Multiplication Rule

If *A* and *B* are two independent events, then

$$P(A \text{ and } B) = P(A)*P(B)$$

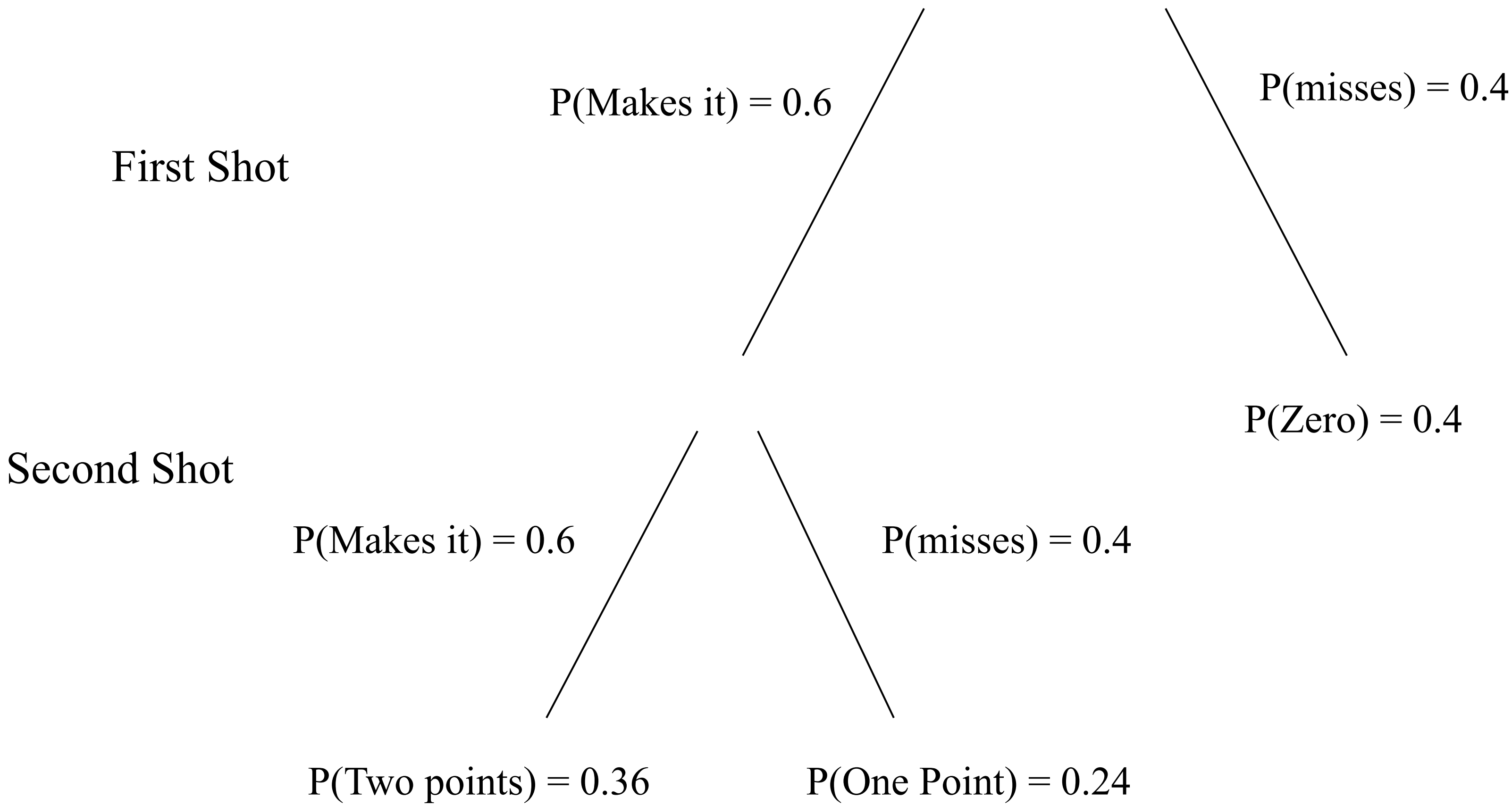
If *A* and *B* are two dependent events, then

$$P(A \text{ and } B) = P(A)*P(B \mid A)$$

AKA **The Conditional Independence Rule**

Let's revisit the one and one situation: A player with a free throw percentage of 60% goes to the line for a one and one. If he makes the first shot, he gets a second. If he misses the first shot, the ball is live. What is the most likely outcome: Zero, One point, or Two points.

