Limits

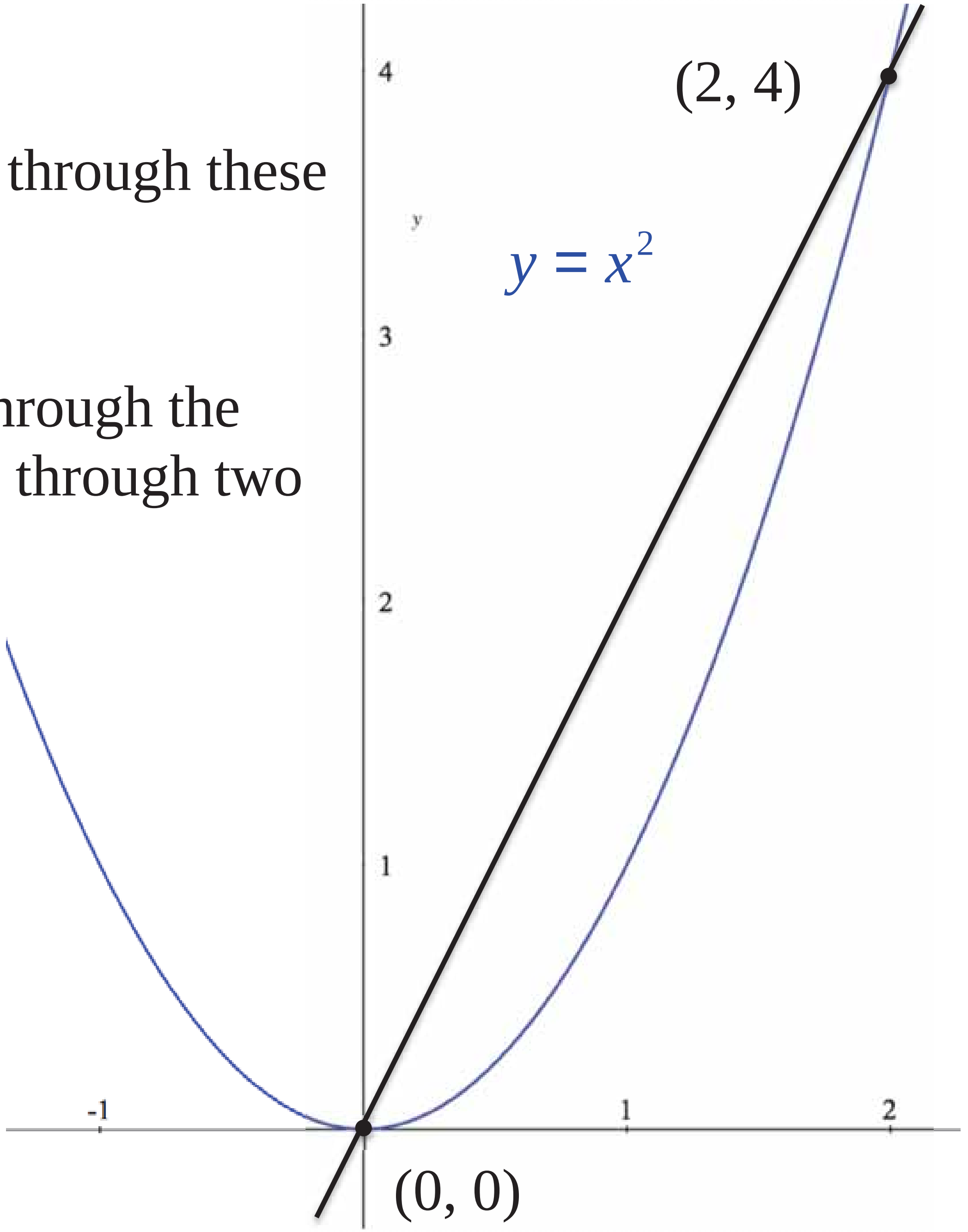
What is the slope of the line through these two points?

$$m = \frac{4-0}{2-0} = 2$$

This is called a *secant line* through the curve $y = x^2$ because it passes through two points on the curve.

