Probability Rules:

- The sum of the probabilities for all possible outcomes in a sample space is 1.

- The probability of an outcome is a number between 0 and 1 inclusive. An outcome that always happens has probability 1. An outcome that never happens has probability 0.

- The probability of an outcome occurring equals 1 minus the probability that it doesn't occur.

- The probability that two mutually exclusive (disjoint) events occur is 0.

Strategies for Solving Probability Problems:

Draw a picture of the situation -

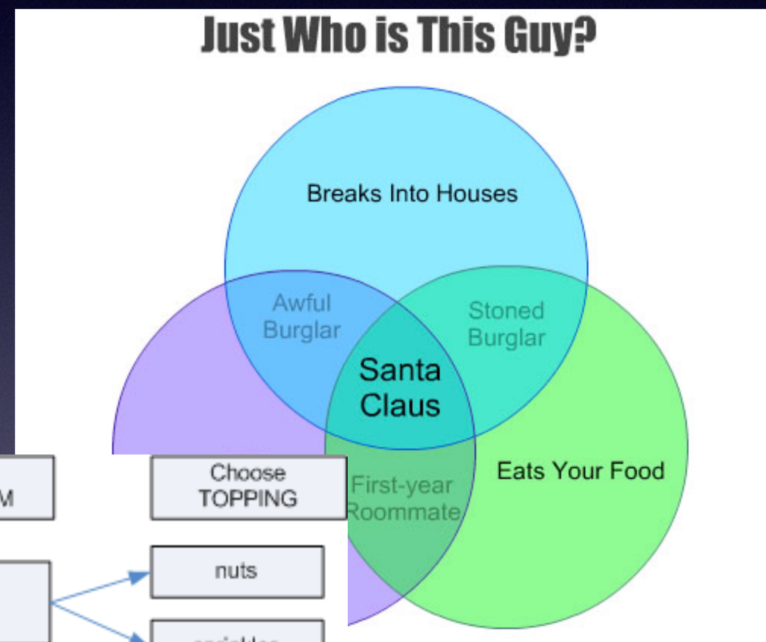
- Table/Charts

		Breakfast		
		Yes	No	Total
Sex	Male	66	66	132
	Female	125	74	199
			140	331

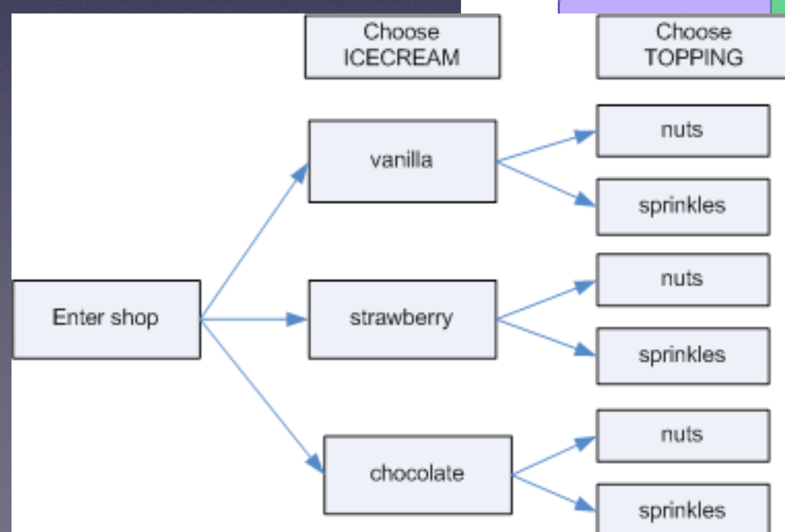
White	Black	Red	Silver	Gold
0.46	0.22	0.09	0.11	0.12

- Formulas

- Venn Diagram



- Tree Diagram



Formulas

Is there a formula on the AP formula sheet that applies?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Intersection in both equations - sometimes you will need to use both equations to solve one problem

Is there a formula/idea that is not on the AP formula sheet that applies?