To do this, make a tree diagram:



Multiplication Rule

If A and B are two independent events, then

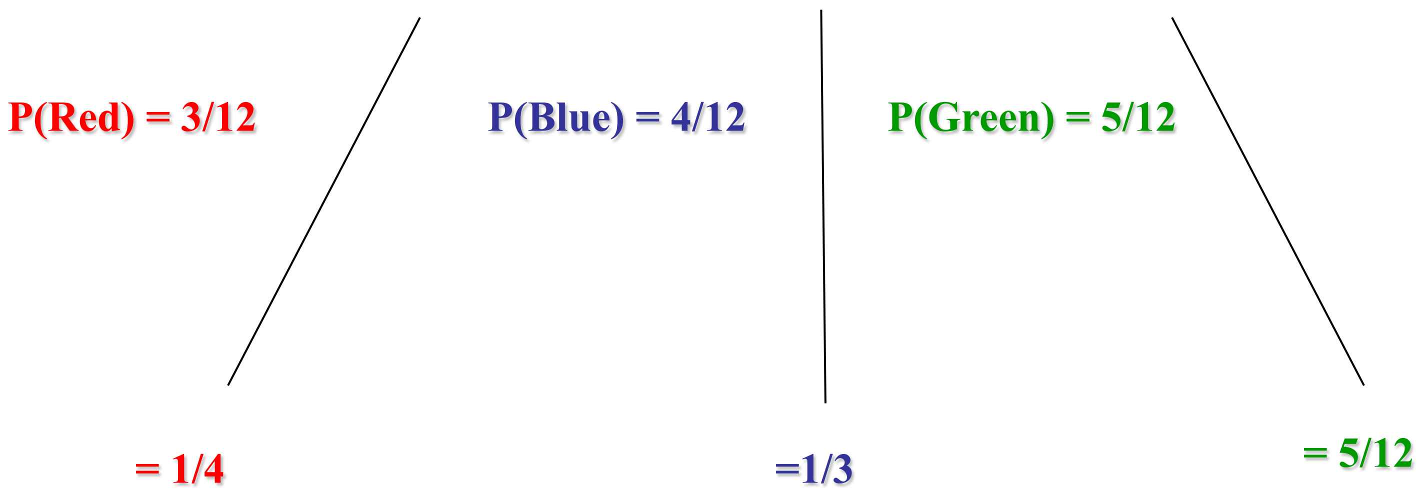
$$P(A \text{ and } B) = P(A)*P(B)$$

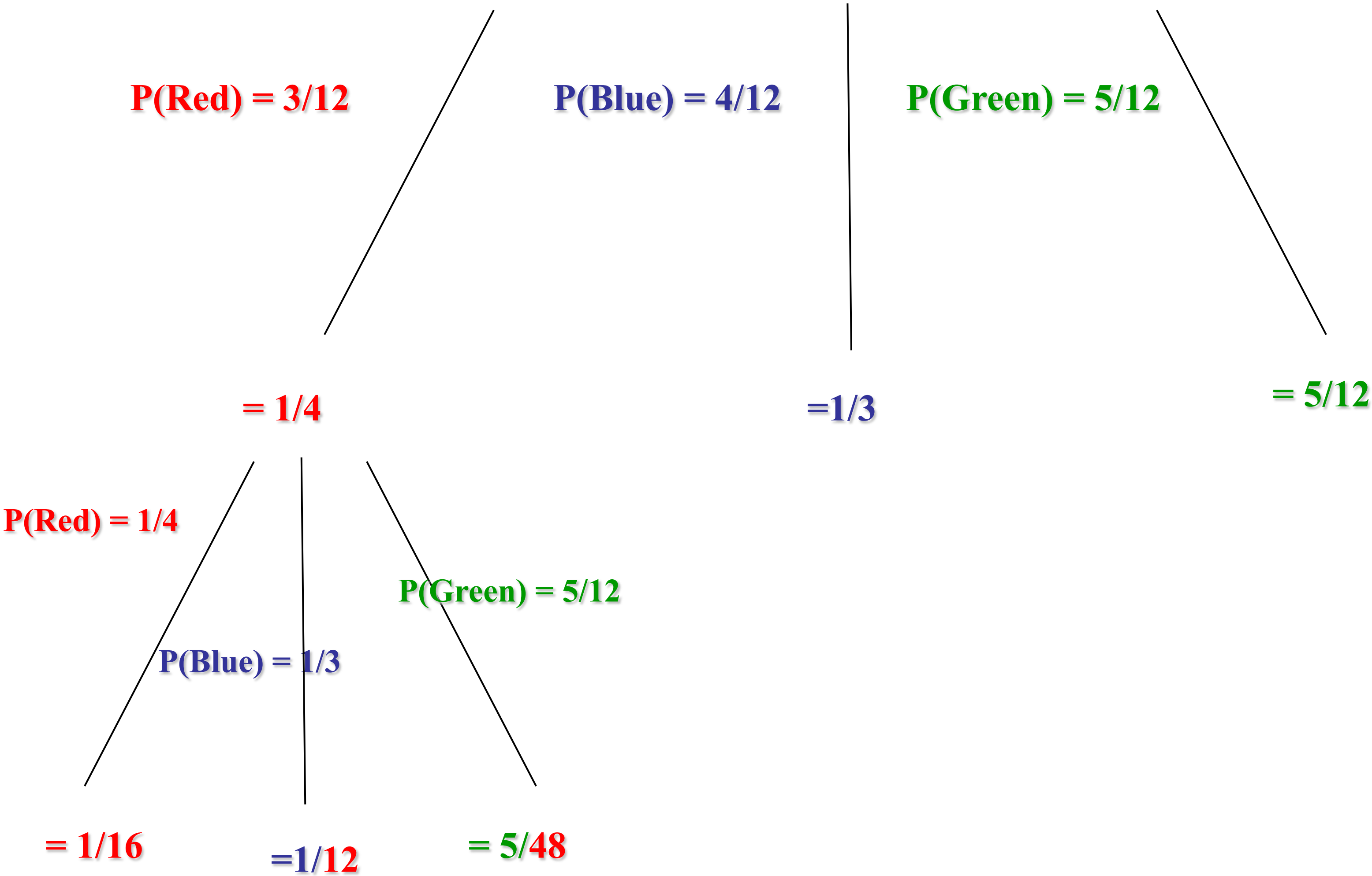
If A and B are two dependent events, then