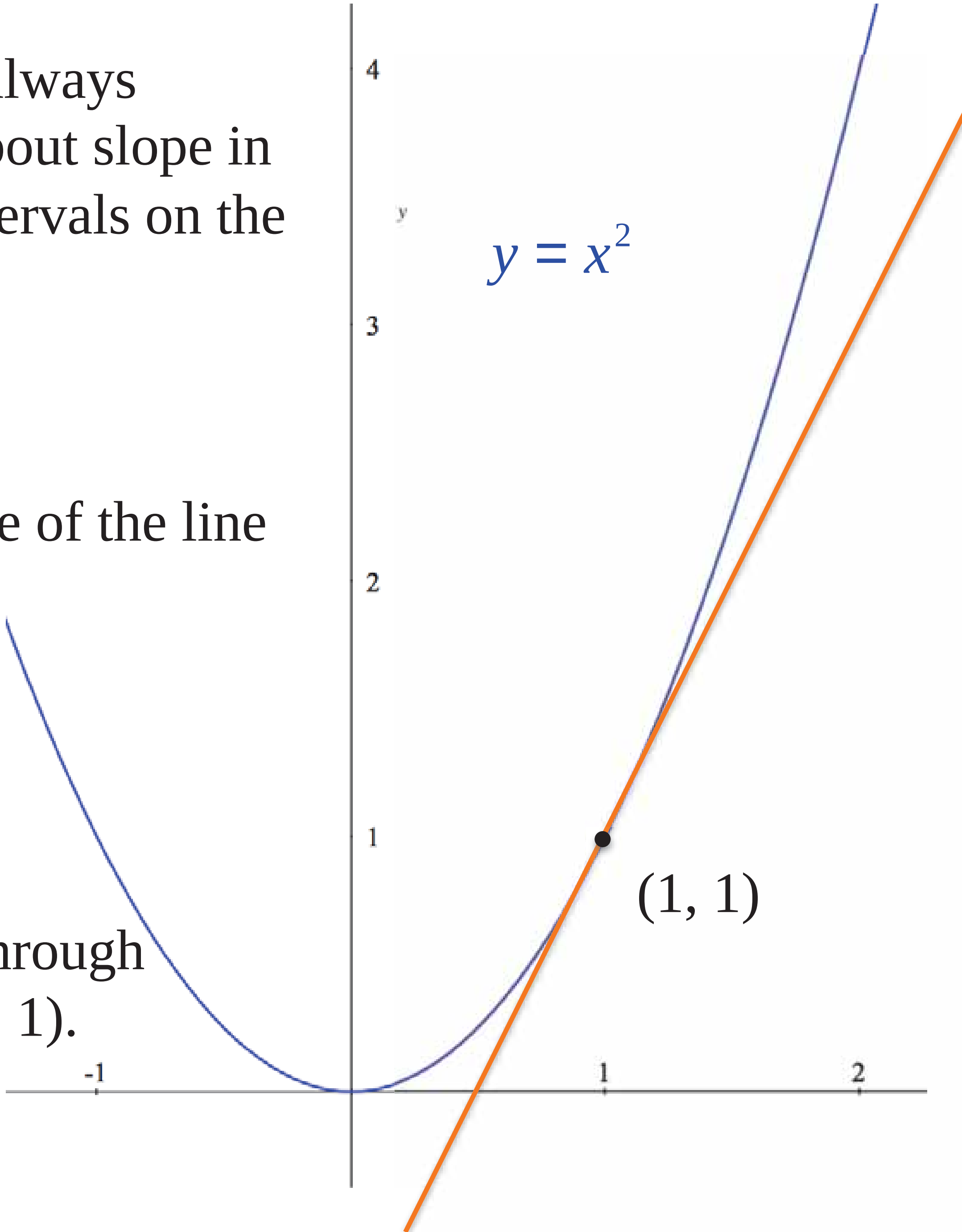
Finding the slope of a line is easy

What about finding the slope of a *curve*?



Since the slope of a curve is always changing, we can only talk about slope in terms of specific points or intervals on the curve

For example, what is the slope of the line through this point?



This is called a *tangent line* through the curve $y = x^2$ at the point (1, 1).

Since the slope of a curve is always changing, we can only talk about slope in terms of specific points or intervals on the curve