If events are disjoint, then $P(A \cap B) = 0$

If events are independent, then $P(A|B) = P(A)$ or

$$P(A \cap B) = P(A)P(B)$$

Use these formulas when appropriate, i.e. based on what information is given

If events are independent, then

$$P(A|B)=P(A) \qquad \text{or} \qquad P(A\cap B)=P(A)P(B)$$

A fair coin is tossed twice. The four possible outcomes are listed below

		2nd toss	
	Two coin tosses	H_2	T_2
1st toss	H_1	H_1H_2	H_1T_2
	T_1	T_1H_2	T_1T_2

$P(H_1)=$ probability of 1st toss heads $=\frac{1}{2}$

$P(H_1\cap H_2)=$ probability of both tosses being heads

$=\frac{1}{4}$ only one of the four possibilities is two heads

$P(H_2 \mid H_1)=$ probability of 2nd toss being heads given that the 1st was heads

$=\frac{1}{2}$ two cases of heads on first toss with one outcome being heads on the second toss

$$P(H_2 \mid H_1)=P(H_2)=\frac{1}{2}$$

Since the conditional probability is the same, the events are independent

If events are independent, then

$$P(A|B) = P(A) \qquad \text{or} \qquad P(A \cap B) = P(A)P(B)$$

A fair coin is tossed twice. The four possible outcomes are listed below

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	Two coin tosses	H_2	T_2
1st toss	H_1	H_1H_2	H_1T_2
	T_1	T_1H_2	T_1T_2

$$P(H_2 \mid H_1) = P(H_2) = \frac{1}{2}$$

We can also look at it this way:

$$P(H_2 \mid H_1) = \frac{P(H_1 \cap H_2)}{P(H_1)}$$

$$\frac{P(H_1 \cap H_2)}{P(H_1)} = \frac{\text{only one way it can happen}}{\text{two of the four possibilities has a first toss heads}} = \frac{1/4}{1/2} = \frac{1}{2}$$

