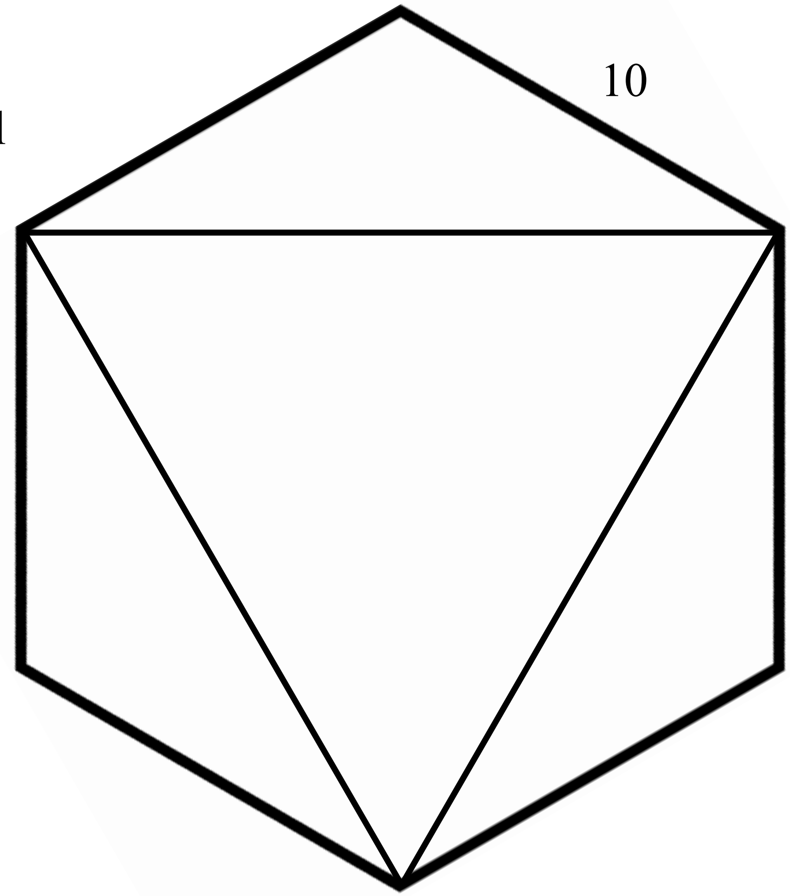

Since the triangles all have sides of length 10…

20

for each diagonal

Given the regular hexagon of side length 10, find the lengths of all the interior diagonals

These diagonals also form one big equilateral triangle so if we find one length we've found the other two..



Given the regular hexagon of side length 10, find the lengths of all the interior diagonals

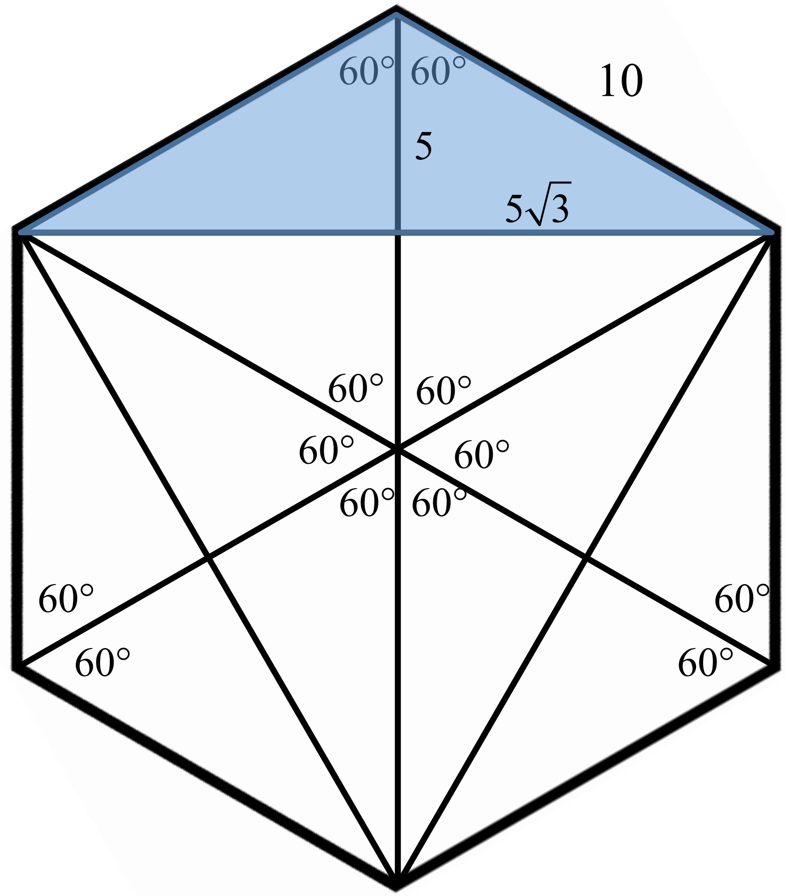
What can we say about this triangle?

