Gracie and Vivian, planning a big SI fundraiser (actually it's just to pay for the food at their table), build a 60 meter high-dive platform in the middle of the field. After charging admission for prime bleacher seats, they then "persuade" Mr Murphy to be the first high diver.

Owen and Garo observe Mr Murphy's "dive" closely enough to form an equation for his height. They find the equation to be given by:

$$h(t) = 60 - 4.9t^2$$

and the graph is given by...

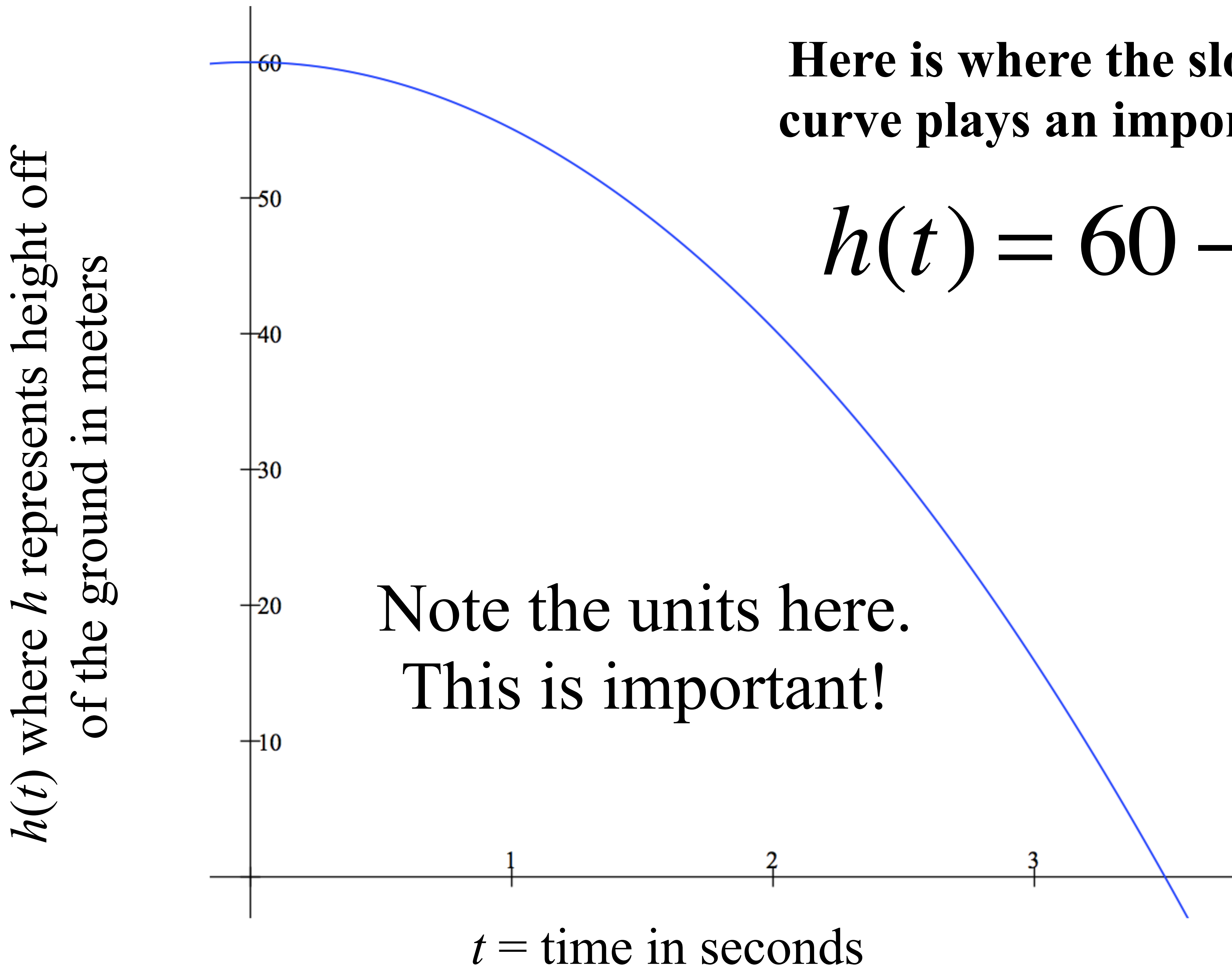
How can we find Mr. Murphy's velocity at 1 second?

at 2 seconds?

when he hits the water?

