Position & Velocity

Unit 5-5

Daniel designs and builds a large human size shooter for the Robotics team and "convinces" Mr. Murphy to be the first test subject. Roman figures the equation for Mr. Murphy's height to be...

$$h(t) = 128t - 16t^2$$

1) When does Mr. Murphy reach his maximum height?

At the top of the graph as we have seen before

But how do we find the top of the graph? Calculator? What about without the calculator?

At his maximum height, Mr. Murphy stops meaning his velocity is 0

And since we know that $h'(t) = v(t)$

We can take the derivative and find when it equals 0

2) Where does Mr. Murphy reach his maximum height?

We can just plug our answer to #1 into h(t)

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$$h(t) = 128t - 16t^2$$

1) When does Mr. Murphy reach his maximum height?

$$h'(t) = v(t) = 128 - 32t = 0$$

$$128 = 32t \qquad t = 4 \text{ seconds}$$

2) Where does Mr. Murphy reach his maximum height?

$$h(4) = 128(4) - 16(4)^2 = 256 \text{ feet}$$

Niamh and Mia decide that the launch would be better from a platform that is 112 feet high. With a slightly smaller initial velocity, the equation becomes

$$h(t) = 112 + 96t - 16t^2$$

1) When does Mr. Murphy reach his maximum height?

$$h'(t) = v(t) = 96 - 32t = 0 \qquad 96 = 32t$$

$$t = 3 \text{ seconds}$$

2) Where does Mr. Murphy reach his maximum height?

$$h(3) = 112 + 96(3) - 16(3)^2 = 256 \text{ feet}$$

3) How fast is Mr. Murphy falling when he lands?

$$h(t) = 112 + 96t - 16t^2 = 0 = -16(t^2 - 6t - 7)$$

$$= -16(t - 7)(t + 1) \qquad t = \cancel{-1}, 7$$

$$t = 7 \text{ seconds} \qquad v(7) = 96 - 32(7) = -128 \text{ feet/second}$$

Leaving math class, Alex and Ilaria are arguing over food. As they walk through the piazza, Alex grabs the food and runs. Ilaria starts chasing her towards the building. Given the door to the classroom to be where x = 0 and the equation for Alex's path while evading Ilaria is given by the equation below,