It's isosceles and…

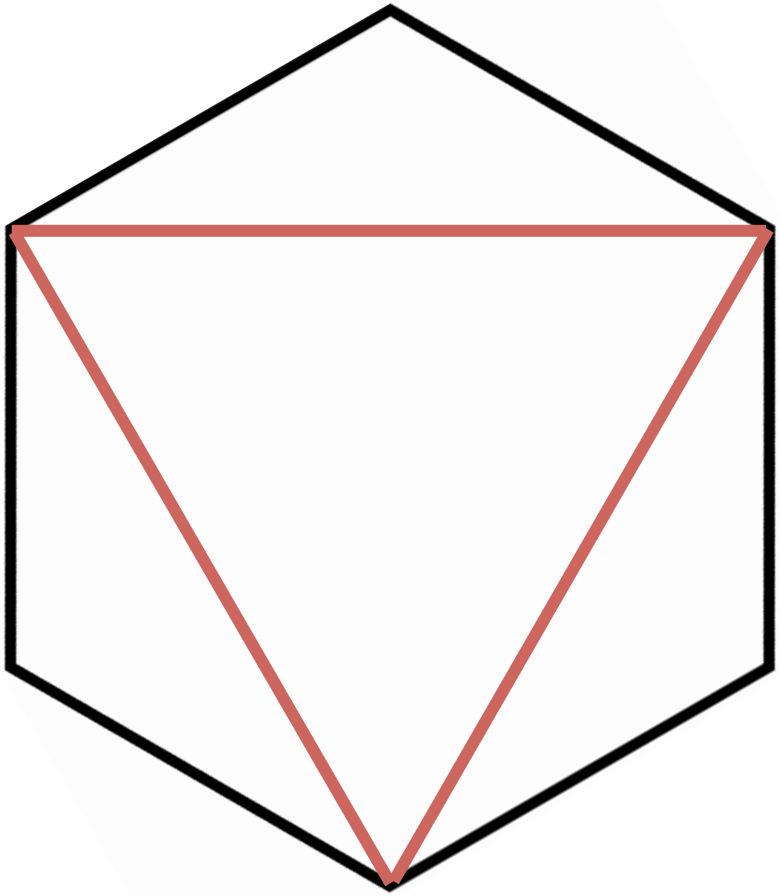
…it's altitude bisects it into two right triangles.