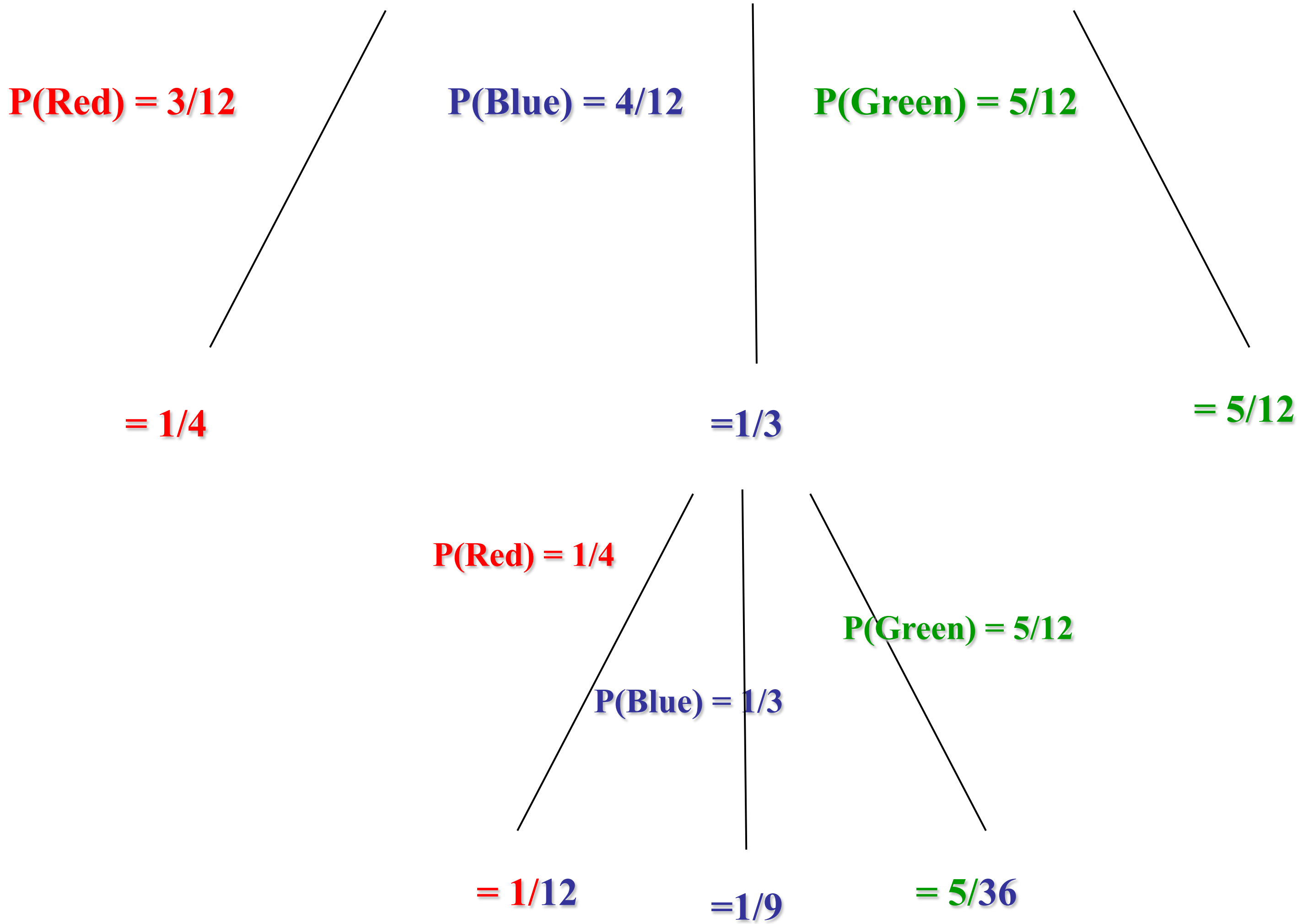
$$P(A \text{ and } B) = P(A)*P(B \mid A)$$