Here is where the slope of the curve plays an important role.

$$h(t) = 60 - 4.9t^2$$

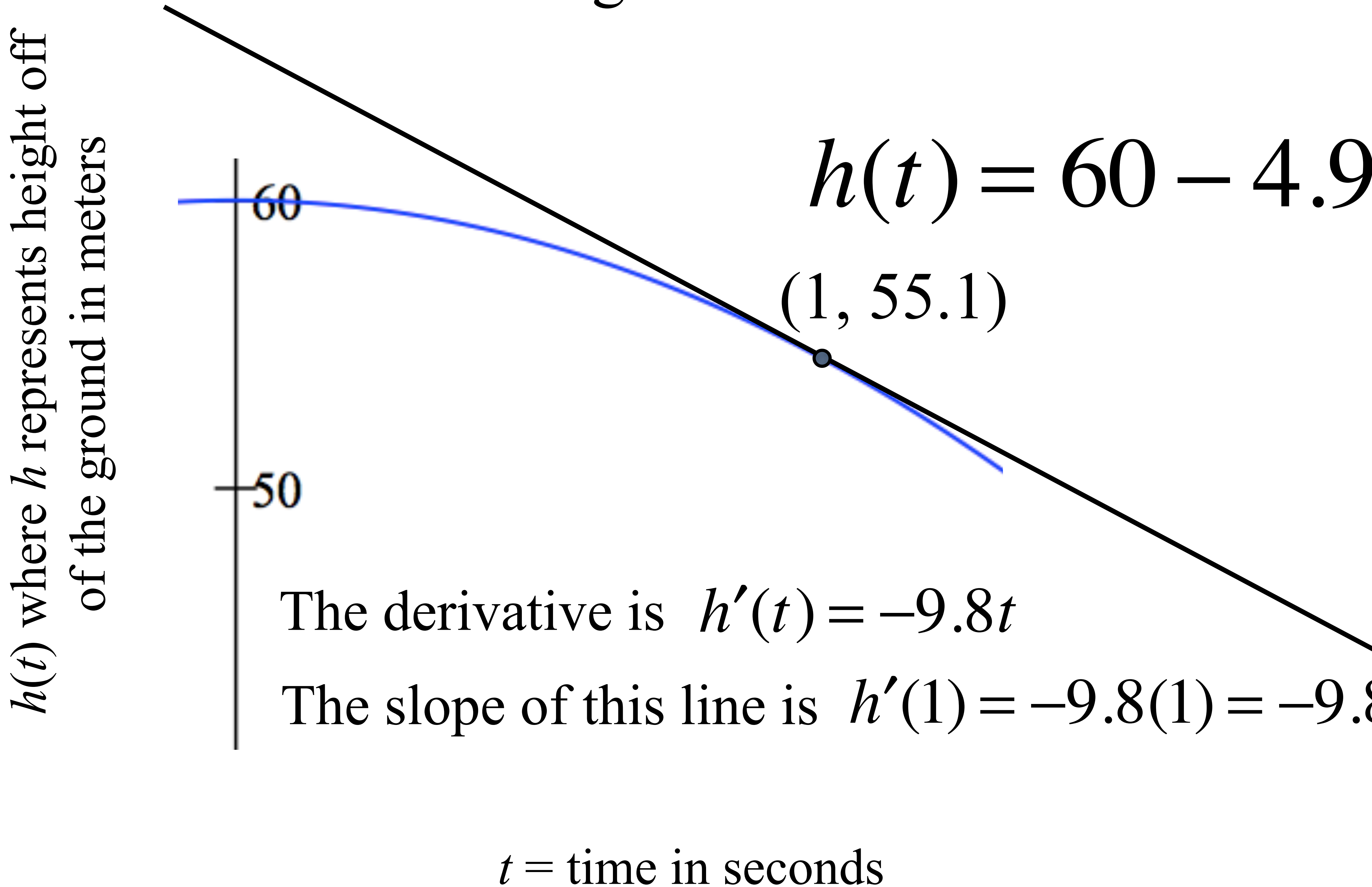


Note the units here.
This is important!