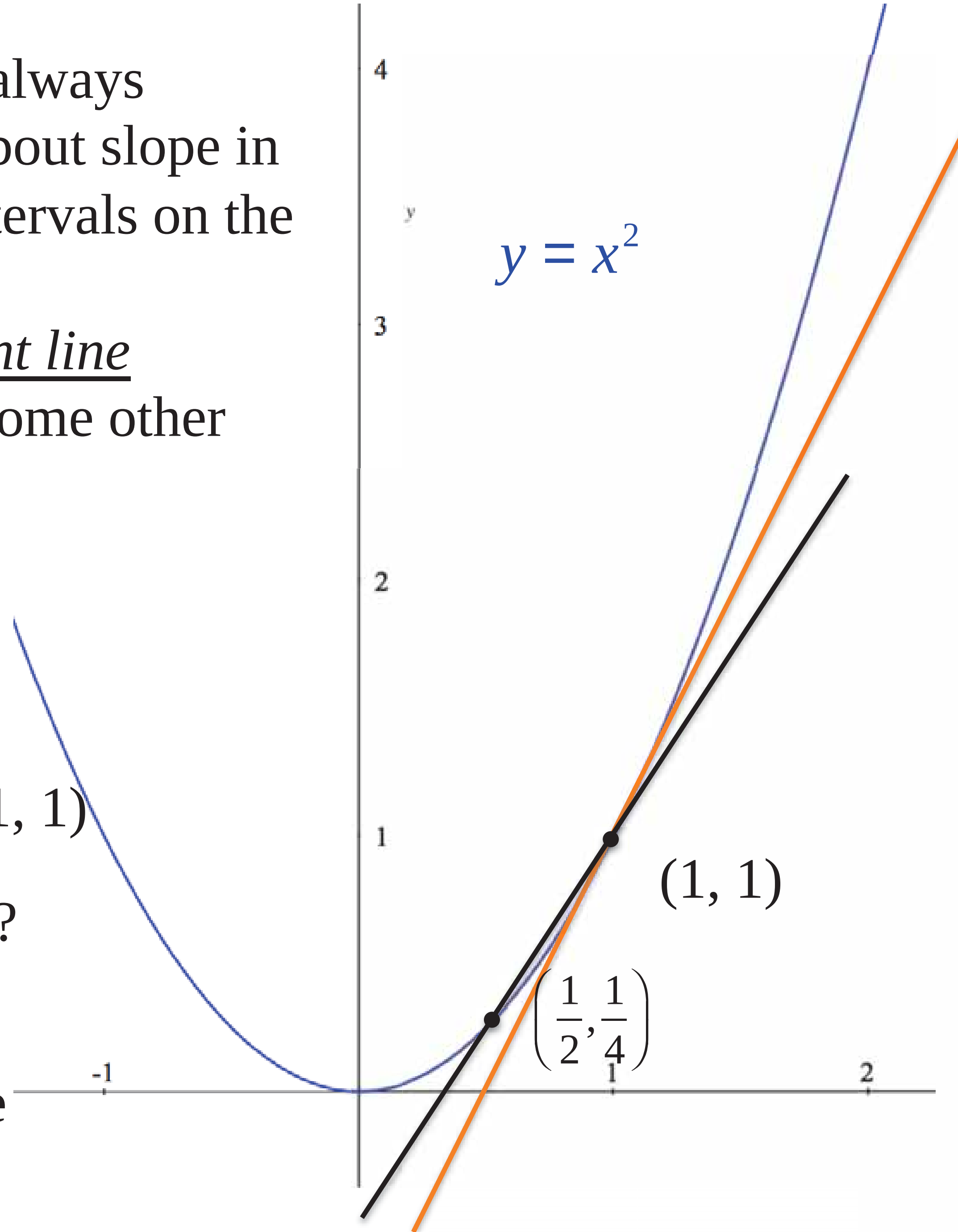
Let's start by drawing a *secant line* through the point (1, 1) and some other point close to it.

$$m = \frac{1-\frac{1}{4}}{1-\frac{1}{2}} = \frac{3}{2}$$

This is close to the slope at (1, 1)

How much closer can we get?

That's where limits come in. We'll come back to this slope of a curve idea later…



What exactly is a limit?

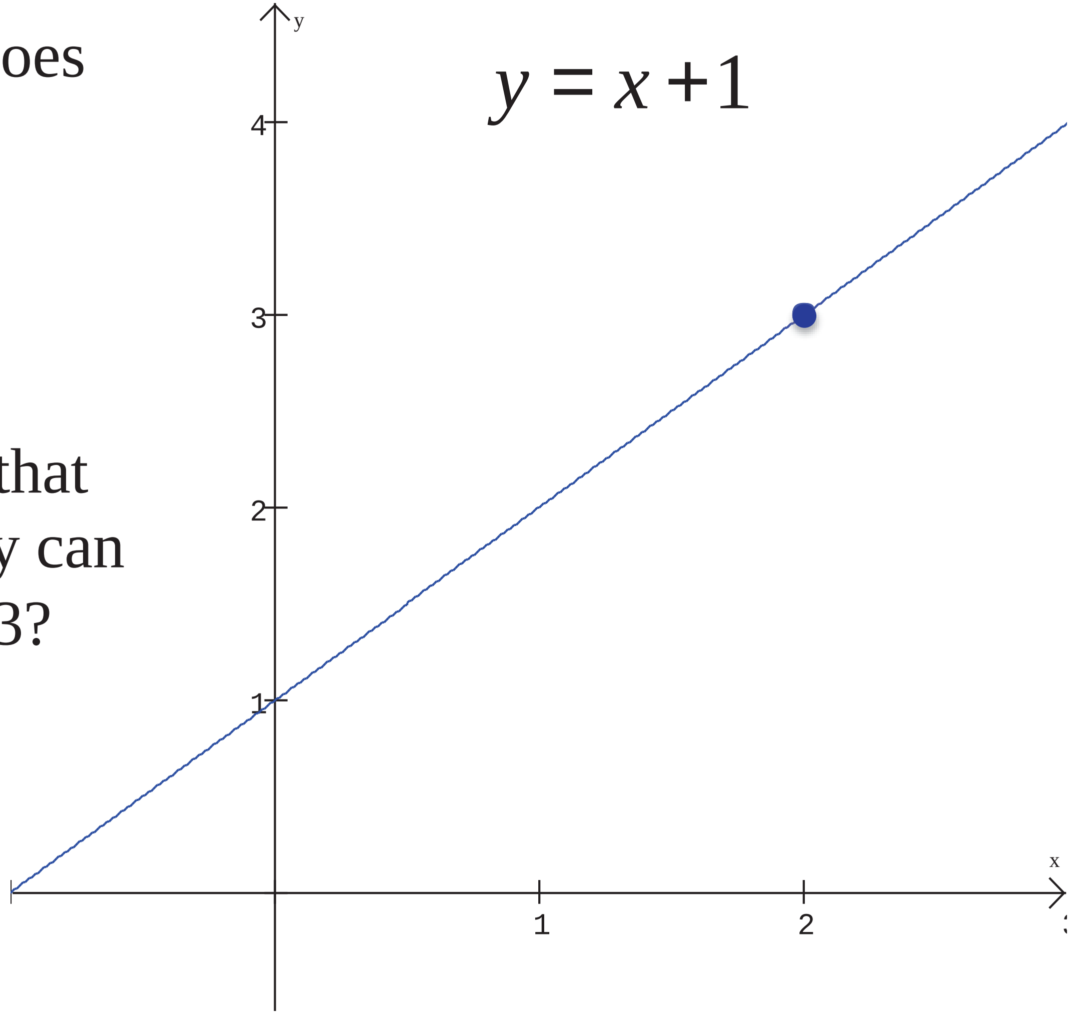
…and what can we do with limits?