More
Tree Diagrams

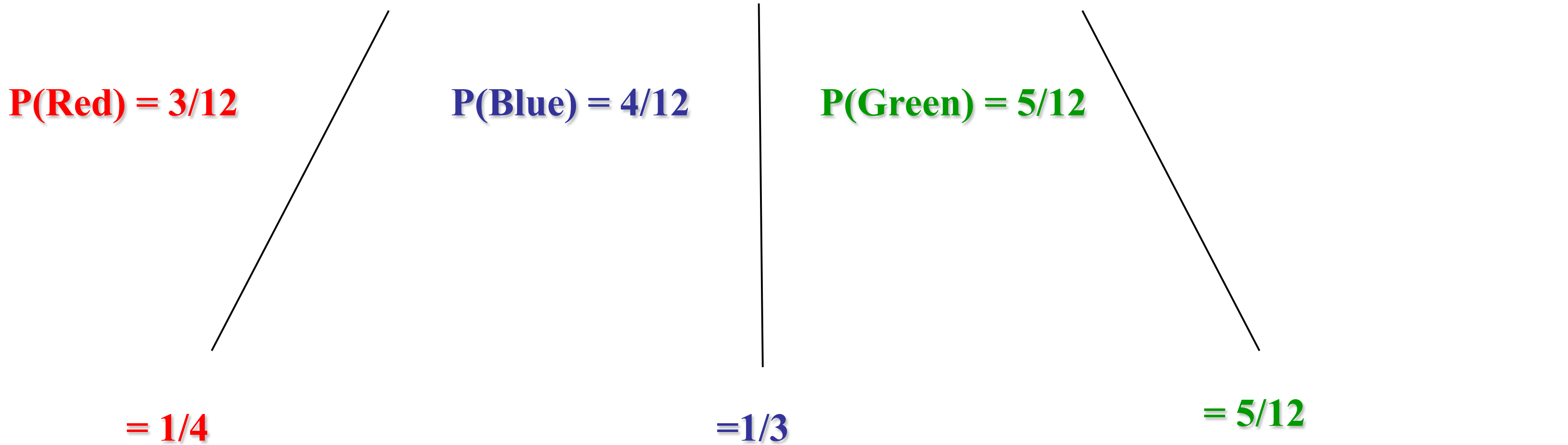
A jar has 3 red marbles, 4 blue marbles, and 5 green marbles. If you draw two marbles with replacement(meaning you put the first one back after drawing it), find all possible outcomes using a tree diagram.

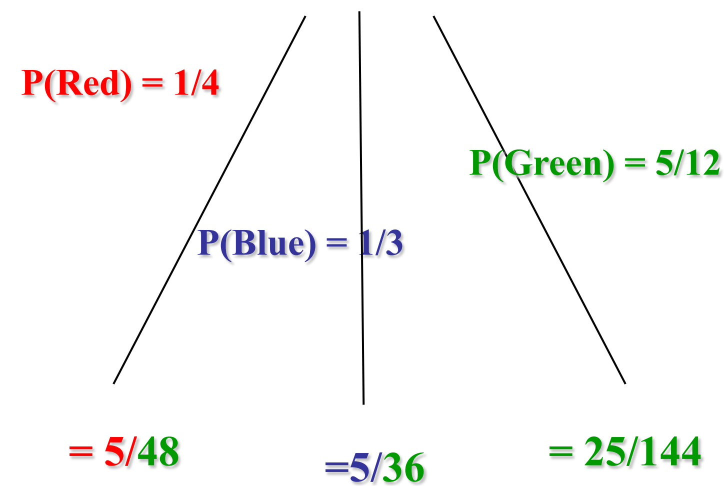



P(Red) = 3/12
P(Blue) = 4/12
P(Green) = 5/12
= 1/4
=1/3
= 5/12
P(Red) = 1/4
P(Green) = 5/12
P(Blue) = 1/3
= 1/16
=1/12
= 5/48



P(Red) = 3/12
P(Blue) = 4/12
P(Green) = 5/12
= 1/4
=1/3
= 5/12
P(Red) = 1/4
P(Blue) = 1/3
P(Green) = 5/12
= 1/12
=1/9
= 5/36





Note that these are *independent events* because we put the first marble back

For example: the probability of two green marbles is just the product of each one's probability

$$P(G) = \frac{5}{12} \qquad P(2G) = \frac{5}{12}\cdot\frac{5}{12} = \frac{25}{144}$$

