# Extremes of Radical Functions

Standard 5c: Apply the Chain Rule to differentiate radical functions

Standard 5d: Find the critical values and extremes of radical functions

Find the domain of $y = \sqrt{16x - x^3}$

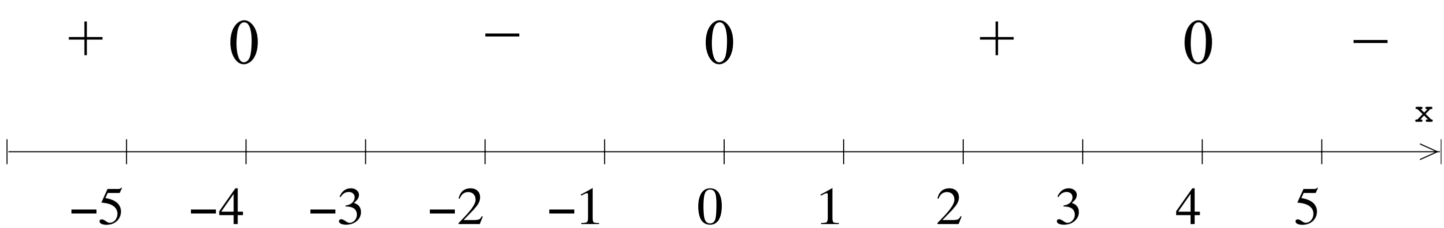
$16x - x^3 \geq 0$

What values of $x$ give us something with a real number square root?

$x(16 - x^2) \geq 0$

$x(4 - x)(4 + x) \geq 0$

Now make a sign pattern number line




$x \leq -4$

$0 \leq x \leq 4$

Differentiate: $y = \sqrt{16x - x^3}$

$$y = \left(16x - x^3\right)^{\frac{1}{2}}$$

Re-write with an exponent

$$y' = \frac{1}{2}\left(16x - x^3\right)^{-\frac{1}{2}}\left(16 - 3x^2\right)$$

Don't forget the inside

Now find the critical points (Where $y'$ is 0 or undefined)

$$y' = \frac{\left(16 - 3x^2\right)}{2\sqrt{16x - x^3}} = 0$$

or where $16 - 3x^2 = 0$

$$16 = 3x^2$$

$$\frac{16}{3} = x^2$$

$$x = \pm\frac{4}{\sqrt{3}}$$

$$x = \pm\frac{4}{\sqrt{3}} \approx \pm 2.309$$

But the only value that works here is…

Because the other value is not in the domain

$$x = \frac{4}{\sqrt{3}}$$

Why?

Recall that the domain is $x \le -4$, $0 \le x \le 4$

Now find the critical points (Where $y'$ is 0 or undefined)

$$y' = \frac{(16 - 3x^2)}{2\sqrt{16x - x^3}} = \textit{undefined} \text{ or where}$$

$$16x - x^3 = 0$$

$$x(16 - x^2) = 0$$

$$x = 0, \pm 4$$

So the critical values here are

$$x = -4, 0, \frac{4}{\sqrt{3}}, 4$$

So how do we do this again?

- Find the domain of the radical function
- Differentiate (don't forget the Chain Rule)
- Find the critical points
  - where the derivative is 0 (numerator)
  - where the derivative is undefined (denominator)
- Check the critical points against the domain
- Make a sign pattern to locate the minima and maxima