Remember that we can now find the slope of this curve at any point using the derivative

Let’s draw the tangent line at $t = 1$ second

$$h(t) = 60 - 4.9t^2$$

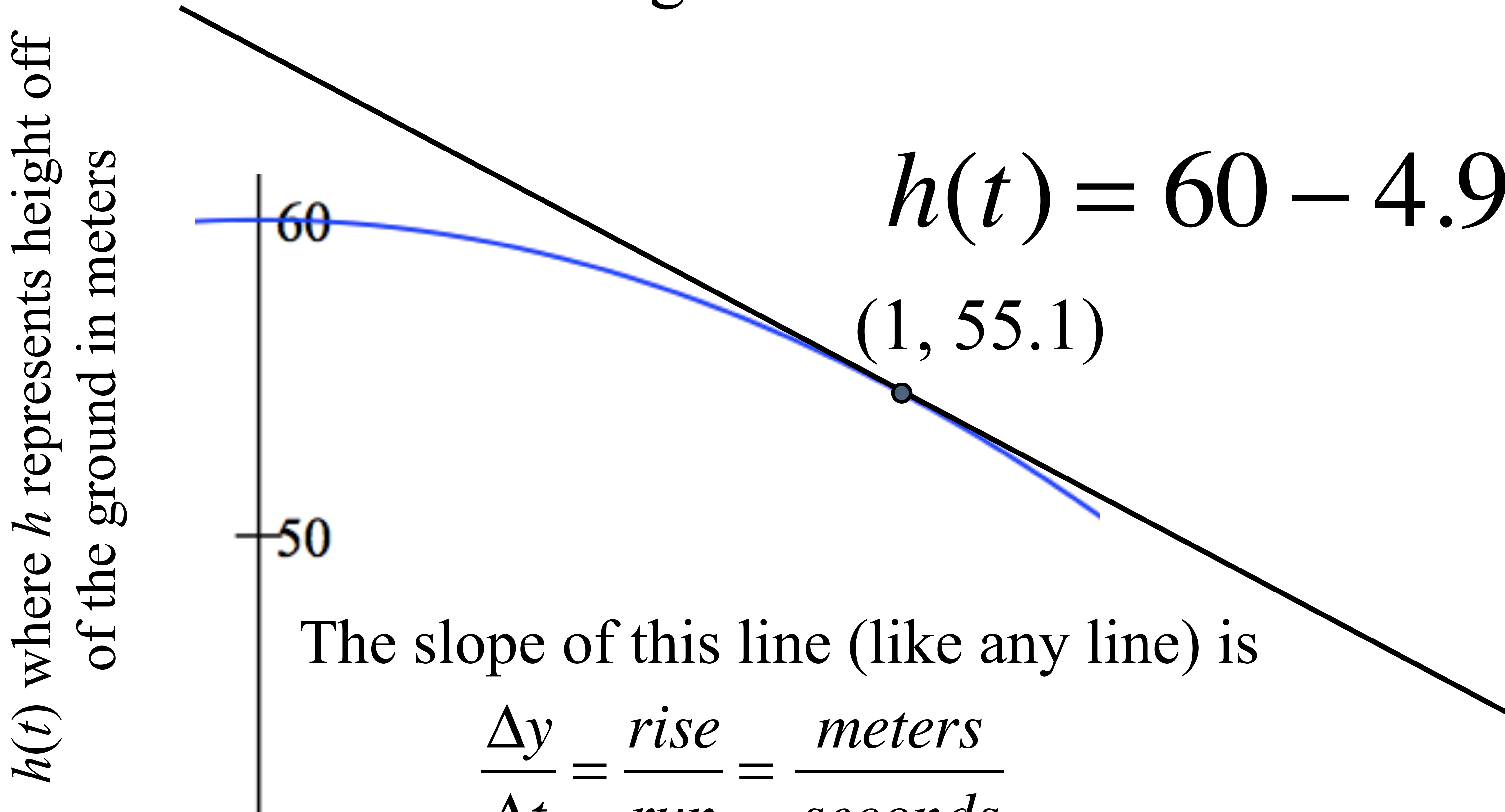


The derivative is $h'(t) = -9.8t$

The slope of this line is $h'(1) = -9.8(1) = -9.8$

Remember that we can now find the slope of this curve at any point using the derivative