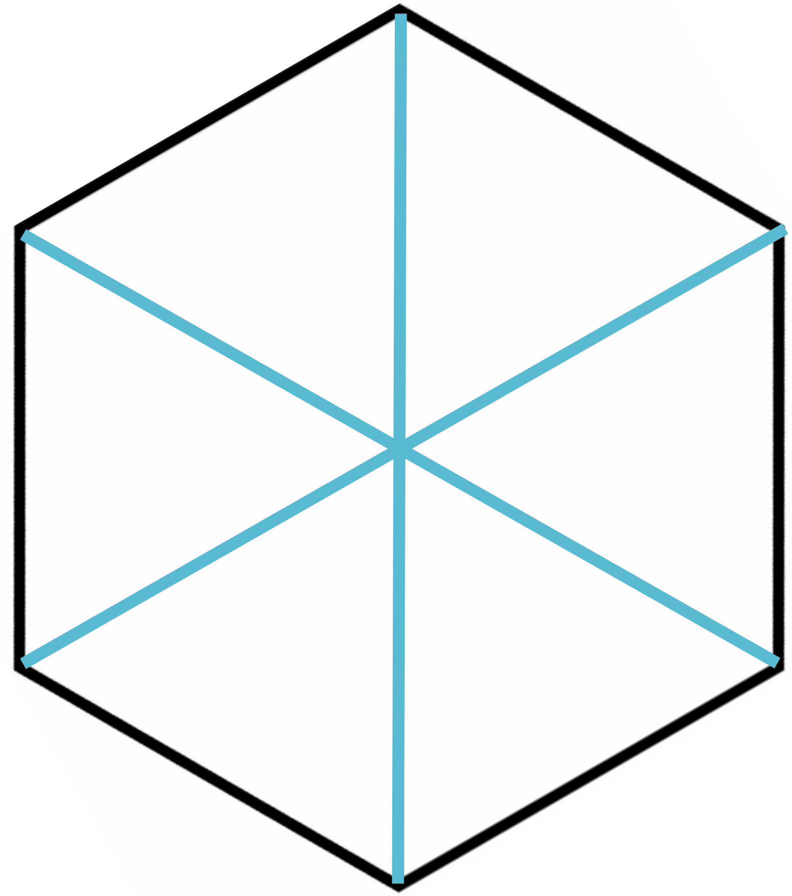
In this case they are both 30-60-90 triangles.

So let's find the side lengths using this ratio.



Given the regular hexagon of side length 10, find the lengths of all the interior diagonals