And one more thing…

So the critical values here are

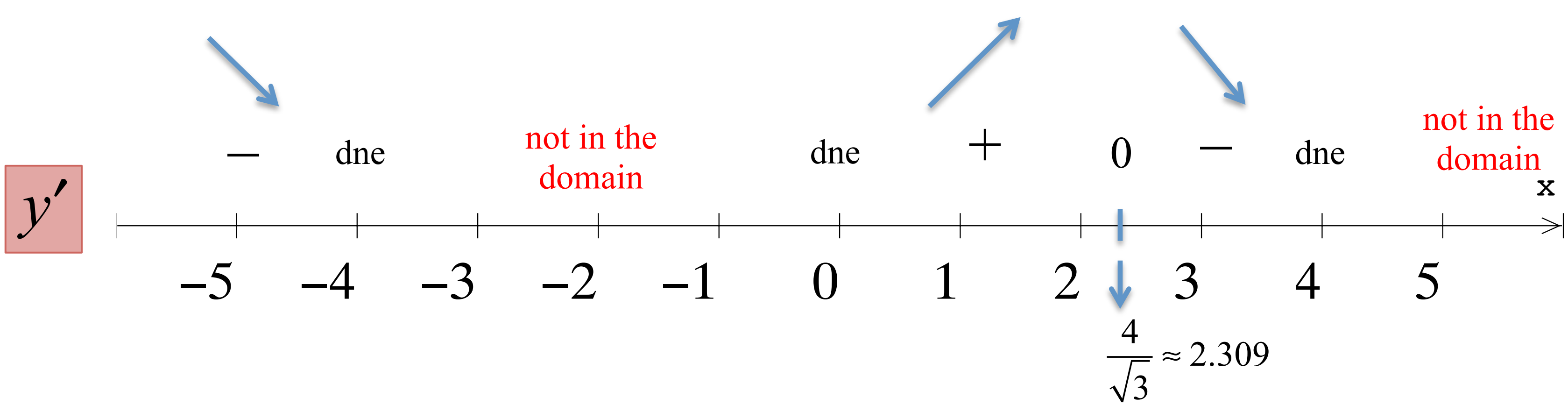
$$x = -4, 0, \frac{4}{\sqrt{3}}, 4$$

So where are the maxima and minima?

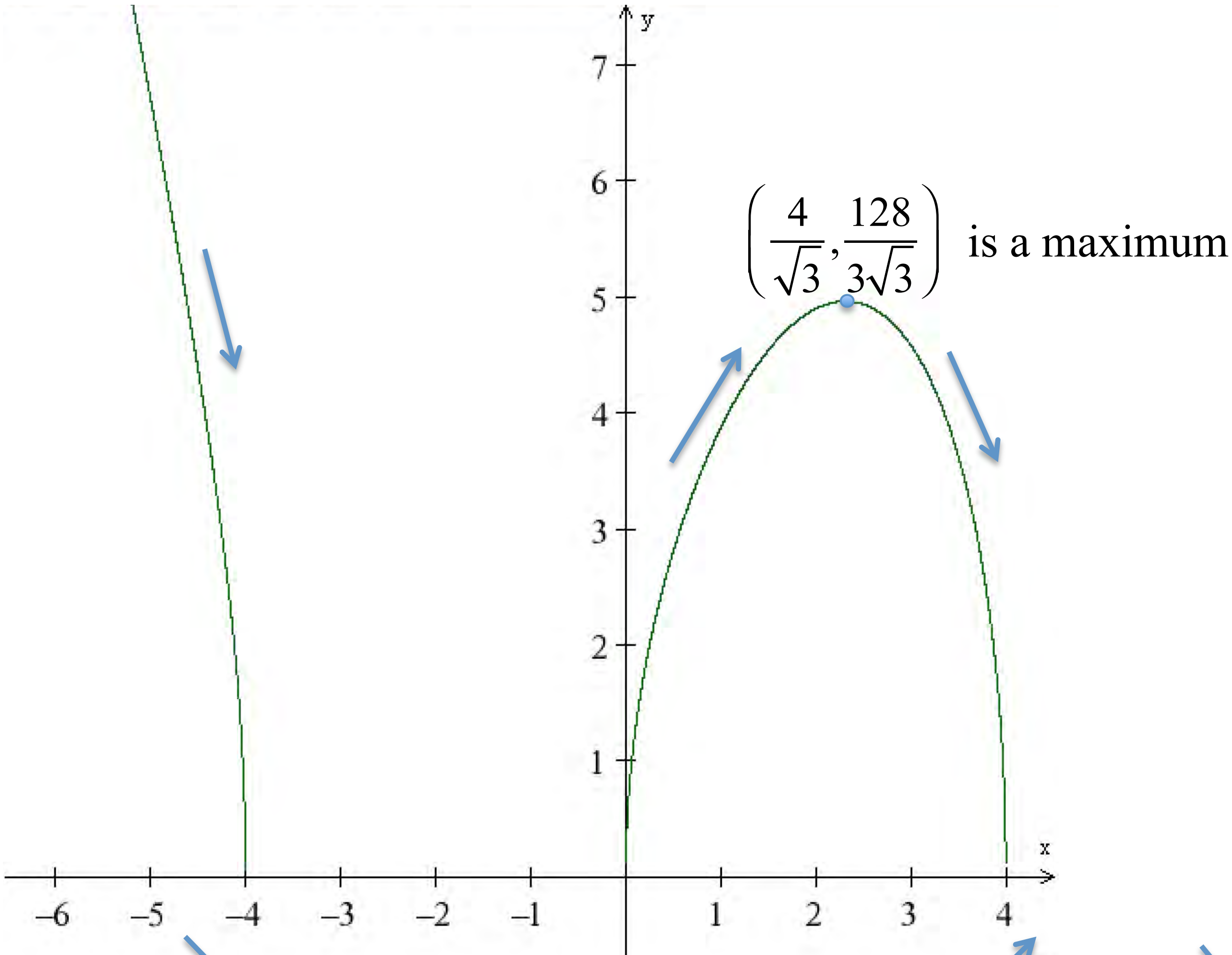
Here is where we will need to make the sign pattern