Let's draw the tangent line at $t = 1$ second



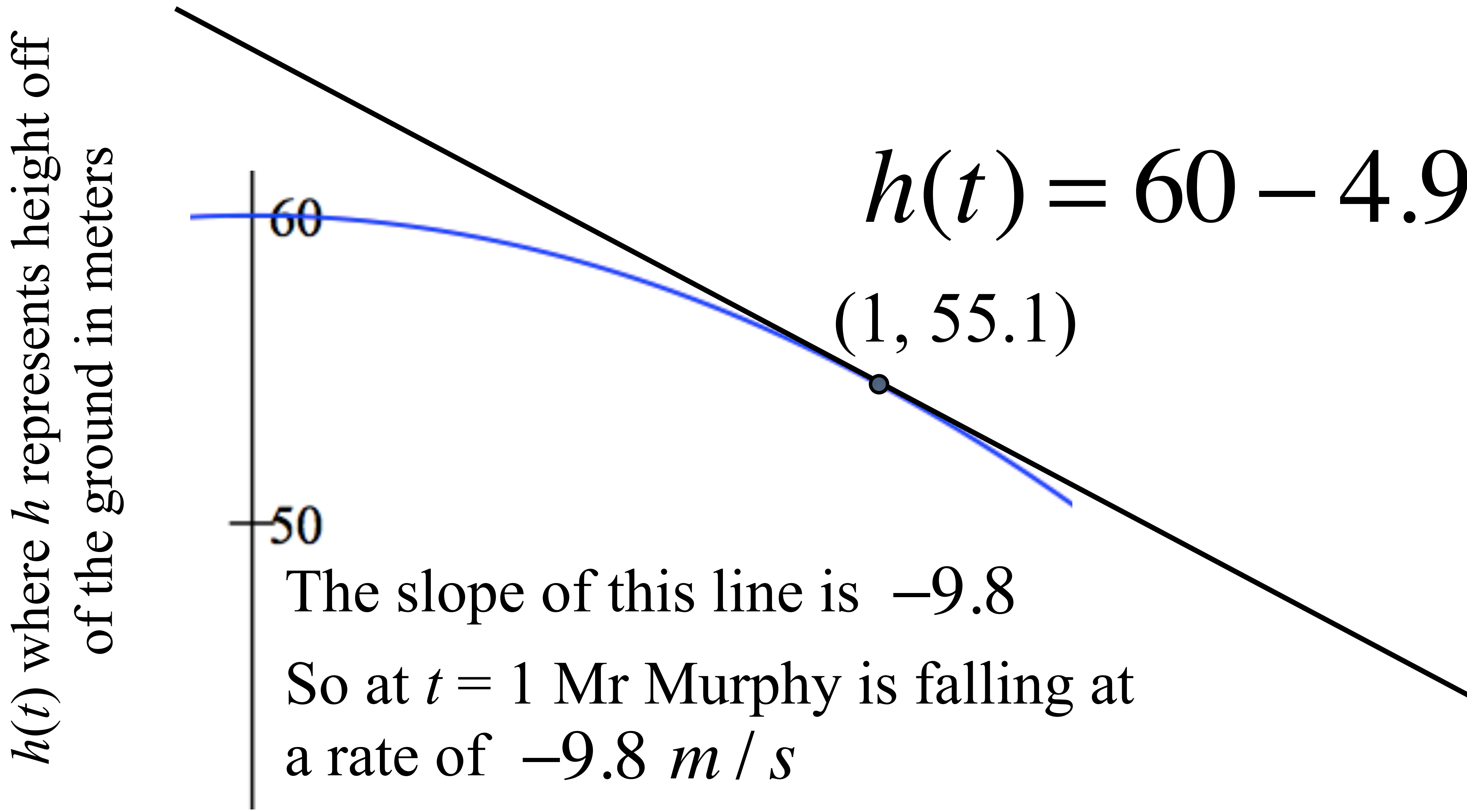
$$h(t) = 60 - 4.9t^2$$

The slope of this line (like any line) is

$$\frac{\Delta y}{\Delta t} = \frac{rise}{run} = \frac{meters}{seconds}$$

t = time in seconds

$\frac{meters}{seconds}$ or meters per second are the units for **Velocity!!!**



$$h(t) = 60 - 4.9t^2$$

The slope of this line is -9.8

So at $t = 1$ Mr Murphy is falling at a rate of $-9.8\ m/s$

t = time in seconds

So the lesson here is:

Velocity is the first derivative of position

$$\frac{meters}{seconds}$$

$$h'(t) = v(t)$$

Let's look at some free fall equations from Physics, shall we?