…we'll answer the first question now but the second one will have to wait until later

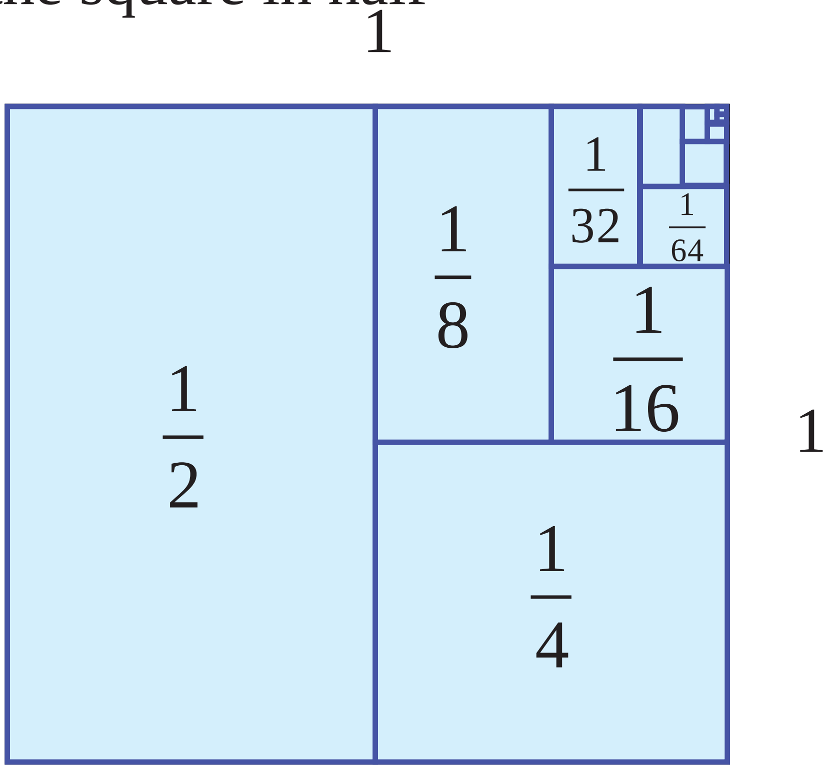
As *x* approaches 2, what does *y* approach?

3

But why would we ask that question when we already can see that it approaches 3?



Given a square of area 1, divide the square in half



Since the area of all of these slices of the square add up to the area of the square…

But since we could actually keep doing this forever, we won't ever actually reach 1.

But the limit is 1

If I stand in the middle of a room and walk forward 5 feet, then every step I take afterwards is half the distance of the previous step, what is the ***limit*** of the distance I will cover?

$$5+\left(\frac{1}{2}\right)5+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)5+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)5\ldots$$

First Step

Second Step

Third Step

Fourth Step

$$5\left[1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{3}\ldots\right]=10$$