$$x(t) = t^4 - 2t^3 - 3t^2 + 4t$$

x is measured in meters and t is measured in seconds

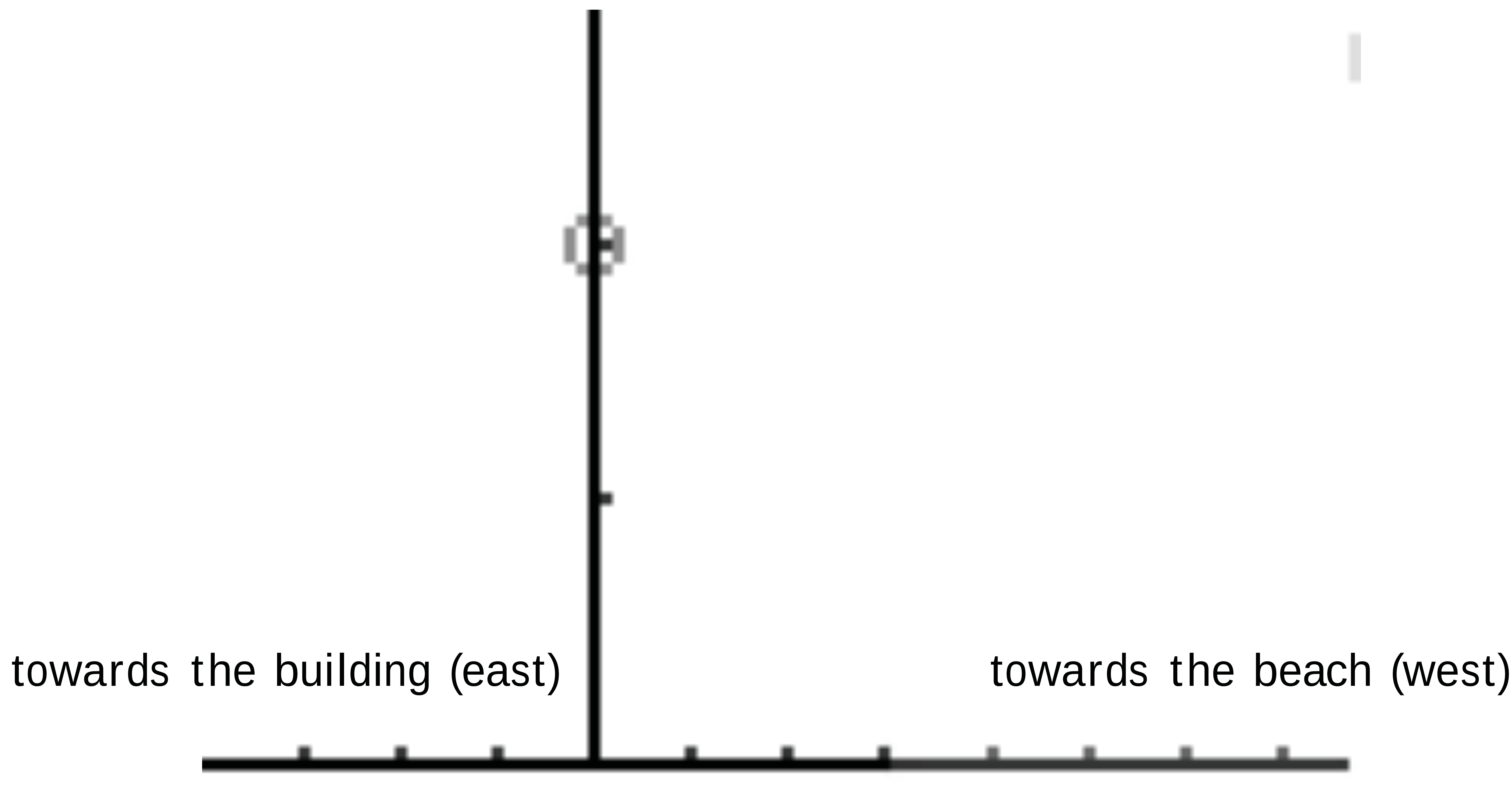
in which the direction towards the building (east) is considered negative and the direction back towards the beach (west) is considered positive

1) Which way does Alex run first?

In other words, was her initial velocity positive or negative?

$$x'(t) = v(t) = 4t^3 - 6t^2 - 6t + 4 \qquad v(0) = 4 > 0$$

Answer: Alex was running towards the beach


towards the building (east)
towards the beach (west)

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2) When does Alex change directions?

In other words, when did her velocity change signs?

$$v(t) = 4t^3 - 6t^2 - 6t + 4 = 0$$

Using Synthetic Substitution we get... $t = -1, \frac{1}{2}, 2$

Since we are measuring time we won't count any negative values for t

$t = \frac{1}{2}, 2$ seconds

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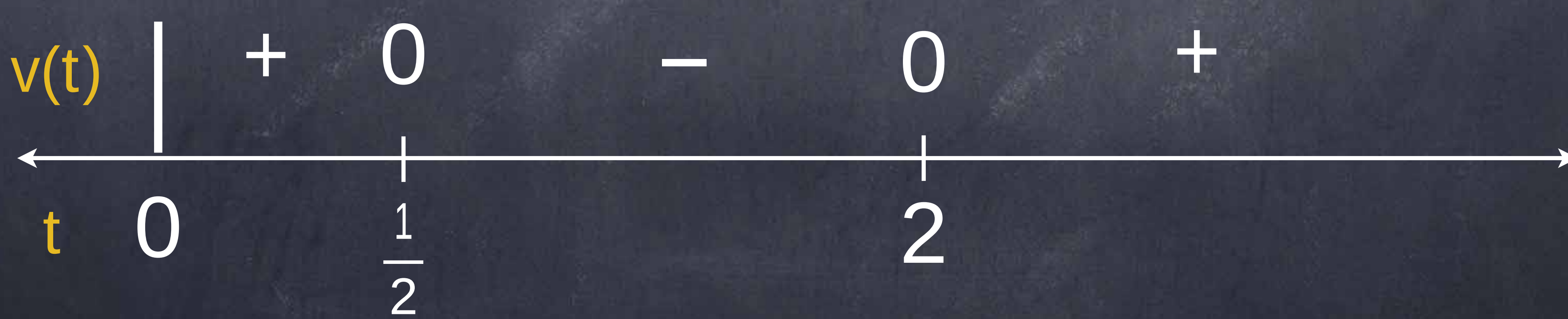
2) When does Alex change directions?

