Conditional Probability

Example: Drawing a King or a Heart

Drawing a card from the top of a randomly shuffled deck, each card has a probability of $\frac{1}{52}$ of being drawn.

$P(\text{K})$ = Probability of drawing a king is $\frac{4}{52} = \frac{1}{13}$

because…

4 Kings

52 Cards

Example: Drawing a King or a Heart

Drawing a card from the top of a randomly shuffled deck, each card has a probability of $\frac{1}{52}$ of being drawn.

$P(\heartsuit)$ = Probability of drawing a heart is $\frac{13}{52} = \frac{1}{4}$

because…

13 Hearts

52 Cards

Example: Drawing a King or a Heart

So what is the probability of drawing a King *or* a Heart?

P(K) = Probability of drawing a king is

$$\frac{4}{52} = \frac{1}{13}$$

While P(♥) = Probability of drawing a heart is

$$\frac{13}{52} = \frac{1}{4}$$

?

Example: Drawing a King or a Heart

So what is the probability of drawing a King *or* a Heart?

Note that there are 17 cards but…

2 King of Hearts

$P(\text{K or } \heartsuit) = 4/52 + 13/52 - 1/52$

$P(\text{K or } \heartsuit) = 16/52$

So we end up with this formula:

P(K or ♥) = P(K) + P(♥) – P(K and ♥)

…and that leads us to the general formula:

The Probability of the Event {*A* or *B*} when *A* and *B* are Not Disjoint

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If we already knew that the card was a Heart what would the probability be that it is a King?

This is called **Conditional Probability.** In other words, what is the probability that the card is a King given that it is a Heart?

The notation for **Conditional Probability** is that the probability of A given that B is true can be given by $P(A \mid B)$

In this case we will be finding $P(K \mid \heartsuit)$

$$P(K \mid \heartsuit) = \frac{P(K \cap \heartsuit)}{P(\heartsuit)}$$

$P(K \cap \heartsuit) \longrightarrow$ King and Heart

$P(\heartsuit) \longrightarrow$ Heart

$$P(K \mid \heartsuit) = \frac{1/52}{13/52} = \frac{1}{13}$$

If we already knew that the card was a Heart what would the probability be that it is a King?

This is called **Conditional Probability.** In other words, what is the probability that the card is a King given that it is a Heart?

This makes sense because there are 13 hearts and one of them is a king

$$P(K \mid \heartsuit) = \frac{1/52}{13/52} = \frac{1}{13}$$

13 Hearts

52 Cards

The Conditional Probability of the Event {*A* given that *B* has occured} when *A* and *B* are Not Disjoint

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Probability of *A* given *B* has occured

Probability of *A* and *B* divided by the probability of *B*

So are the king and the heart *independent events*?

Notice what happens when we compare two particular probabilities here:

$$P(K) = \frac{1}{13} \qquad P(K \mid \heartsuit) = \frac{1}{13}$$

The fact that these two probabilities are equal shows that drawing a king is independent of drawing a king given that the card is a heart.

$$P(K \mid \heartsuit) = P(K)$$

The Conditional Probability of the Event {*A* given that *B* has occured} when *A* and *B* are Not Disjoint

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Probability of *A* given *B* has occured

Probability of *A* and *B* divided by the probability of *B*

So Conditional Probability has this corollary:

If the two events are independent

$$P(A \mid B) = P(A)$$

Suppose you are choosing at random from only the married women

P(Age 30-64 | Married Women) = The probability that among the married women, someone age 30-64 will be chosen

$$P(30-64 \mid Married) = \frac{P(Married\ \&\ 30-64)}{P(Married)} = \frac{43,808/103,870}{59,920/103,870}$$

Age and marital status of women in the U. S. (thousands)

	Age			
	18-29	**30-64**	**65 and over**	**Total**
Married	7,842	43,808	8,270	59,920
Never Married	13,930	7,184	751	21,865
Widowed	36	2,523	8,385	10,944
Divorced	704	9,174	1,263	11,141
Total	22,512	62,689	18,669	103,870

Source: Data for 1999 from the 2000 Statistical Abstract of the U.S.

Suppose you are choosing at random from only the married women

P(Age 30-64 | Married Women) = 43,808/59,920

Another way to look at it:

Since this condition means that we are only drawing from married women, we can also just change the denominator when considering the probability.

Age and marital status of women in the U. S. (thousands)

	Age			
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