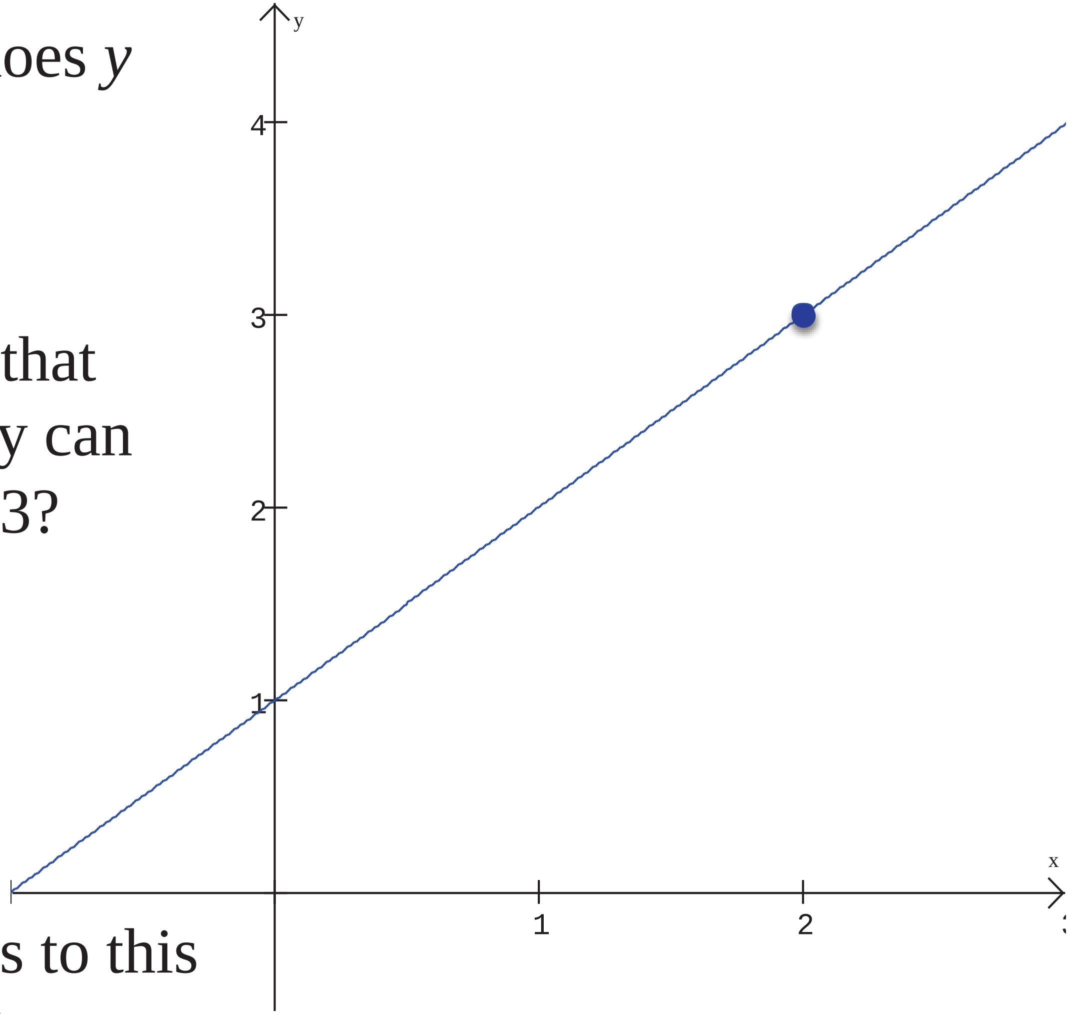
1

As x approaches 2, what does y approach?

3

But why would we ask that question when we already can see that it approaches 3?

$$y = \frac{x^2 - x - 2}{x - 2}$$



Notice what happens to this function at 2

$$y = \frac{0}{0}$$

Which is called an Indeterminate Form of a Number (pg 73)

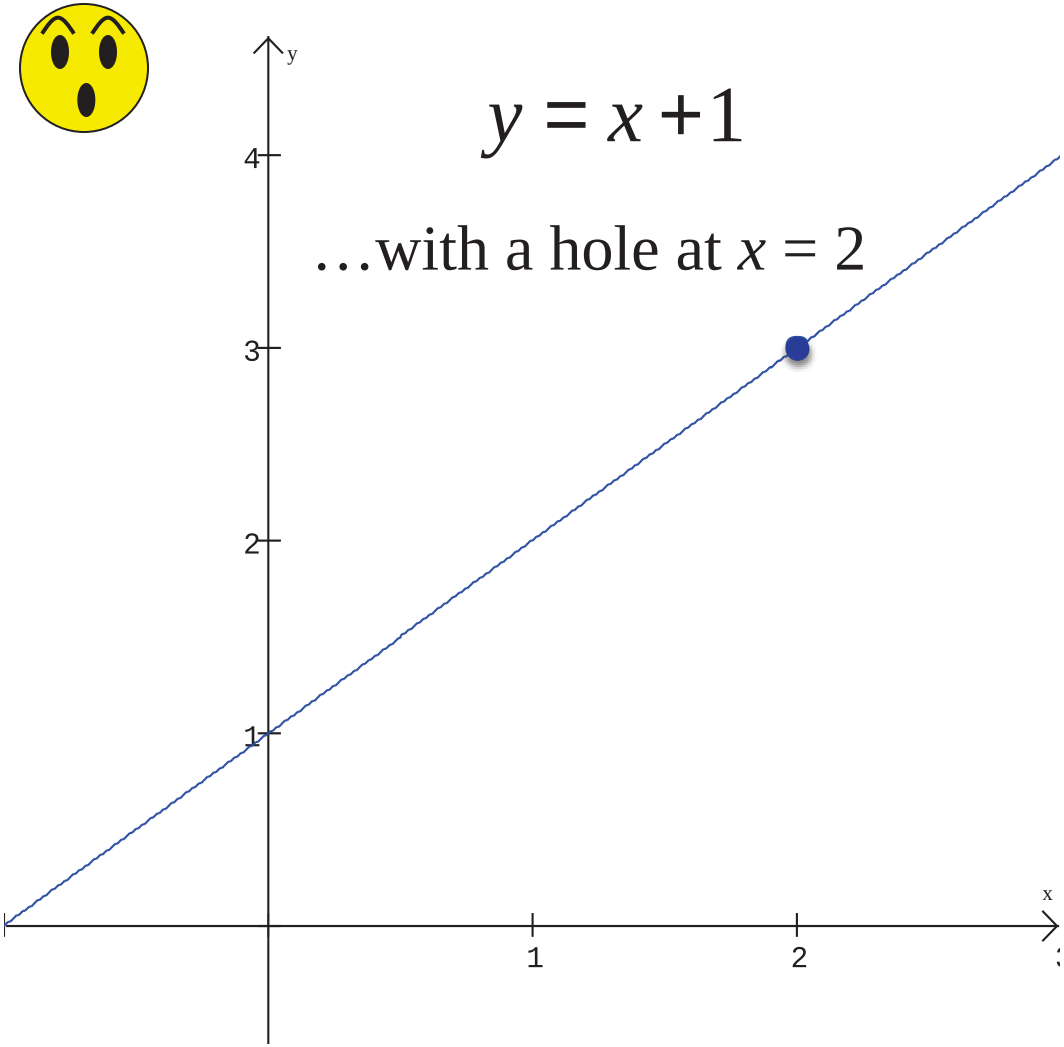
So what do we do?