But first let's confirm that she did change directions at those points



We will have to confirm such things in the future...you'll see why

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3) Where does Alex change directions? $t = \frac{1}{2}, 2$ seconds

$$x\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^4 - 2\left(\tfrac{1}{2}\right)^3 - 3\left(\tfrac{1}{2}\right)^2 + 4\left(\tfrac{1}{2}\right) = 1.0625$$

1.0625 meters towards the beach.

$$x(2) = 2^4 - 2(2)^3 - 3(2)^2 + 4(2) = -4$$

4 meters away from the door to room 423

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4) What is Alex's acceleration at these points?

$$v(t) = 4t^3 - 6t^2 - 6t + 4 \qquad v'(t) = a(t)$$

$$a(t) = 12t^2 - 12t - 6$$

$$a\left(\tfrac{1}{2}\right) = 12\left(\tfrac{1}{2}\right)^2 - 12\left(\tfrac{1}{2}\right) - 6 = -9 \text{ meters/sec}^2$$

$$a(2) = 12(2)^2 - 12(2) - 6 = 18 \text{ meters/sec}^2$$

Now Annie and Elise also start arguing over food. As they walk through the piazza, Elise grabs the food and runs. Annie starts chasing her towards the building. Jill and Dylan time the chase over an 8 second period. Given the door to the classroom to be where x = 0 and the equation for Elise' path while evading Annie is given by the equation below,

