

Remember these from last time?

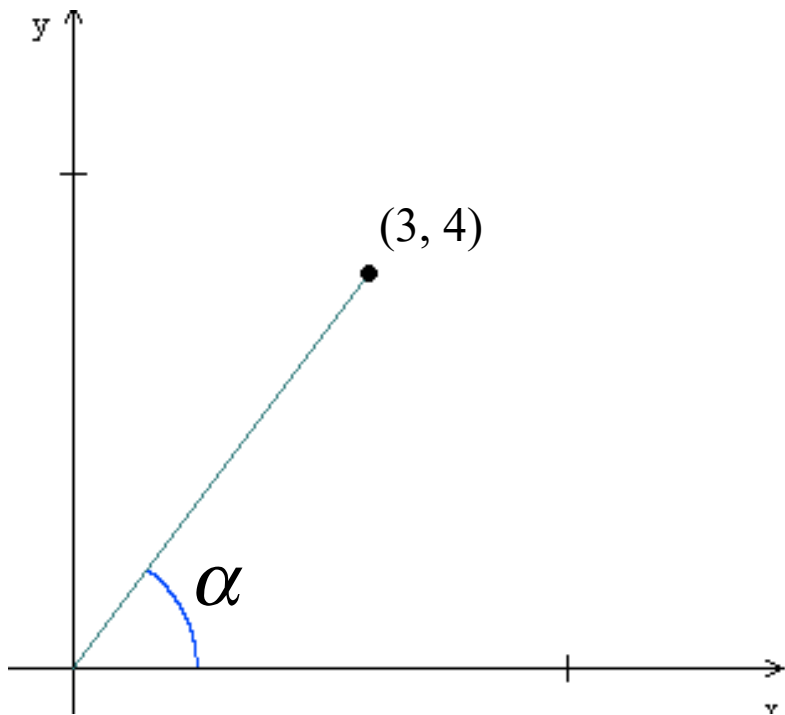
These are called the **Double Angle Identities for Sine & Cosine** (pg 148)

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

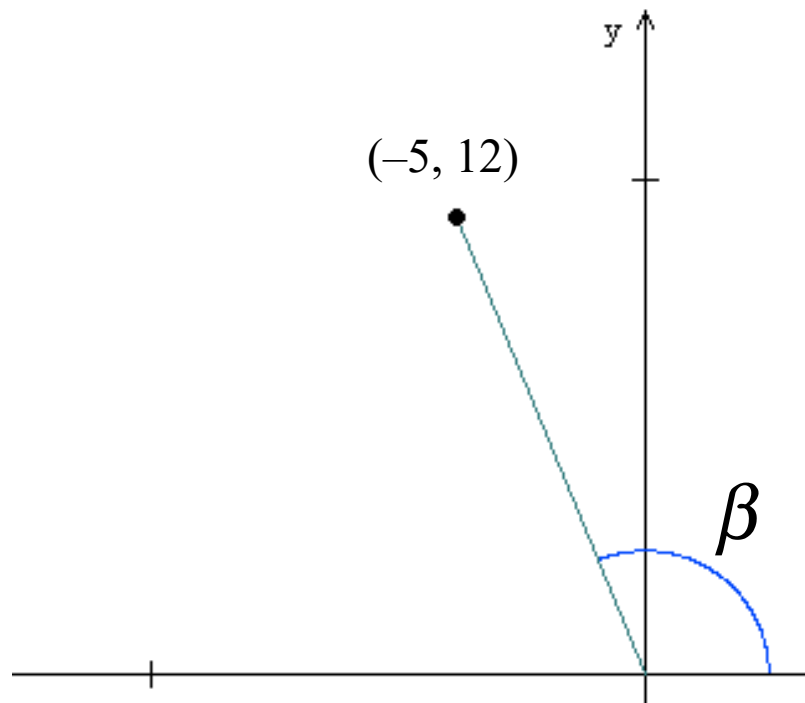
$$\cos(2A) = 2 \cos^2 A - 1$$

$$\cos(2A) = 1 - 2 \sin^2 A$$



$$\sin \alpha = \frac{4}{5}$$

$$\cos \alpha = \frac{3}{5}$$



$$\sin \beta = \frac{12}{13}$$

$$\cos \beta = -\frac{5}{13}$$

Use the answers you have and the composite identities to solve the given problems

$$\sin \alpha = \frac{4}{5} \qquad \cos \alpha = \frac{3}{5} \qquad \sin \beta = \frac{12}{13} \qquad \cos \beta = -\frac{5}{13}$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$$

$$\sin(2\beta) = 2 \sin \beta \cos \beta = 2 \cdot \frac{12}{13} \cdot \left(-\frac{5}{13}\right) = -\frac{120}{169}$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

$$\cos(2\beta) = \cos^2 \beta - \sin^2 \beta = \left(-\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$$

This is consistent with  $2\alpha$  being a 2<sup>nd</sup> quadrant angle

This is consistent with  $2\beta$  being a 3<sup>rd</sup> quadrant angle

The **Double Angle Identity for Cosine** can be manipulated like this:

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$

$$\cos(2A) = \cos^2 A - 1 + \cos^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\frac{\cos(2A) + 1}{2} = \cos^2 A$$

$$\cos A = \pm \sqrt{\frac{1 + \cos(2A)}{2}}$$

The **Double Angle Identity for Cosine** can be manipulated like this:

$$\frac{\cos(2A) + 1}{2} = \cos^2 A$$

$$\cos A = \pm \sqrt{\frac{1 + \cos(2A)}{2}}$$

$$\cos A = \pm \sqrt{\frac{1 + \cos(2A)}{2}}$$

$$\sin A = \pm \sqrt{\frac{1 - \cos(2A)}{2}}$$

The **Half Angle Identities**

# The Half Angle Identities

$$\cos A = \pm \sqrt{\frac{1 + \cos(2A)}{2}}$$

$$\sin A = \pm \sqrt{\frac{1 - \cos(2A)}{2}}$$

These Identities become essential in Calculus especially without the radical as below

$$\cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$