So as long as the two events are independent

$$P(A \cap B) = P(A)P(B)$$

Note that probability of the second green marble is not affected by the first if they are *independent*

$$P(B \mid A) = P(B)$$

Multiplication Rule

If A and B are two independent events, then

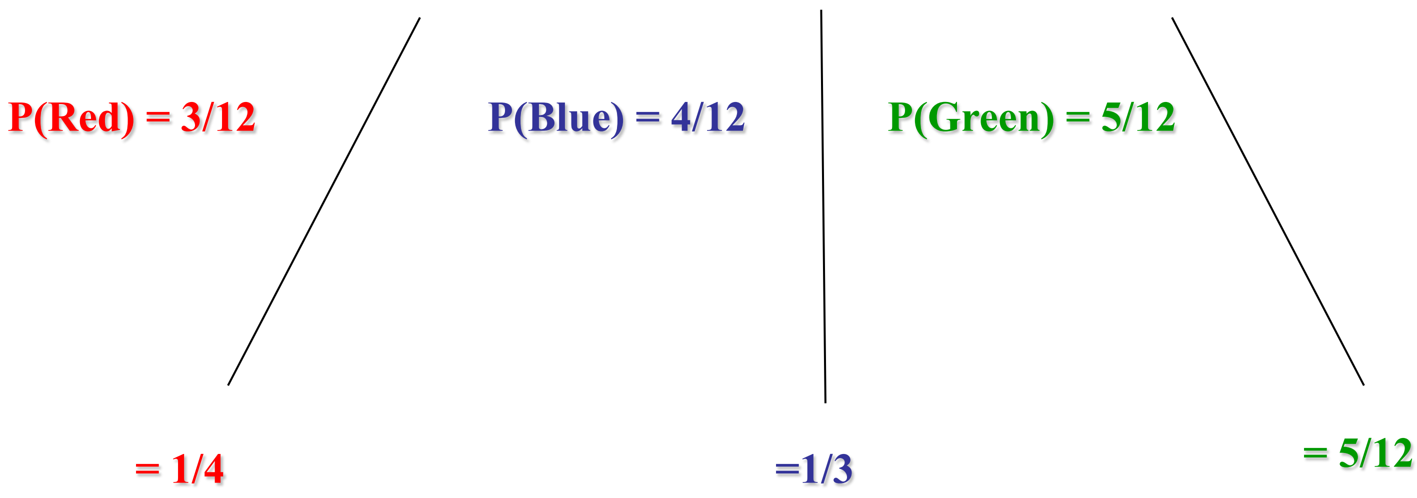
$$P(A \text{ and } B) = P(A)*P(B)$$

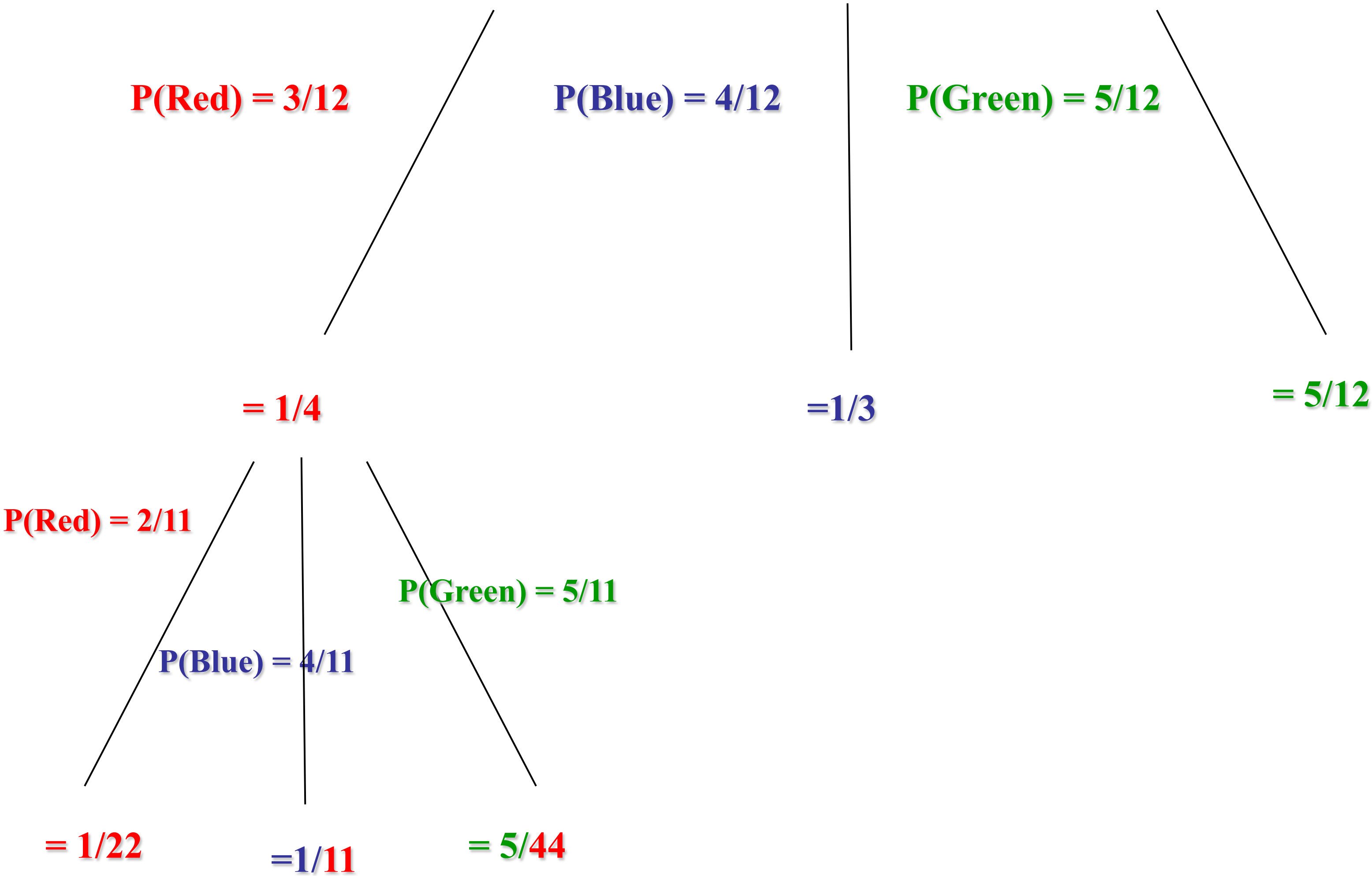
If A and B are two dependent events, then