$$y = \frac{x^2 - x - 2}{x - 2}$$

$$y = \frac{(x-2)(x+1)}{x-2}$$

$$y = (x+1)$$

$$= 3$$



$y = x + 1$

…with a hole at $x = 2$

And what does the graph look like?

So what do we do?

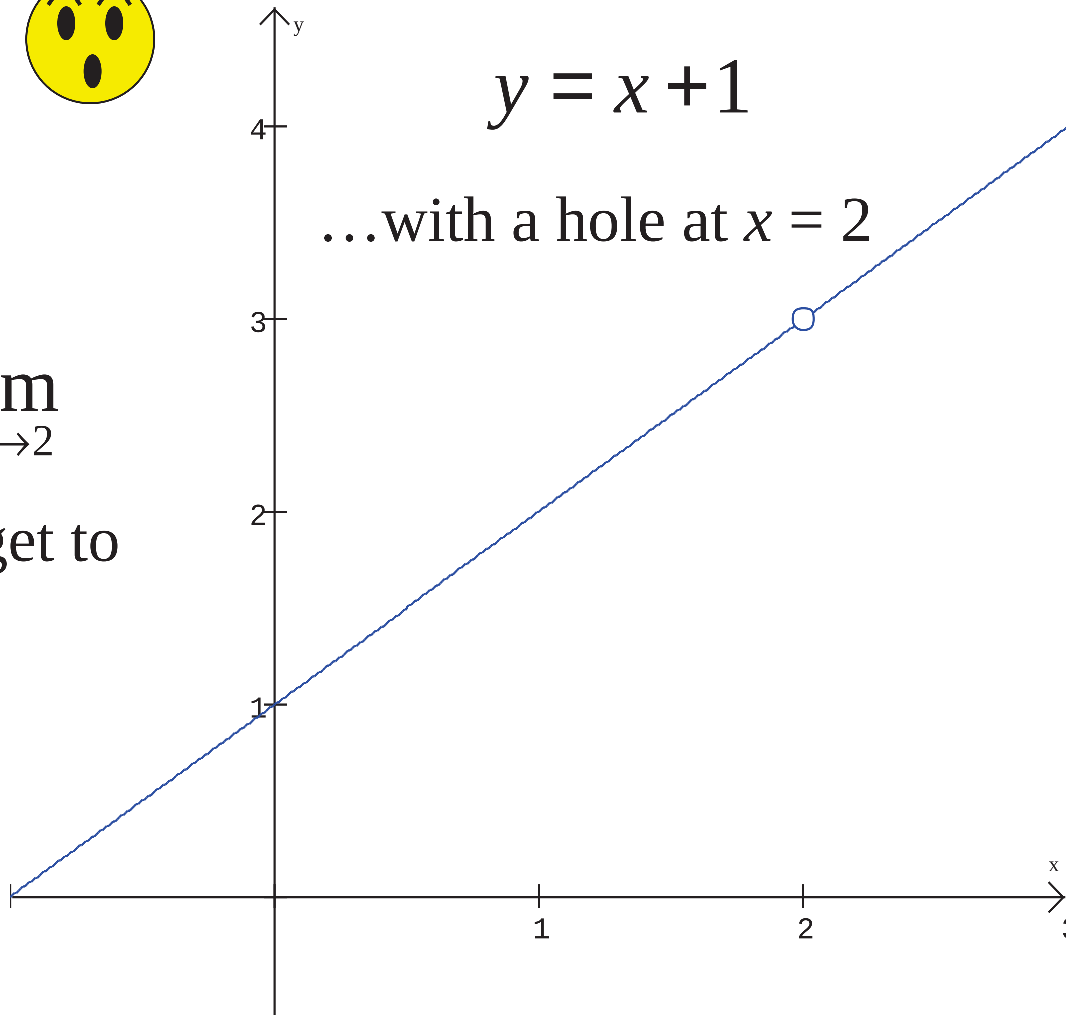
$$\lim_{x\to 2}\frac{x^2-x-2}{x-2}$$

Before you ask what $\lim\limits_{x\to 2}$ means, wait because I'll get to it…

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$$

$$=\lim_{x\to 2}(x+1)$$

$$=3$$



Limit notation: $\lim_{x \to a} f(x) = L$

"The limit of f of x as x approaches a is L."