$$x(t) = \frac{t^4}{4} - 3t^3 + 10t^2 - 12t + 16$$

x is measured in feet and t is measured in seconds

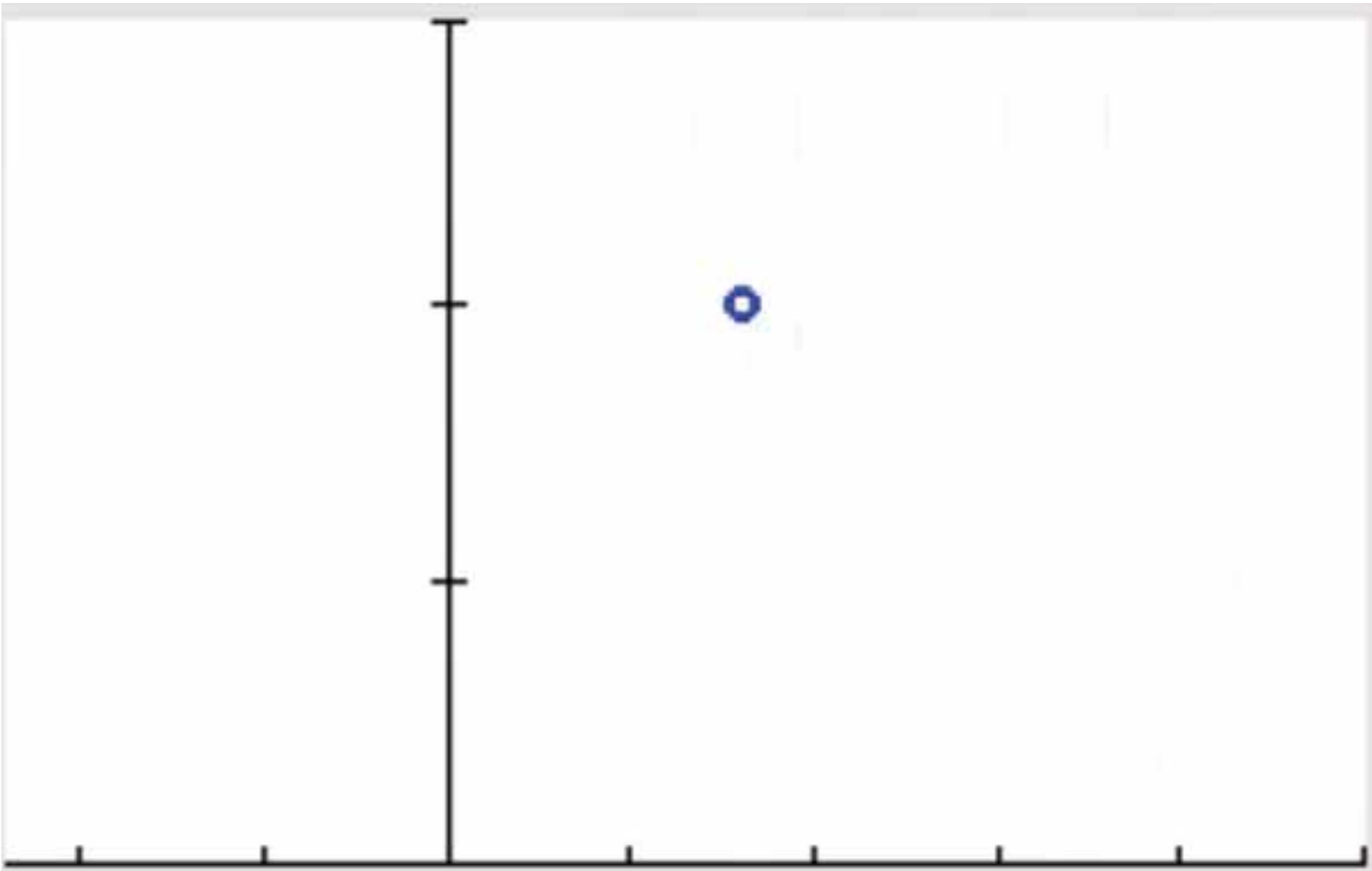
in which the direction towards the building (east) is considered negative and the direction back towards the beach (west) is considered positive

1) Where is Elise when she starts running?

x(0) = 16 feet west of the door

2) When and where does Elise change directions?

Let's take a look at a diagram



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1) Where is Elise when she starts running?

x(0) = 16 feet west of the door

2) When and where does Elise change directions?

$$v(t) = t^3 - 9t^2 + 20t - 12 = 0$$

Synthetic division will give us... $(t-1)(t-2)(t-6) = 0$

She changed directions at t = 1, 2, and 6 seconds

x(1) = 11.25 feet

x(2) = 12 feet

x(6) = -20 feet

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in which the direction towards the building is considered negative and the direction back towards the beach is considered positive

3) Over what intervals of t is Elise running towards the beach?

$$v(t) = t^3 - 9t^2 + 20t - 12 = 0$$

$$(t-1)(t-2)(t-6) = 0$$

Synthetic division will give us...

She changed directions at t = 1, 2, and 6 seconds

t	0		1		2		6		8
v(t)		–	0	+	0	–	0	+	

$1 < t < 2$ or $6 < t < 8$ seconds

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4) When does Elise pass the door and in what direction is she running when she does?

$$x(t) = \frac{t^4}{4} - 3t^3 + 10t^2 - 12t + 16 = 0$$

Using your calculator, the times are?

$t = 4$ *and* 7.197 seconds

$$v(t) = t^3 - 9t^2 + 20t + 16$$

$$v(4) = 4^3 - 9(4)^2 + 20(4) + 16 = -12 \text{ ft/sec}$$

$$v(7.197) = 7.197^3 - 9(7.197)^2 + 20(7.197) + 16 = 66.550 \text{ ft/sec}$$

At t = 4 she's running towards the building. At t = 7.197 she's running towards the beach