The Power Rule

The short-cut you've been waiting for

$$f'(x) = \lim_{h \to 0} \frac{f\left(x+h\right) - f(x)}{h}$$

So when

$f(x) = x^2$

$$\lim_{h \to 0} \frac{\left(x+h\right)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0}(2x+h) = 2x$$

or when

$f(x) = x^3$

$$\lim_{h \to 0} \frac{\left(x+h\right)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} =$$

$$= \lim_{h \to 0}(3x^2 + 3xh + h^2) = 3x^2$$

or even when

$f(x) = x^4$

$$\lim_{h \to 0} \frac{\left(x+h\right)^4 - x^4}{h} = \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h} =$$

$$\lim_{h \to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h} = 4x^3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

So when

$f(x) = x^2$ gives us $f'(x) = 2x$

$f(x) = x^3$ gives us $f'(x) = 3x^2$

$f(x) = x^4$ gives us $f'(x) = 4x^3$

There seems to be a pattern here

$f(x) = x^n$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = nx^{n-1}$$

The Power Rule

$$\frac{d}{dx} x^n = nx^{n-1}$$

A quick review of exponents

$$\frac{1}{x^n} = x^{-n} \qquad \sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}} \qquad \frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

Steps to get you there:

1. Write everything as a power
2. Take the derivative
3. Rewrite all negative powers as positive
4. Convert rational powers to radicals

Shall we try an example or two?

A quick review of exponents

$$\frac{1}{x^n} = x^{-n}$$

$$\sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}}$$

$$\frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

$$f(x) = x^2 - 3x + 4$$

$$g(x) = 2x^4 + x$$

$$f'(x) = 2x - 3$$

$$g'(x) = 8x^3 + 1$$

A quick review of exponents

$$\frac{1}{x^n} = x^{-n}$$

$$\sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}}$$

$$\frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

Steps to get you there:

1. Write everything as a power
2. Take the derivative
3. Rewrite all negative powers as positive
4. Convert rational powers to radicals

$$f(x) = \frac{2}{x^2} + \sqrt[3]{x}$$

$$f(x) = 2x^{-2} + x^{1/3}$$

$$f'(x) = -4x^{-3} + \frac{1}{3}x^{-2/3}$$

$$f'(x) = -\frac{4}{x^3} + \frac{1}{3x^{2/3}}$$

$$f'(x) = -\frac{4}{x^3} + \frac{1}{3\sqrt[3]{x^2}}$$

A quick review of exponents

$$\frac{1}{x^n} = x^{-n}$$

$$\sqrt[r]{x^p} = \left(\sqrt[r]{x}\right)^p = x^{\frac{p}{r}}$$

$$\frac{1}{\sqrt[r]{x^p}} = \frac{1}{\left(\sqrt[r]{x}\right)^p} = x^{-\frac{p}{r}}$$

The Power Rule still applies with all of these

$$h(x) = -\frac{4}{x^3} + \sqrt{x}$$

$$h(x) = -4x^{-3} + x^{\frac{1}{2}}$$

$$h'(x) = 12x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$h'(x) = \frac{12}{x^4} + \frac{1}{2x^{\frac{1}{2}}}$$

$$h'(x) = \frac{12}{x^4} + \frac{1}{2\sqrt{x}}$$