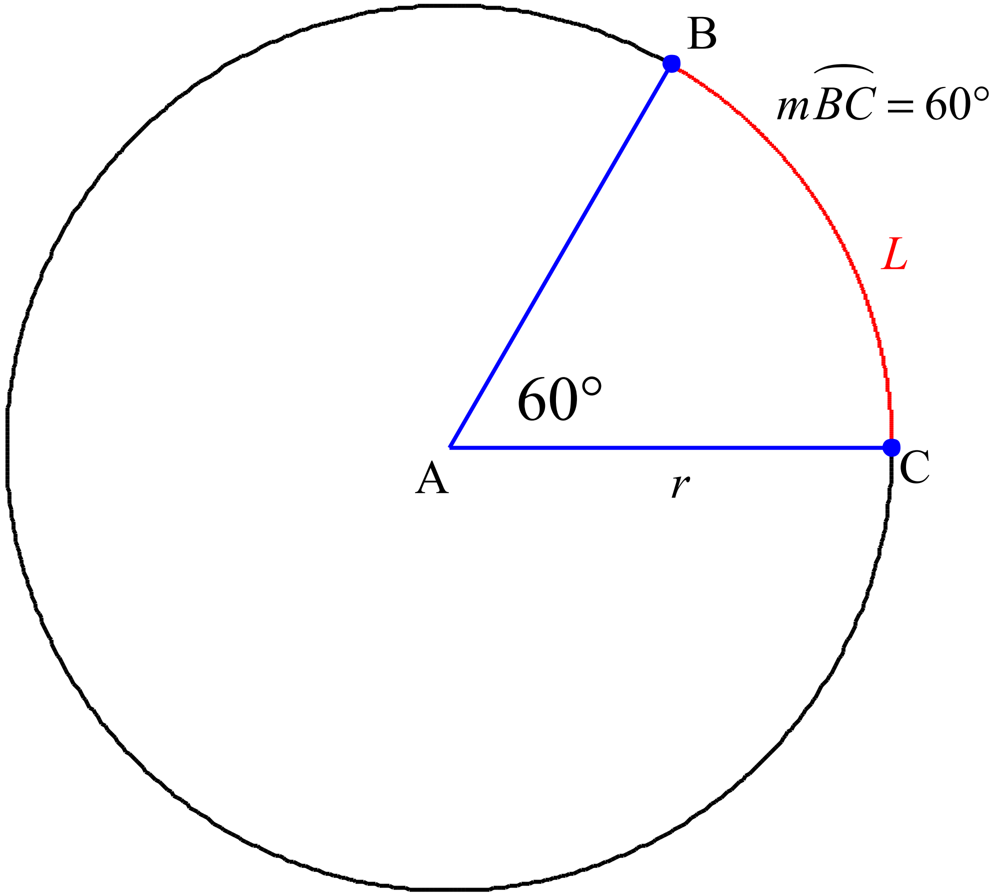
$10\sqrt{3}$

for each diagonal

So the lengths of these diagonals are…

20

for each diagonal




Just a few gentle reminders:

$$m\overset{\frown}{BC} = m\angle BAC$$

So if

$$m\angle BAC = 60°$$

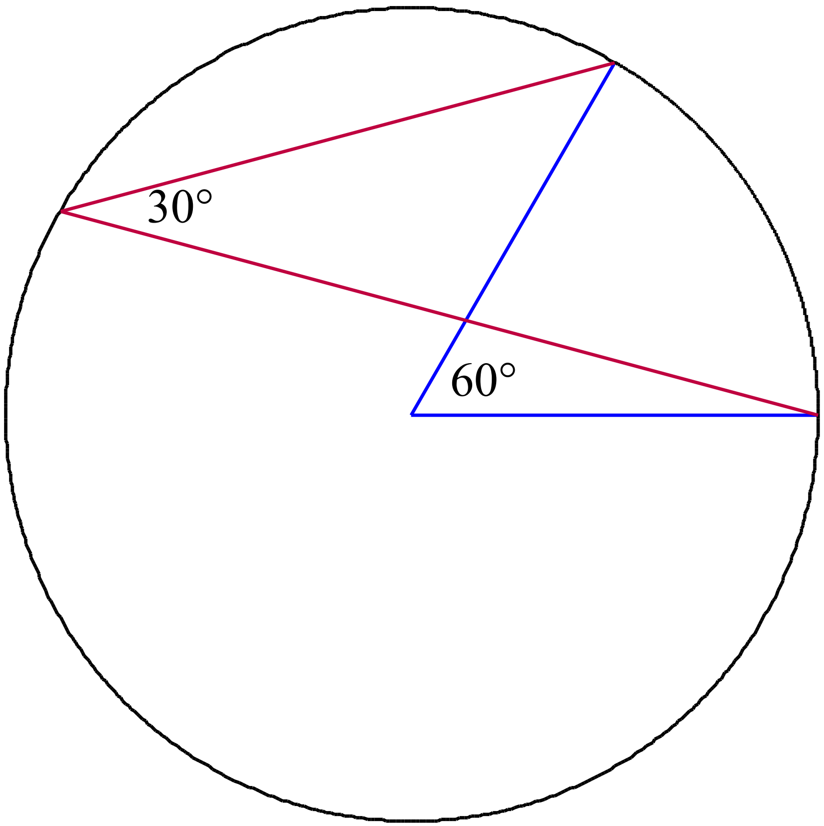
$$m\overset{\frown}{BC} = 60°$$

But when we talk about *arc length*

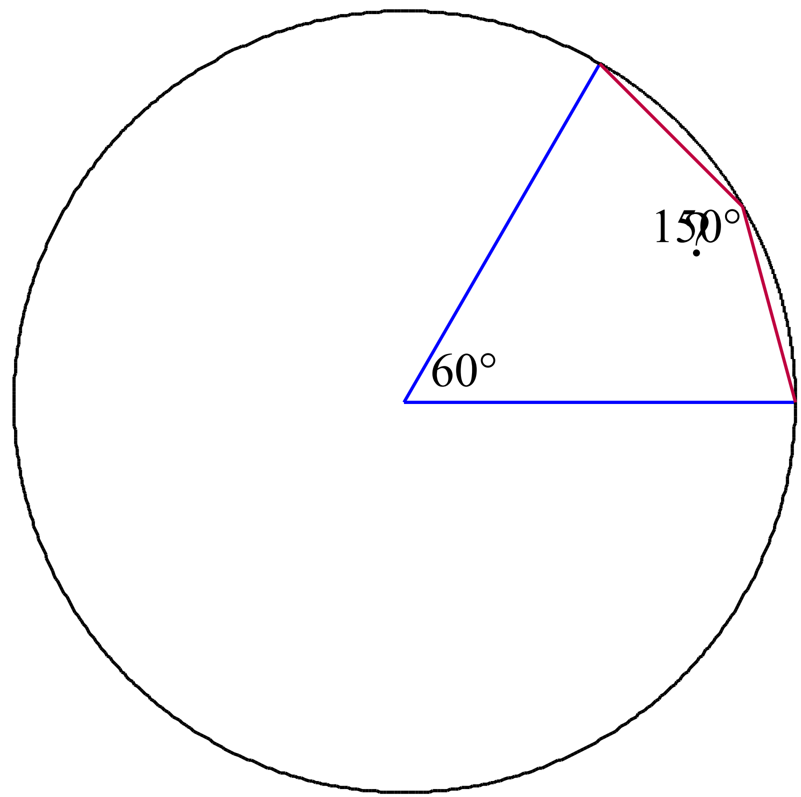
$$L = 2\pi r\left(\frac{60°}{360°}\right)$$