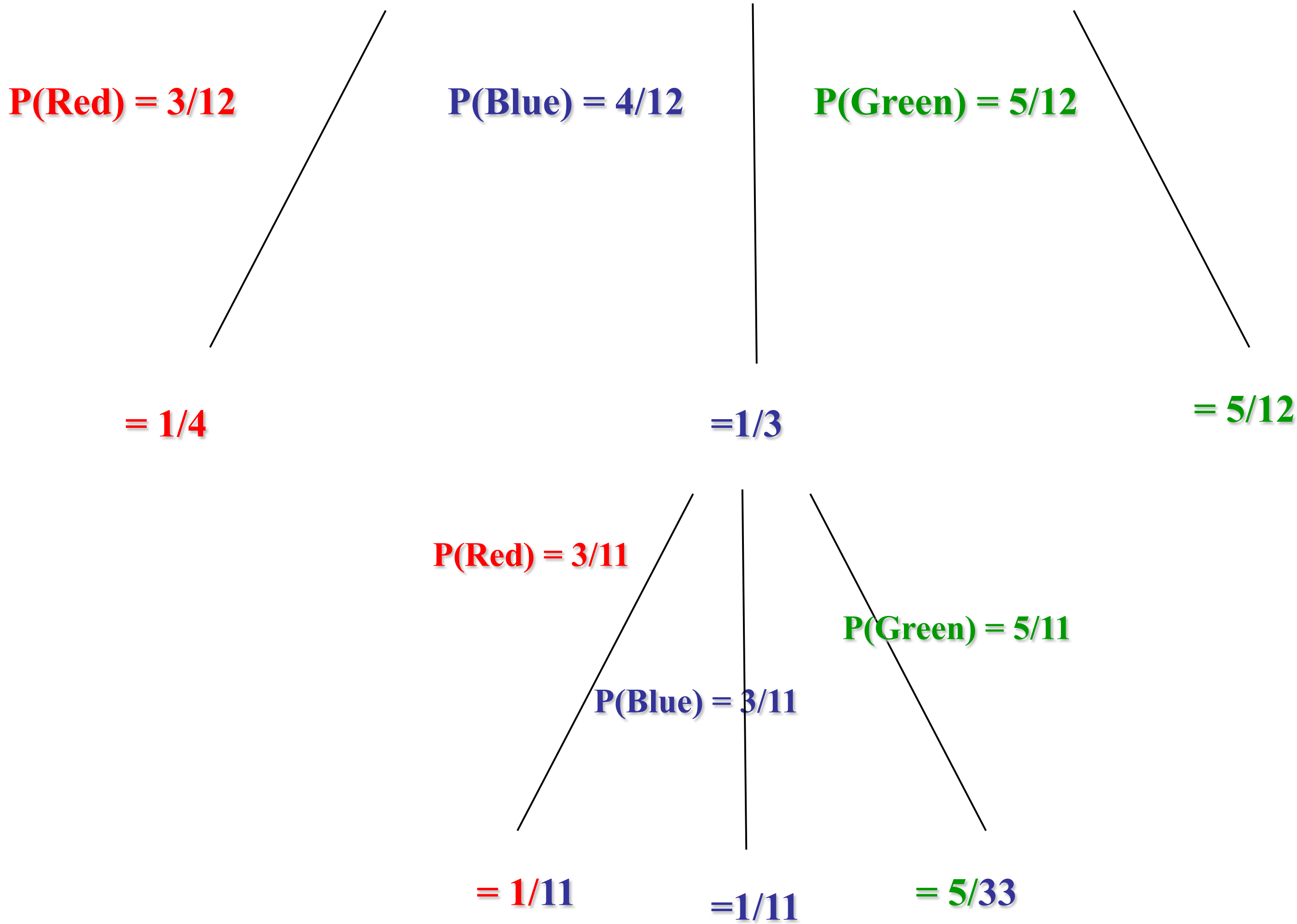
$$P(A \text{ and } B) = P(A)*P(B \mid A)$$

