The Power Rule

The short-cut you've been waiting for

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

So when $f(x) = x^2 \qquad \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} (2x+h) = 2x$

or when

$$f(x) = x^{3} \qquad \lim_{h \to 0} \frac{\left(x+h\right)^{3} - x^{3}}{h} = \lim_{h \to 0} \frac{x^{3} + 3x^{2}h + 3xh^{2} + h^{3} - x^{3}}{h} = \lim_{h \to 0} \frac{h(3x^{2} + 3xh + h^{2})}{h} =$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$$

or even when $f(x) = x^{4} \lim_{h \to 0} \frac{(x+h)^{4} - x^{4}}{h} = \lim_{h \to 0} \frac{x^{4} + 4x^{3}h + 6x^{2}h^{2} + 4xh^{3} + h^{4} - x^{4}}{h} = \lim_{h \to 0} \frac{h(4x^{3} + 6x^{2}h + 4xh^{2} + h^{3})}{h} = 4x^{3}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

So when

 $f(x) = x^2$ gives us f'(x) = 2x $f(x) = x^3$ gives us $f'(x) = 3x^2$

$$f(x) = x^4$$
 gives us $f'(x) = 4x^3$

There seems to be a pattern here

$$f(x) = x^n$$
 $f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = nx^{n-1}$

The Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{1}{x^n} = x^{-n} \qquad \qquad \sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = \frac{p}{x^r} \qquad \qquad \frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

Steps to get you there:

- 1. Write everything as a power
- 2. Take the derivative
- 3. Rewrite all negative powers as positive
- 4. Convert rational powers to radicals

Shall we try an example or two?

$$\frac{1}{x^n} = x^{-n} \qquad \qquad \sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}} \qquad \qquad \frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

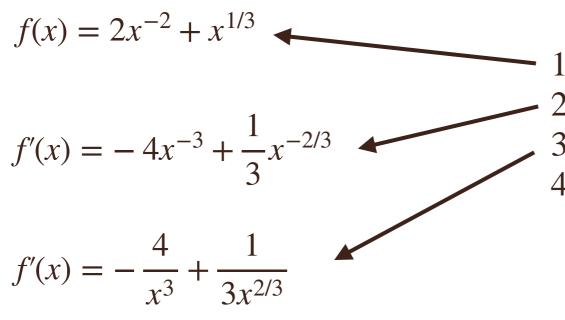
$$f(x) = x^2 - 3x + 4 \qquad \qquad g(x) = 2x^4 + x$$

$$f'(x) = 2x - 3 \qquad \qquad g'(x) = 8x^3 + 1$$

$$\frac{1}{x^n} = x^{-n} \qquad \qquad \sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = \frac{p}{x^r} \qquad \qquad \frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

Steps to get you there:



 $f(x) = \frac{2}{x^2} + \sqrt[3]{x}$

- 1. Write everything as a power
- 2. Take the derivative
- 3. Rewrite all negative powers as positive
- 4. Convert rational powers to radicals

$$f'(x) = -\frac{4}{x^3} + \frac{1}{3\sqrt[3]{x^2}}$$

$$\frac{1}{x^n} = x^{-n} \qquad \qquad \sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}} \qquad \qquad \frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

$$h(x) = -\frac{4}{x^3} + \sqrt{x}$$

$$h(x) = -4x^{-3} + x^{\frac{1}{2}}$$

$$h'(x) = 12x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$h'(x) = \frac{12}{x^4} + \frac{1}{2\sqrt{x}}$$

$$h'(x) = \frac{12}{x^4} + \frac{1}{2x^{\frac{1}{2}}}$$