Unlike the arc measurement, arc length will be measured in units of length

And for one more bit of review…

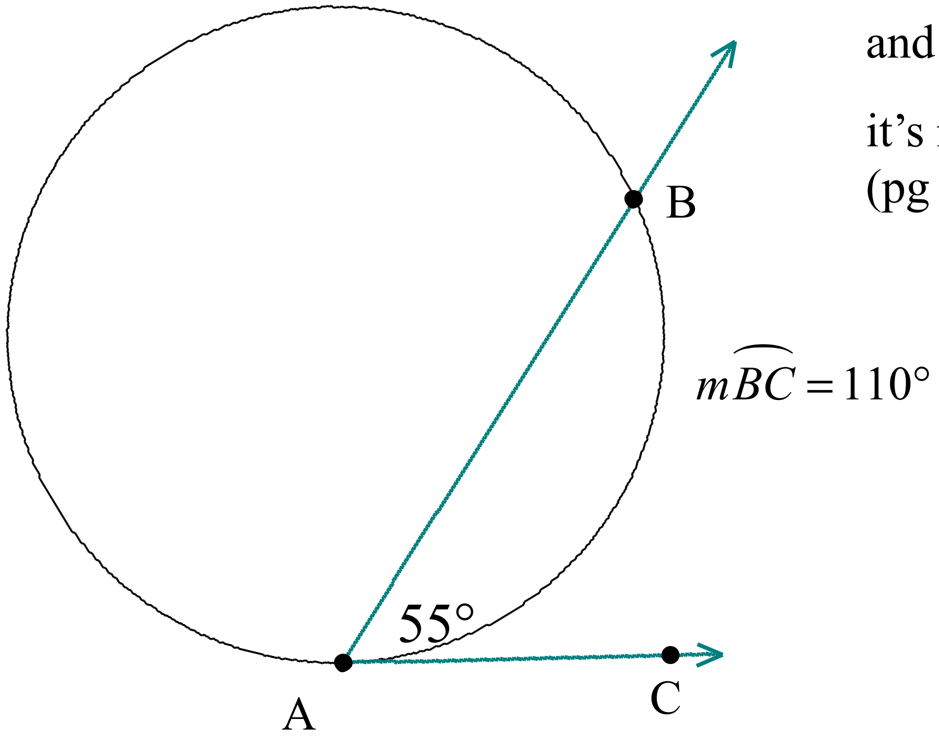



Think about this one…

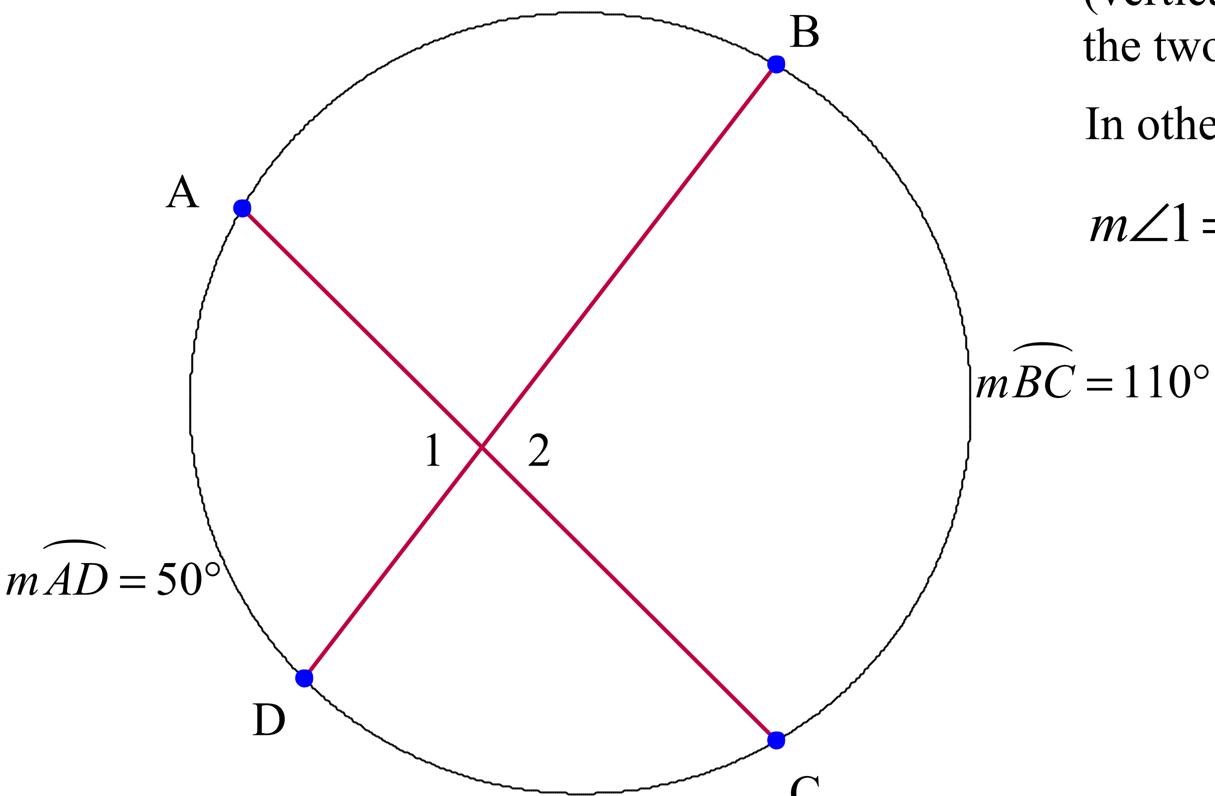


Just like inscribed angles, when an angle is formed by a secant line $\overline{AB}$ and a tangent line, $\overline{AC}$

it's measurement will be half the arc (pg 830)




$m\angle BAC = 55°$




In this case, $m\angle 1 \text{ and } m\angle 2$ (vertical angles) are the average of the two arcs (pg 831)