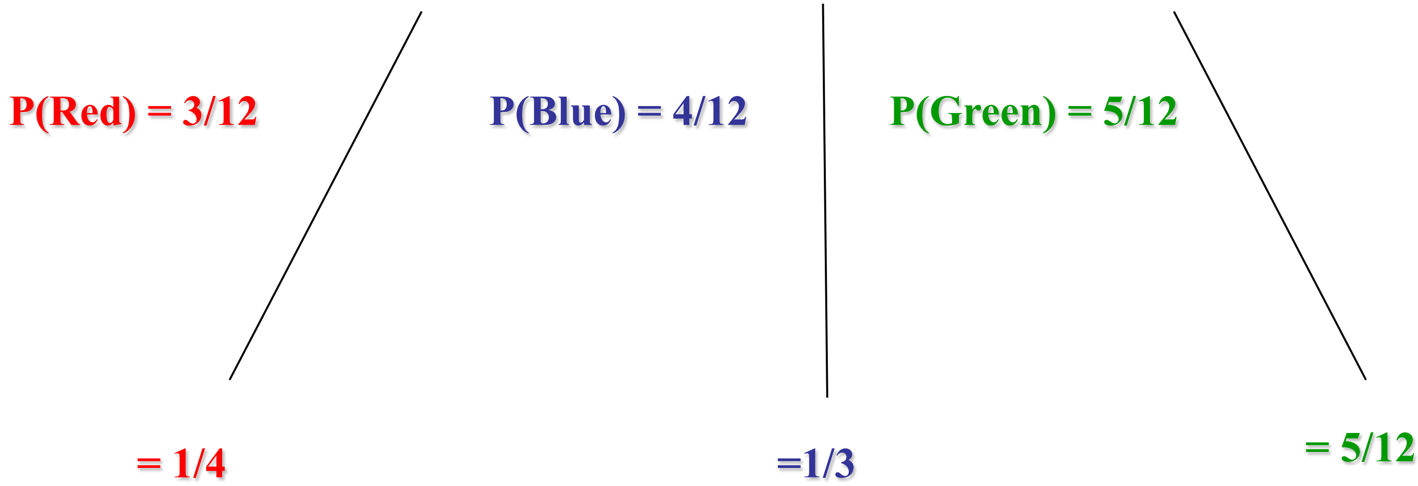
AKA **The Conditional Independence Rule**

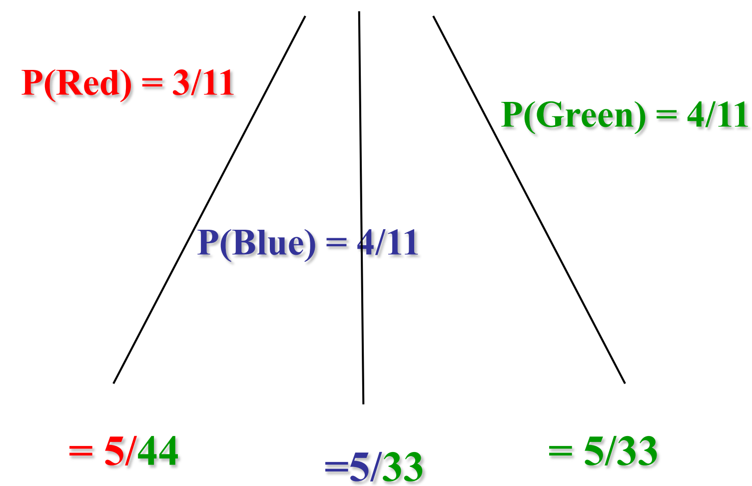
A jar has 3 red marbles, 4 blue marbles, and 5 green marbles. If you draw two marbles <u>without replacement</u>, find all possible outcomes using a tree diagram.