Finding $\lim_{x \to a} f(x)$

1) Plug in a. If it works (meaning you don't get 0/0), you're done. If not…
2) Factor, cancel, and then plug in a

It's that simple

Let's do another one

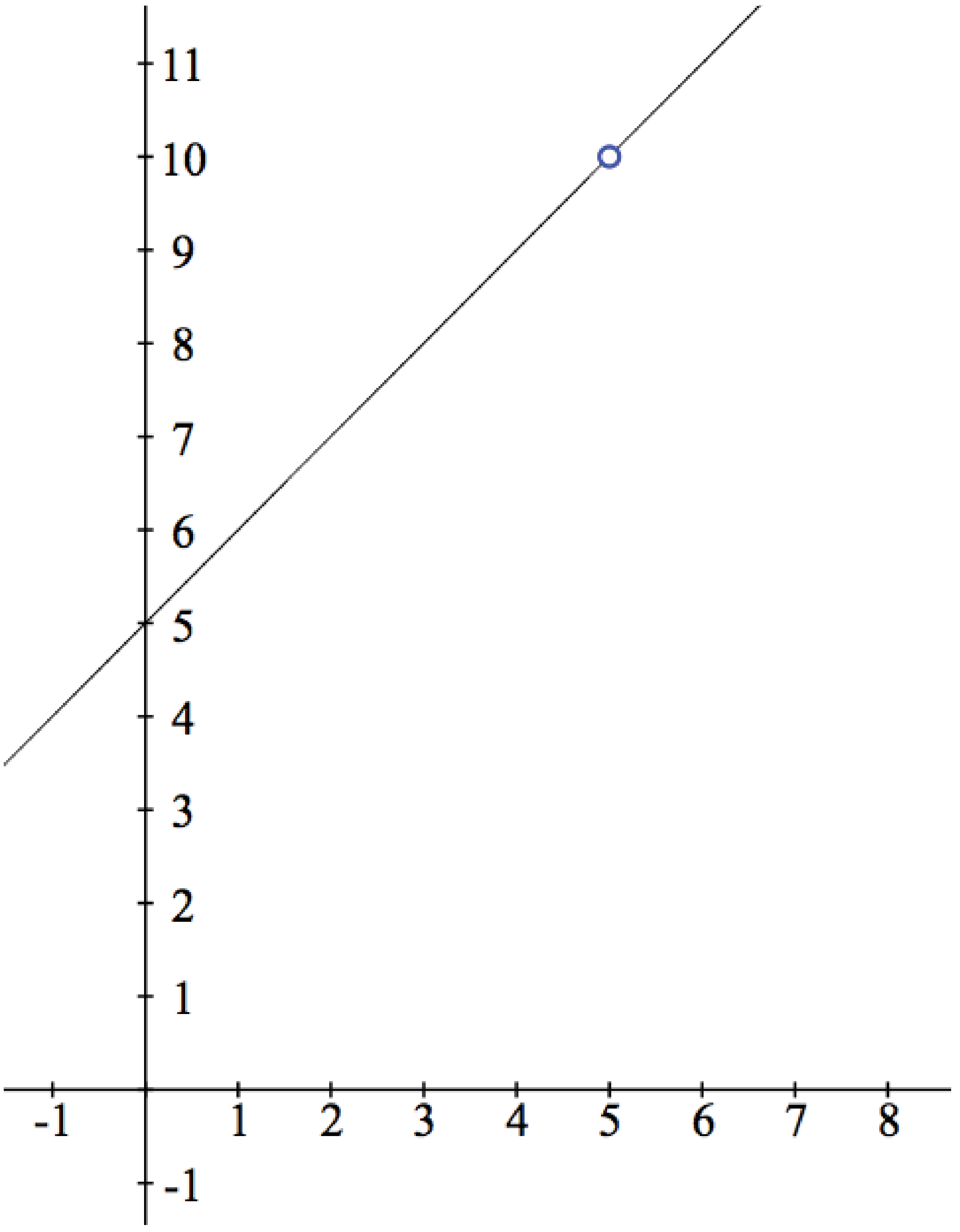
$$\lim_{x\to 5}\frac{x^2-25}{x-5}$$

$$\frac{0}{0}$$

$$\lim_{x\to 5}\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}}$$

$$\lim_{x\to 5} x+5$$

10



Not challenging enough?