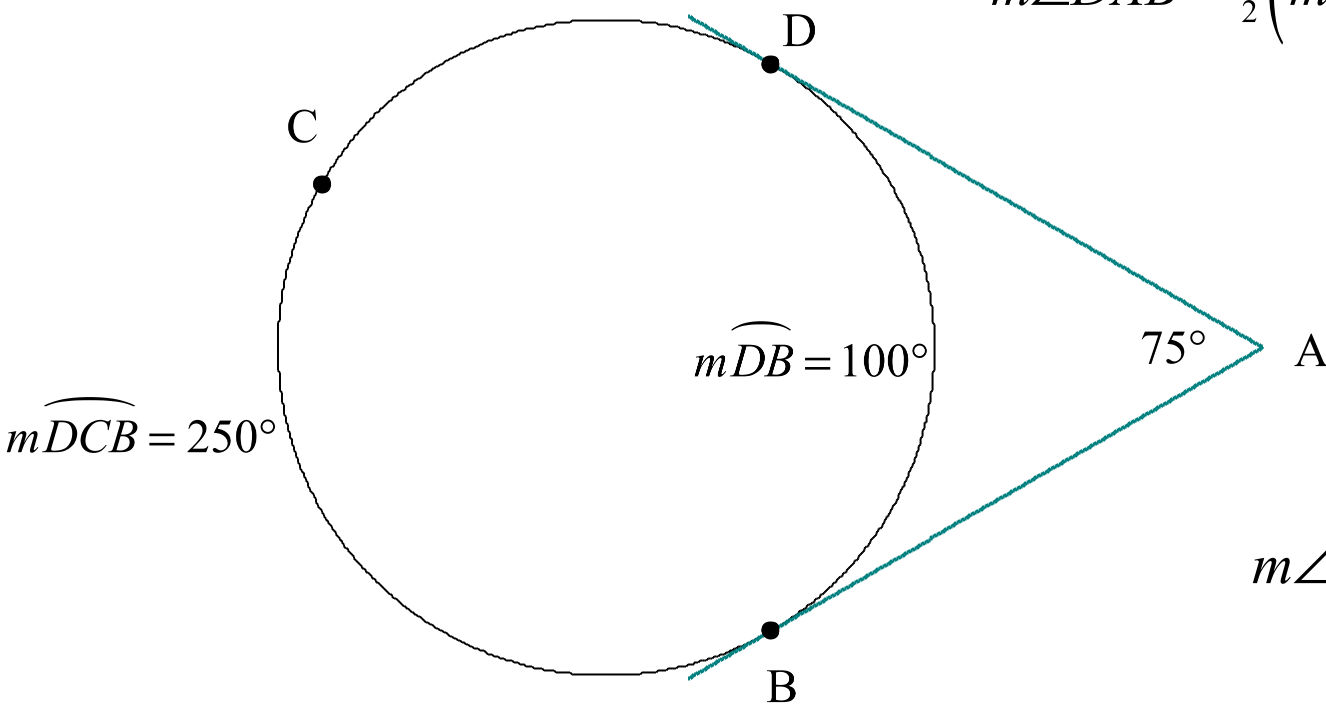
In other words

$$m\angle 1 = \frac{1}{2}\left(m\widehat{AD} + m\widehat{BC}\right)$$

$$m\angle 1 = 80°$$

Once we get to angles formed outside the circle (pg 832) it becomes half the *difference between the outer and inner arc measurements*. In other words,