P(Red) = 3/12
P(Blue) = 4/12
P(Green) = 5/12
= 1/4
=1/3
= 5/12
P(Red) = 2/11
P(Green) = 5/11
P(Blue) = 4/11
= 1/22
=1/11
= 5/44


P(Red) = 3/12
P(Blue) = 4/12
P(Green) = 5/12
= 1/4
=1/3
= 5/12
P(Red) = 3/11
P(Green) = 5/11
P(Blue) = 3/11
= 1/11
=1/11
= 5/33





Note that these are *dependent events* because the second marble's probability depends on the outcome of the first draw.

Recall our formulas

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A)P(B \mid A)$$

For example: the probability of two green marbles is

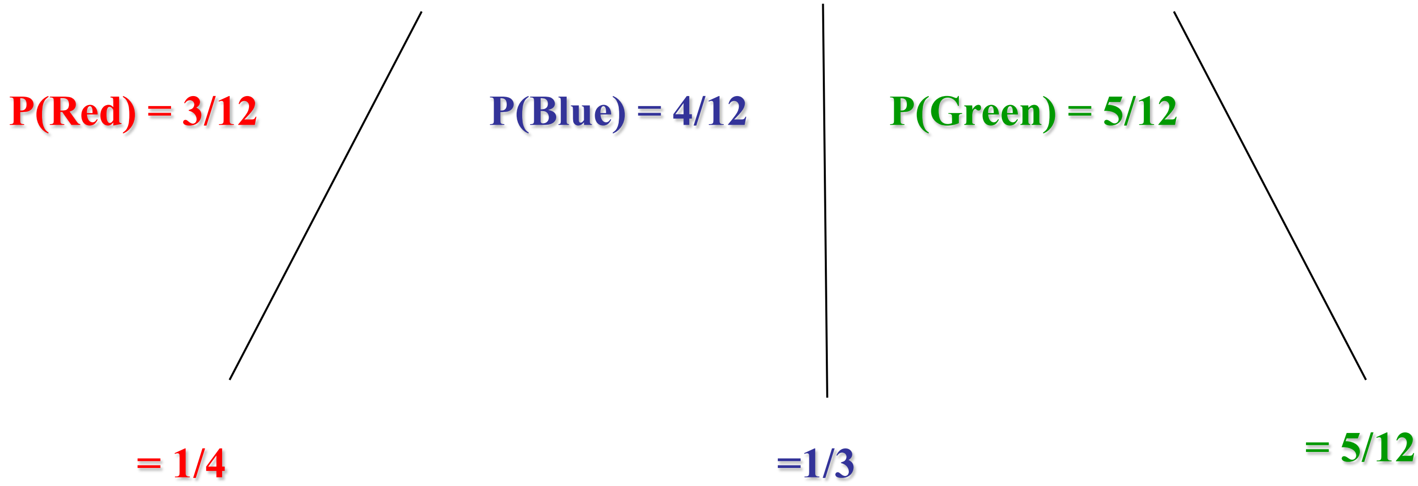
$$P(G_1 \cap G_2) = P(G_1)P(G_2 \mid G_1)$$

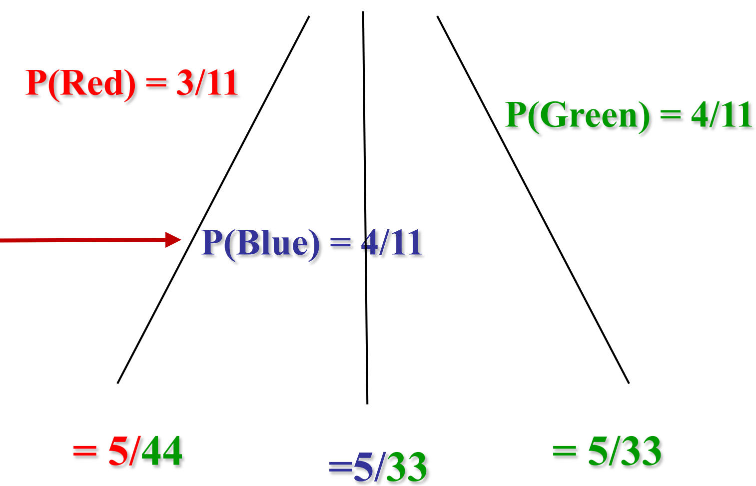
The first marble is green

The second marble is green given that the first one is green

$$P(G_1 \cap G_2) = \frac{5}{12} \cdot \frac{4}{11} = \frac{5}{33}$$

Our answer matches the one in our probability tree





We can see that this works the other way around by looking at the probability of a blue on the second draw given that we had a green on the first.

$$P(B \mid G) = \frac{P(B \cap G)}{P(G)}$$

Consulting our tree branches we get:

$$P(B \mid G) = \frac{5/33}{5/12}$$

$$P(B \mid G) = \frac{\cancel{5}}{33} \cdot \frac{12}{\cancel{5}} = \frac{4}{11}$$

This helps us see how conditional probability and the general multiplication rule go hand in hand

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$