If events are independent, then

$$P(A|B) = P(A) \qquad \text{or} \qquad P(A \cap B) = P(A)P(B)$$

Now that we've established that each coin flip is independent of the others

$H_1H_2H_3$	$H_1H_2T_3$	$H_1T_2H_3$	$H_1T_2T_3$
$T_1H_2H_3$	$T_1H_2T_3$	$T_1T_2H_3$	$T_1T_2T_3$

Let's try the probability of three consecutive heads on three tosses

all eight possibilities are shown here

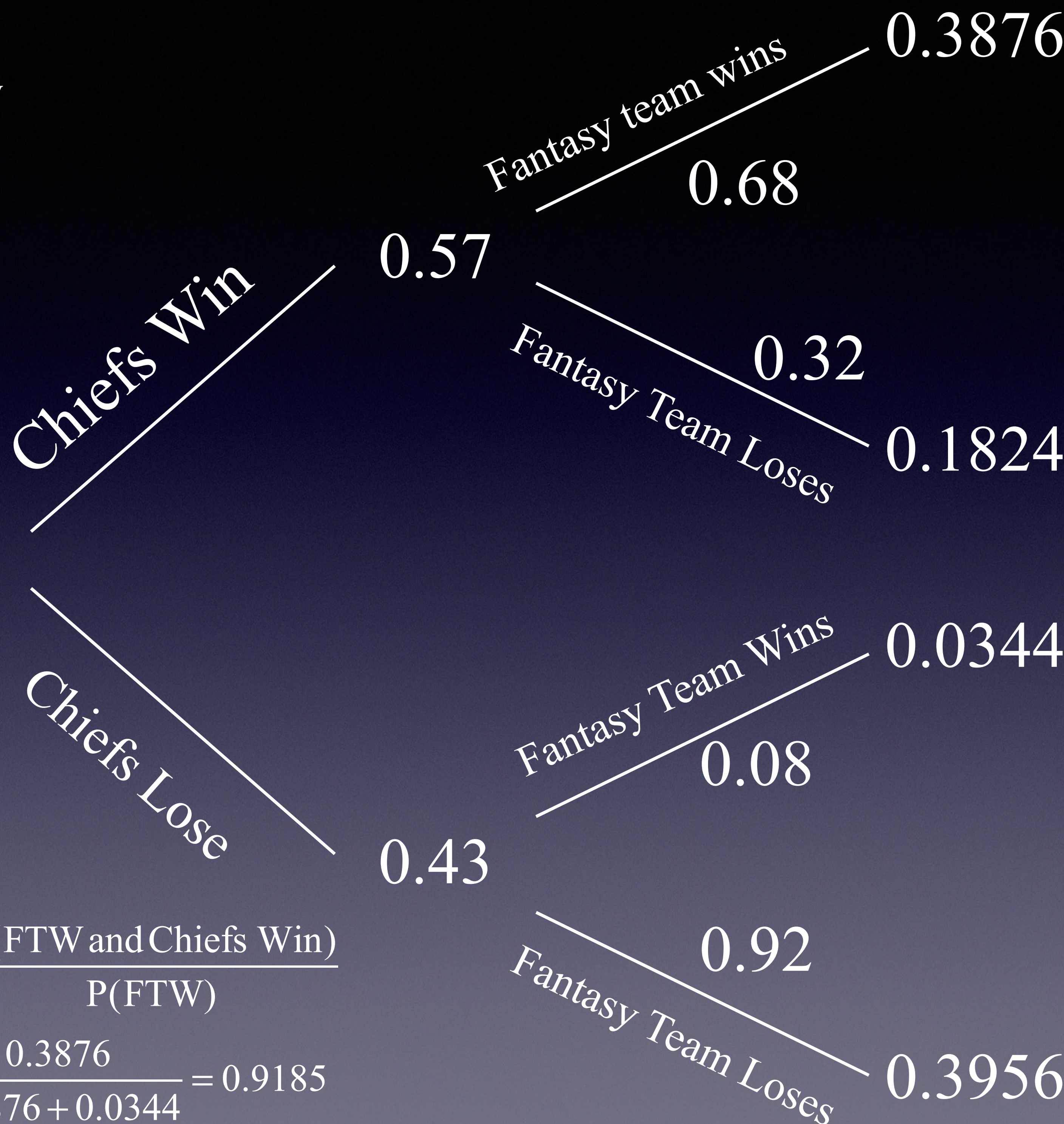
$$P(H_1 \cap H_2 \cap H_3)$$

only one way out of eight possibilities

and since each toss is independent

$$P(H_1 \cap H_2 \cap H_3) = P(H_1)P(H_2)P(H_3) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

The probability of the Chiefs winning this Sunday is given as 0.57. Raven's fantasy team has a probability of 0.68 of winning if KC wins and a probability of 0.08 when KC loses (because she stacked her team with Chiefs players). Use a tree diagram to find the probability that KC won given that Raven's fantasy team won.



$$P(\text{Chiefs Win} \mid \text{FTW}) = \frac{P(\text{FTW and Chiefs Win})}{P(\text{FTW})}$$

$$= \frac{0.3876}{0.3876 + 0.0344} = 0.9185$$

1. A proficiency examination for a certain skill was given to 100 employees of a firm. Forty of the employees were men. Sixty of the employees passed the examination (by scoring above a present level for satisfactory performance.) The breakdown of test results among men and women are shown in the accompanying diagram.

	Male (M)	Female (F)	Total
Pass (P)	24	36	60
Fail (F)	16	24	40
Total	40	60	100

Suppose that an employee is selected at random

from among the 100 who took the examination.

(a) Find the probability that the employee passed, given that he was a man.

$$P(\text{passed}|\text{man}) = \frac{P(\text{passed} \cap \text{man})}{P(\text{man})} = \frac{24/100}{40/100} = \frac{24}{40} = 0.60$$

(b) Find the probability that the employee was a man, given that a passing grade was received.

$$P(\text{man}|\text{passed}) = \frac{P(\text{man} \cap \text{passed})}{P(\text{passed})} = \frac{24/100}{60/100} = \frac{24}{60} = 0.40$$

(c) Are the events passing the exam and male independent?

$$P(\text{passed}|\text{male}) \stackrel{?}{=} P(\text{passed})$$

$$0.60 \stackrel{?}{=} \frac{60}{100}$$

$$0.60 = 0.60 \Rightarrow \text{independent}$$

(d) Are the events passing the exam and female independent?

$$P(\text{passed}|\text{female}) \stackrel{?}{=} P(\text{passed})$$

$$\frac{P(\text{passed} \cap \text{female})}{P(\text{female})} \stackrel{?}{=} P(\text{passed})$$

$$\frac{24/100}{40/100} \stackrel{?}{=} \frac{60}{100}$$

$$0.60 = 0.60 \Rightarrow \text{independent}$$