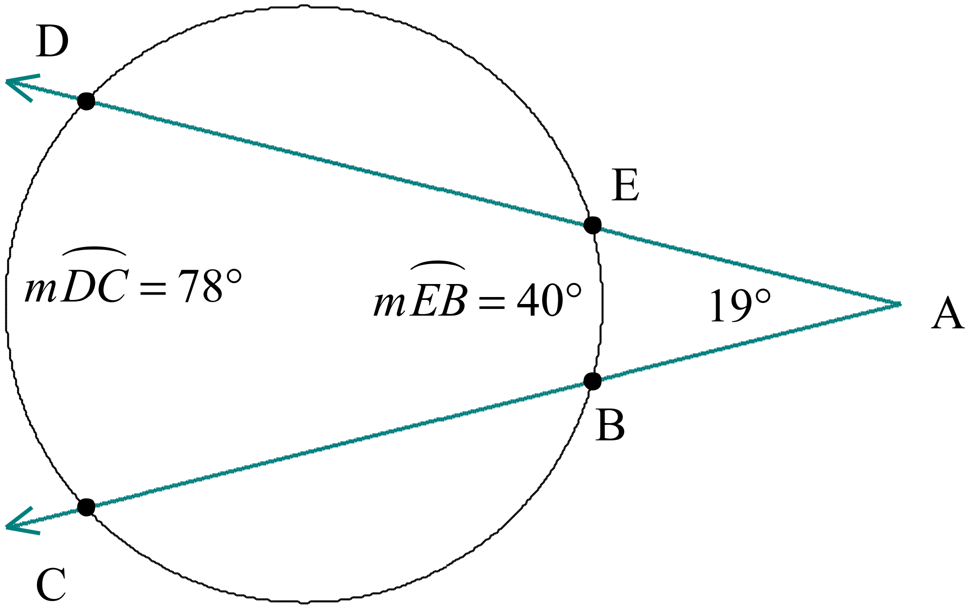
$$m\angle DAB = \frac{1}{2}\left(m\overset{\frown}{DCB} - m\overset{\frown}{DB}\right)$$




$$m\angle DAB = 75°$$

The same idea applies here

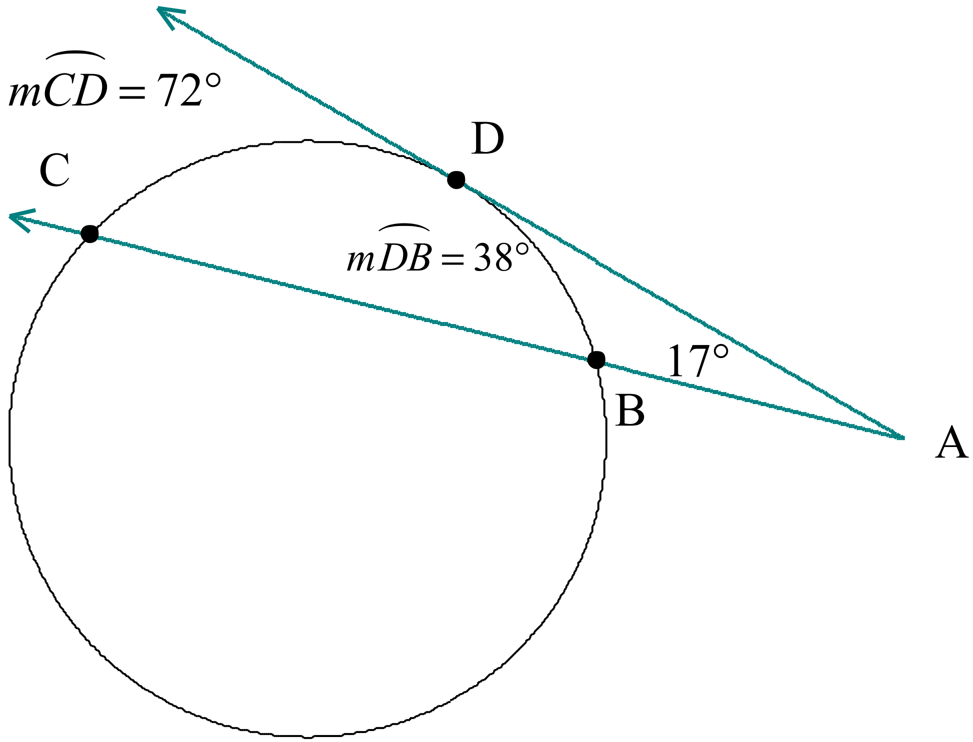
$$m\angle EAB = \tfrac{1}{2}\left(m\overset{\frown}{DC} - m\overset{\frown}{EB}\right)$$




$$m\angle EAB = 19°$$

And here as well

$$m\angle DAB = \frac{1}{2}\left(m\overset{\frown}{CD} - m\overset{\frown}{BD}\right)$$




$$m\angle DAB = 17°$$

Page 833 sums all of this up