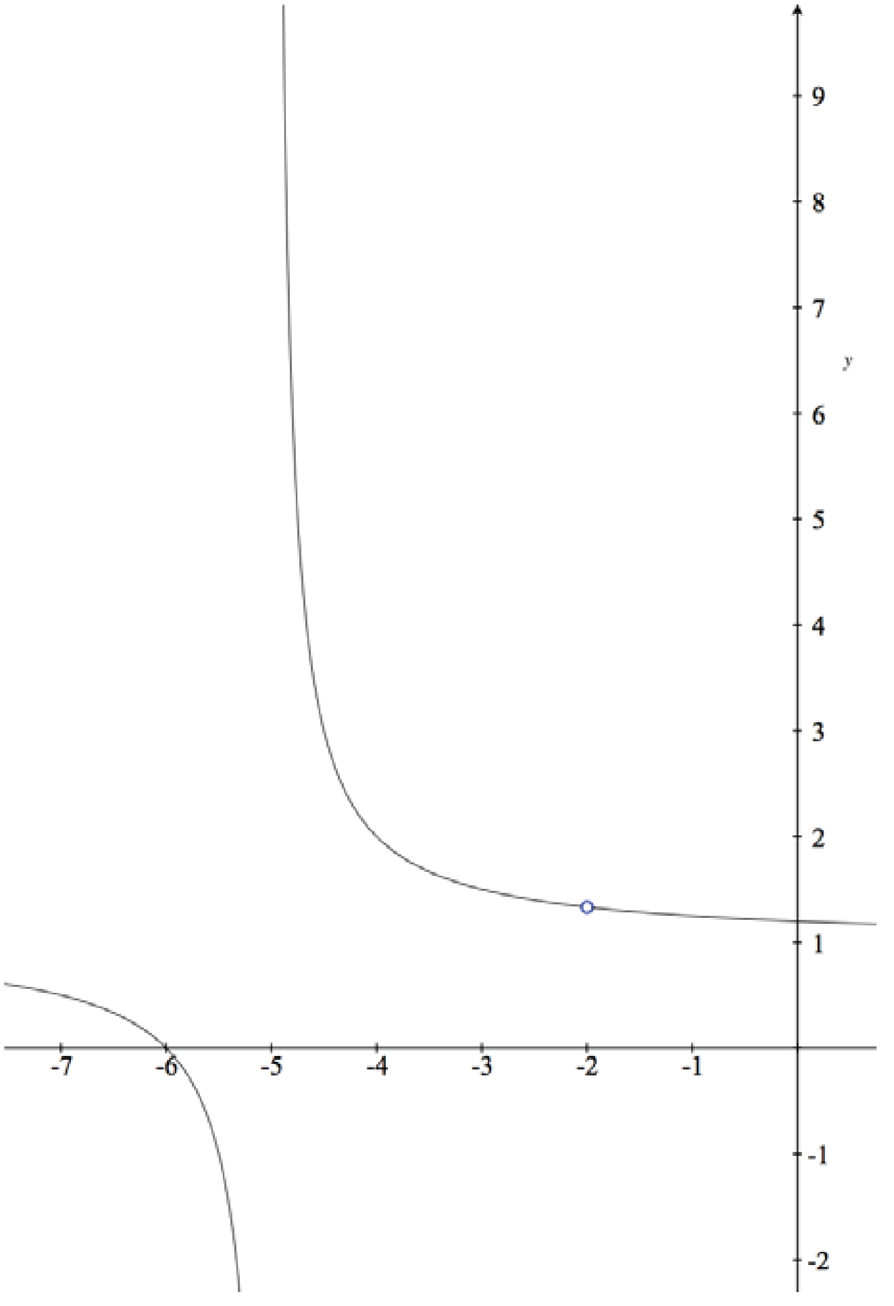
$$\lim_{x \to -2} \frac{x^2 + 8x + 12}{x^2 + 7x + 10}$$

$$\frac{0}{0}$$

$$\lim_{x \to -2} \frac{(x+2)(x+6)}{(x+2)(x+5)}$$

$$\lim_{x \to -2} \frac{(x+6)}{(x+5)}$$

$$\frac{4}{3}$$



Let's try one more just to keep you thinkin'

$$\lim_{x\to 2}\frac{\sqrt{4-x}-\sqrt{2}}{x-2}$$

$$\frac{0}{0}$$

Multiply top and bottom by the conjugate

$$\lim_{x\to 2}\frac{\sqrt{4-x}-\sqrt{2}}{x-2}\text{ i }\frac{\sqrt{4-x}+\sqrt{2}}{\sqrt{4-x}+\sqrt{2}}$$

$$\lim_{x\to 2}\frac{4-x-2}{(x-2)\left(\sqrt{4-x}+\sqrt{2}\right)}$$

$$\lim_{x\to 2}\frac{-x+2}{(x-2)\left(\sqrt{4-x}+\sqrt{2}\right)}$$

$$\lim_{x\to 2}\frac{\cancel{-(x-2)}}{\cancel{(x-2)}\left(\sqrt{4-x}+\sqrt{2}\right)}$$

$$\lim_{x\to 2}\frac{-1}{\left(\sqrt{4-x}+\sqrt{2}\right)}$$

$$-\frac{1}{2\sqrt{2}}$$

Now graph this on your calculator