$$y' = \frac{(16 - 3x^2)}{2\sqrt{16x - x^3}}$$

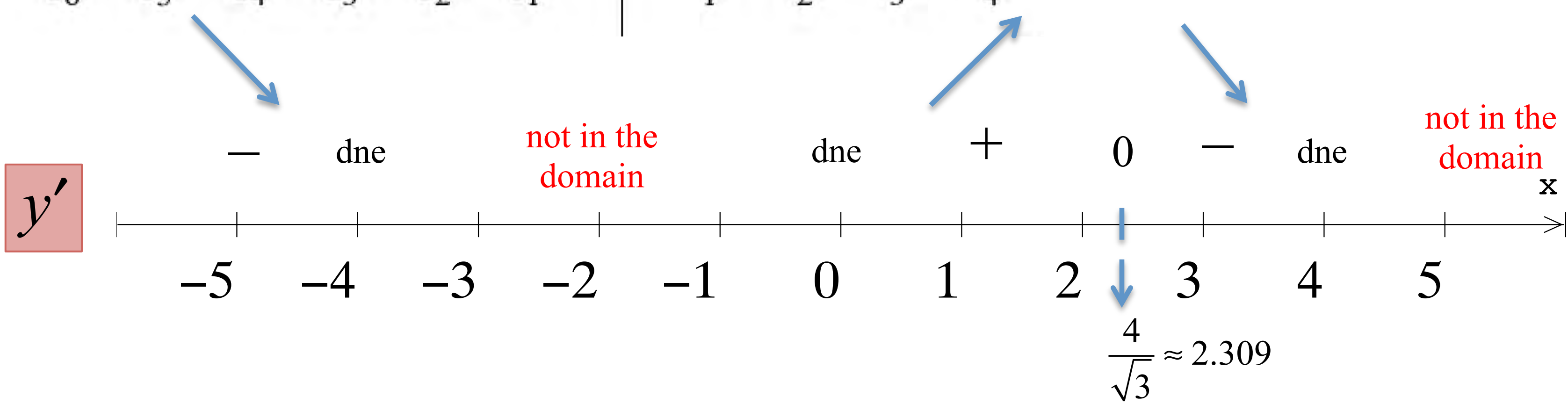
$\left(\frac{4}{\sqrt{3}}, \frac{128}{3\sqrt{3}}\right)$ is a maximum



$$y = \sqrt{16x - x^3}$$



y
x
7
6
5
4
3
2
1
−6
−5
−4
−3
−2
−1
1
2
3
4
$\left(\frac{4}{\sqrt{3}}, \frac{128}{3\sqrt{3}}\right)$ is a maximum

$y'$
−
dne
not in the domain
dne
+
0
−
dne
not in the domain
x
−5
−4
−3
−2
−1
0
1
2
3
4
5
$\frac{4}{\sqrt{3}} \approx 2.309$