- $d_y = v_{iy}\,t + \frac{1}{2}a_y\,t^2$
- $v_{fy} = v_{iy} + a_y\,t$

Apply the Power Rule to d_y … and you get v_{fy}

Keeping in mind that v_{iy} and a_y are both constants…

Constants

$$h(t) = \frac{1}{2}gt^2 + v_0t + h_0$$

$$h'(t) = gt + v_0$$ ← Velocity

What about the second derivative?

$$h''(t) = g$$ ← Acceleration

So the lesson here is:

Velocity is the first derivative of position

$$\frac{meters}{seconds}$$

$$h(t) = \frac{1}{2}gt^2 + v_0 t + h_0$$

$$h'(t) = gt + v_0$$

$$h''(t) = g$$

What about the second derivative? What are the units on a velocity graph?

Acceleration is the second derivative of position

$$\frac{meters/\sec}{seconds}$$

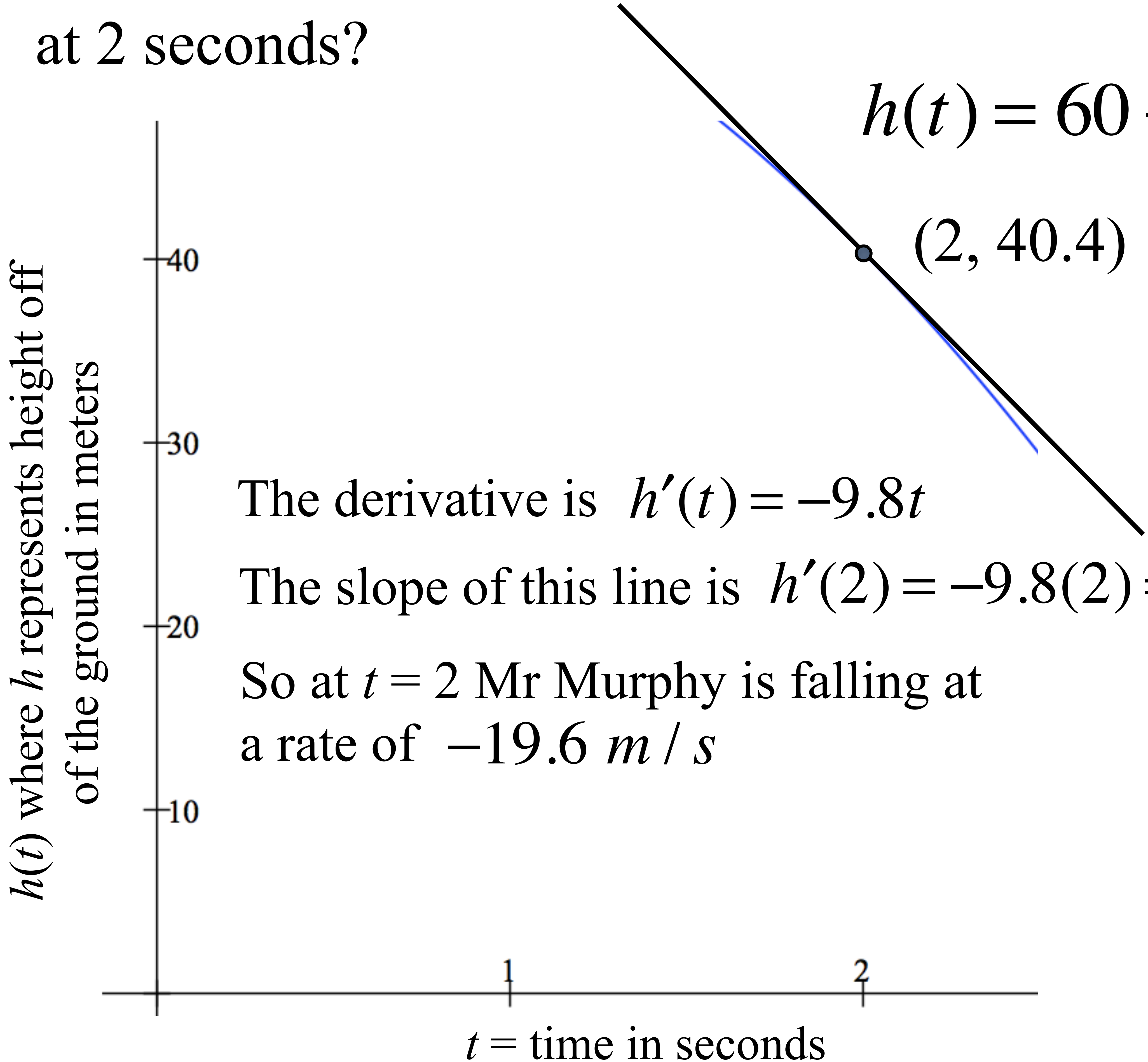
or

$$\frac{meters}{(seconds)^2}$$

$$h''(t) = v'(t) = a(t)$$

at 2 seconds?

$$h(t) = 60 - 4.9t^2$$



The derivative is $h'(t) = -9.8t$

The slope of this line is $h'(2) = -9.8(2) = -19.6$

So at $t = 2$ Mr Murphy is falling at a rate of $-19.6\ m/s$

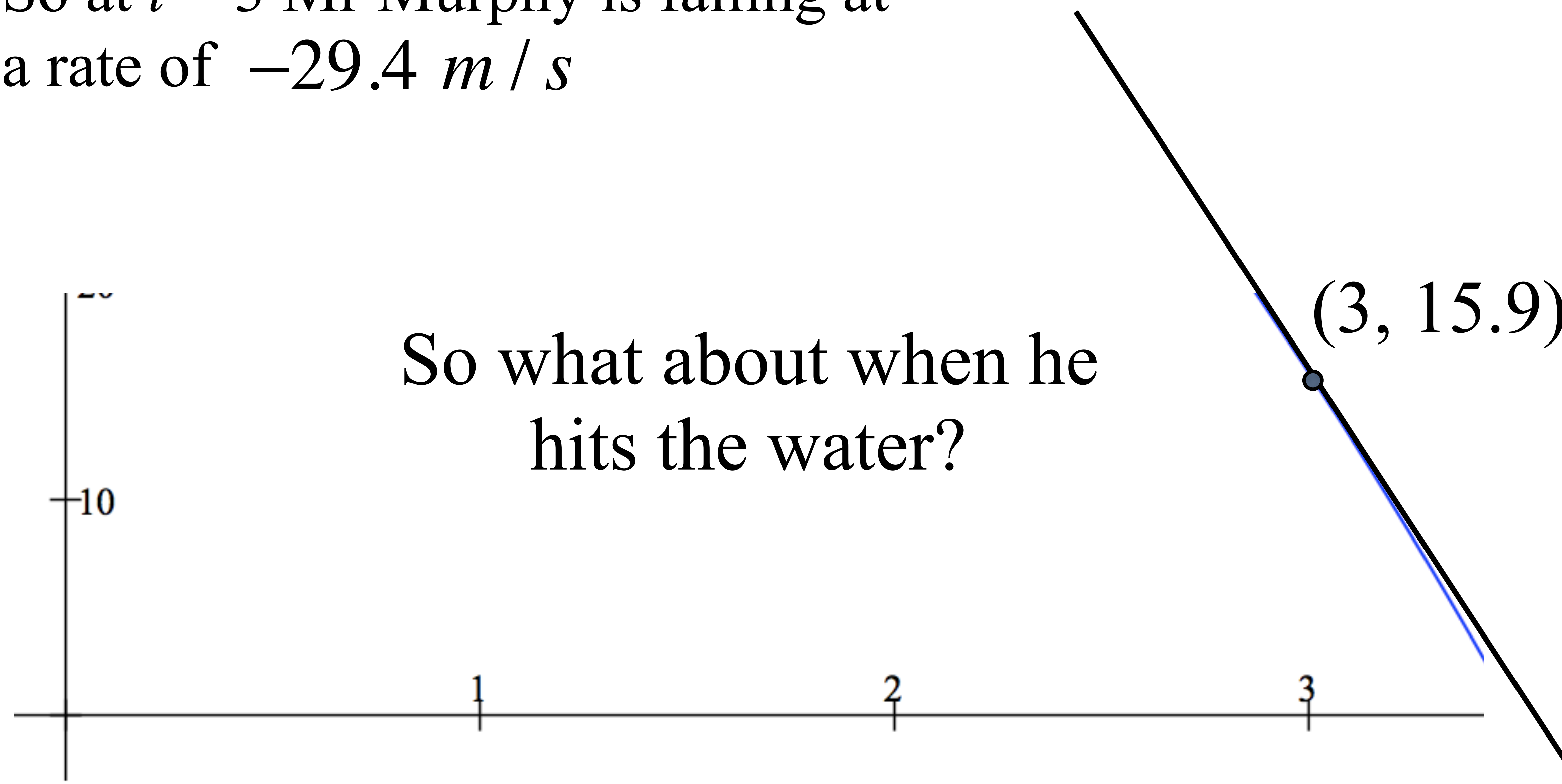
at 3 seconds?

$$h(t) = 60 - 4.9t^2$$

The derivative is $h'(t) = -9.8t$

The slope of this line is $h'(3) = -9.8(3) = -29.4$

So at $t = 3$ Mr Murphy is falling at a rate of $-29.4\ m/s$

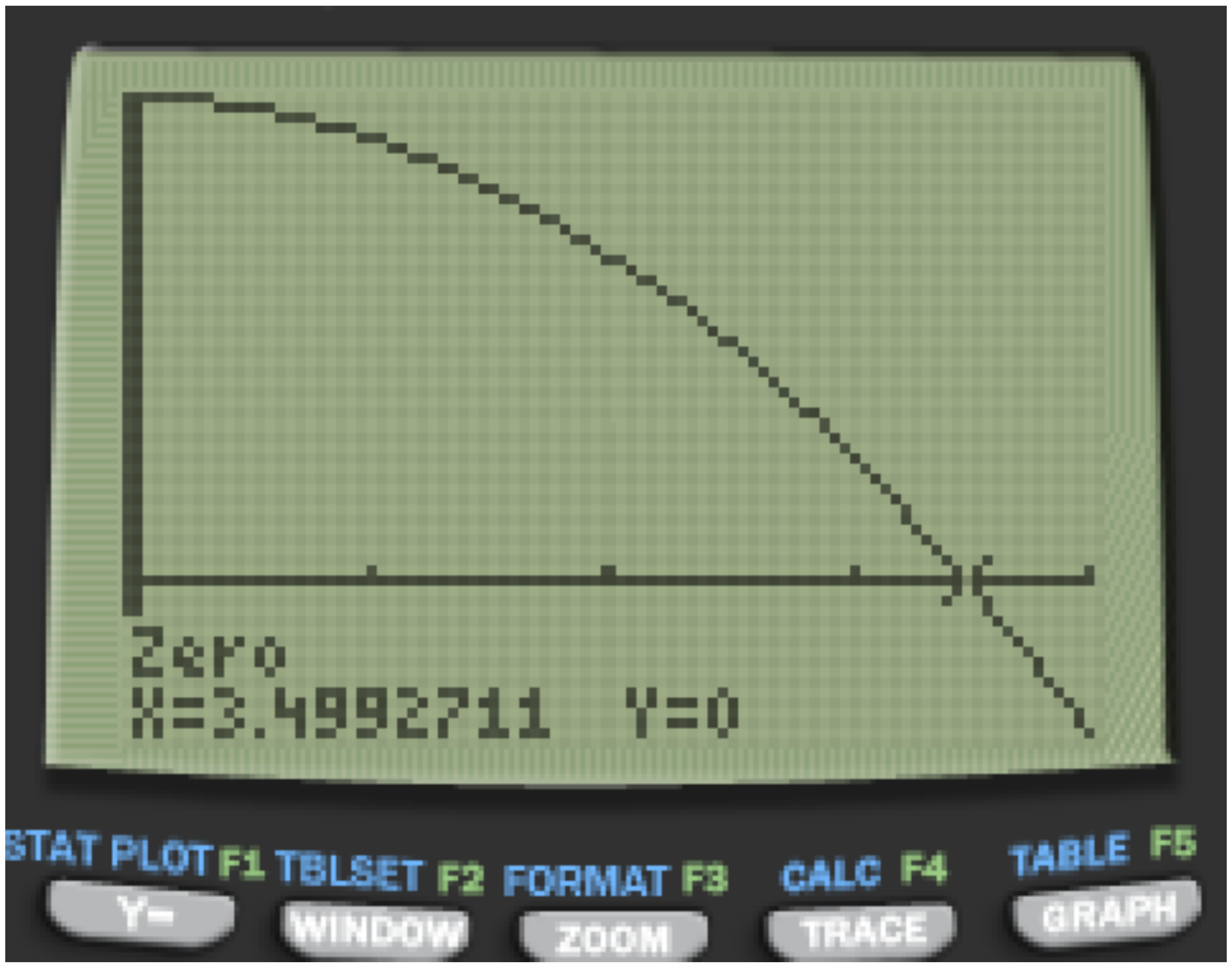


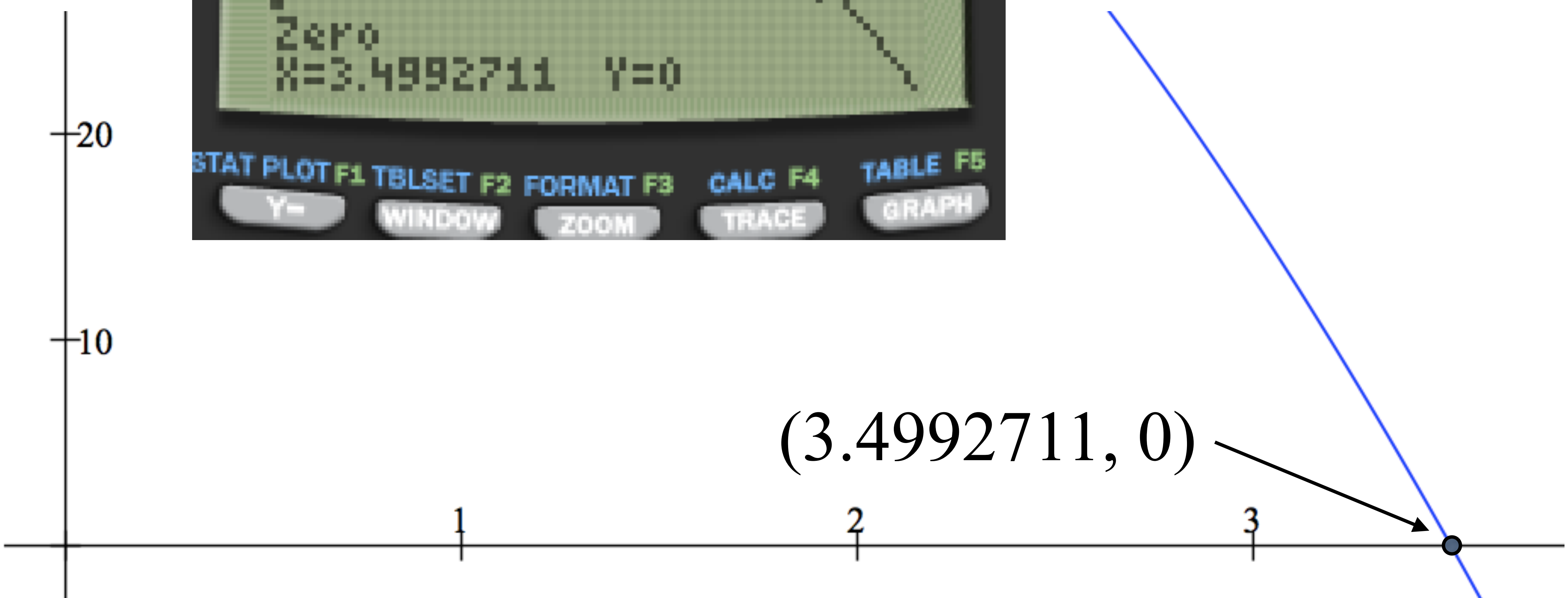
So what about when he hits the water?

The location on the graph at which he hits the water is the *x*-intercept

$$h(t) = 60 - 4.9t^2$$

So let's use the calculator to find the exact location of the *x*-intercept





$$h(t) = 60 - 4.9t^2$$

The derivative is $h'(t) = -9.8t$

The slope of this line is $h'(3) \approx -9.8(3.4992711) \approx -34.2934$

So when he hits the water, Mr Murphy is falling at a rate of

$$-34.2934\ m/s$$

So what about when he hits the water?