It’s all out war between Jacob and Matteo now!

“My Brawl Ball win rate is waaaay higher than yours!”, he shouts at Matteo. Cooler heads prevail when Sam and Lola (mainly to get some peace and quiet in order to resume their more important topics with Sophia and Alexandra) decide to take random samples of their results.

This is called a two proportion *z* test

What’s a two proportion *z* test?

Not much different than a single proportion test.

Let’s start by stating the null and alternate hypotheses for this test?

We start by assuming the two proportions are equal

$$H_0 : p_J = p_M$$

Just like before, the alternate assumes that they are somehow not equal

$$H_a : p_J > p_M$$

Two Sample Hypothesis Tests for Proportions

Note #3: H_0 ALWAYS gets an = ...even if the wording in the problem sounds like it shouldn't

$$H_0: p_1 = p_2 \quad \text{or} \quad p_1 - p_2 = 0$$

Note #1: Use colons

$$H_a: p_1 \begin{array}{c}\neq\\<\\>\end{array} p_2 \quad \text{or} \quad p_1 - p_2 \begin{array}{c}\neq\\<\\>\end{array} 0$$

Note #2: Use only PARAMETERS in your hypothesis...although there will be some problems where we'll use words/sentences

Note #4: The symbol used in the alternate will come from the context of the problem

$\neq$ - two-sided test, equivalent to a Confidence Interval (CI)

$\left.\begin{array}{c}<\\>\end{array}\right\}$ - one-sided test

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After trudging through 70 samples of Jacob's matches and 80 samples of Matteo's, they get the following sample proportions:

Let's check assumptions first

$$\hat{p}_J = \frac{59}{70} \qquad \hat{p}_M = \frac{63}{80}$$

$$n\hat{p}_J \geq 10 \longrightarrow 70\frac{59}{70} \geq 10 \qquad n\hat{p}_M \geq 10 \longrightarrow 80\frac{63}{80} \geq 10$$

$$n(1-\hat{p}_J) \geq 10 \longrightarrow 70\frac{11}{70} \geq 10 \qquad n(1-\hat{p}_M) \geq 10 \longrightarrow 80\frac{17}{80} \geq 10$$

Since we know they play Brawl Starts ***a lot***, this is easily less than 10%

It's all out war between Jacob and Matteo now!

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The formula for the critical value (also on the AP Exam) is shown below

$$z = \frac{(\hat{p}_J - \hat{p}_M) - (p_J - p_M)}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_J} + \frac{1}{n_M}\right)}}$$

Since we presume them to be equal this will always be zero

We have a formula for this

$$\hat{p}_c = \frac{x_J + x_M}{n_J + n_M}$$

$$\hat{p}_c = \frac{59 + 63}{70 + 80} = 0.81\overline{3}$$

$$z = \frac{\left(\frac{59}{70} - \frac{63}{80}\right) - 0}{\sqrt{0.81\overline{3}(1 - 0.81\overline{3})\left(\frac{1}{70} + \frac{1}{80}\right)}} = 0.8681$$

Two Sample Hypothesis Tests for Proportions

Steps in Two Sample Proportion Hypothesis Testing

1. $p_J =$ Jacob's win proportion

   $p_M =$ Matteo's win proportion

2. $H_0 : p_{PF} = p_N$

3. $H_a : \begin{cases} p_J < p_M \\ p_J \neq p_M \\ p_J > p_M \end{cases}$

4. State α.

5. Assumptions:

   1. Random **Independent** Samples
   2. $n_1\hat{p}_1 \geq 10,\ n_1(1-\hat{p}_1) \geq 10$

      $n_2\hat{p}_2 \geq 10,\ n_2(1-\hat{p}_2) \geq 10$
   3. SSSTRP

6. 2 Sample Proportion z Test

7. $df = N/A$

only use $\hat{p}_c$ on Hypothesis Tests, not on CIs

$$\hat{p}_c = \frac{\text{total number of successes}}{\text{total number of trials}}$$

$$\hat{p}_c = \frac{x_{BA} + x_{AA}}{n_{BA} + n_{AA}}$$

8/9. $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \#$$

10. $P - value =$

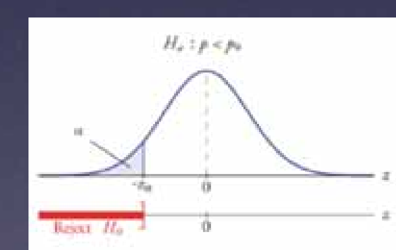
$$P(z > \#) = normalcdf(\#, 1E99, 0, 1)$$
$$P(z < \#) = normalcdf(-1E99, \#, 0, 1)$$

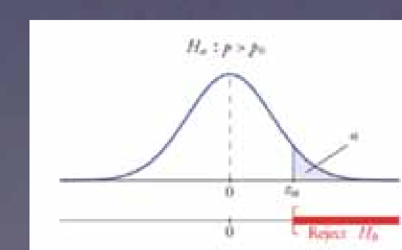
} one-sided tests

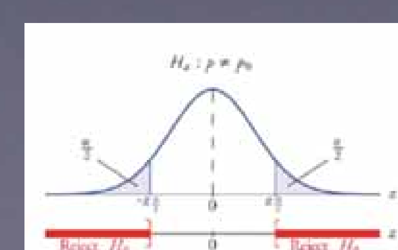
$$2P(z > \#) = 2 * normalcdf(\#, 1E99, 0, 1)$$
$$2P(z < \#) = 2 * normalcdf(-1E99, \#, 0, 1)$$

} two-sided tests

11.







12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0.
    2. Summarize in context discussing H_a.

It's all out war between Jacob and Matteo now!

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$$z = \frac{\left(\frac{59}{70} - \frac{63}{80}\right) - 0}{\sqrt{0.81\overline{3}(1 - 0.81\overline{3})\left(\frac{1}{70} + \frac{1}{80}\right)}} = 0.8681$$

normalcdf(0.8681, 1E99) = 0.1927

This is much bigger than any level of significance, even 0.1

Since our *p-value* is way above any reasonable level of significance, we fail to reject the null hypothesis. We do not have sufficient evidence that Jacob's win rate is better than Matteo's

Does a 95% confidence interval contain the possibility that Jacob is wrong?

Confidence Intervals

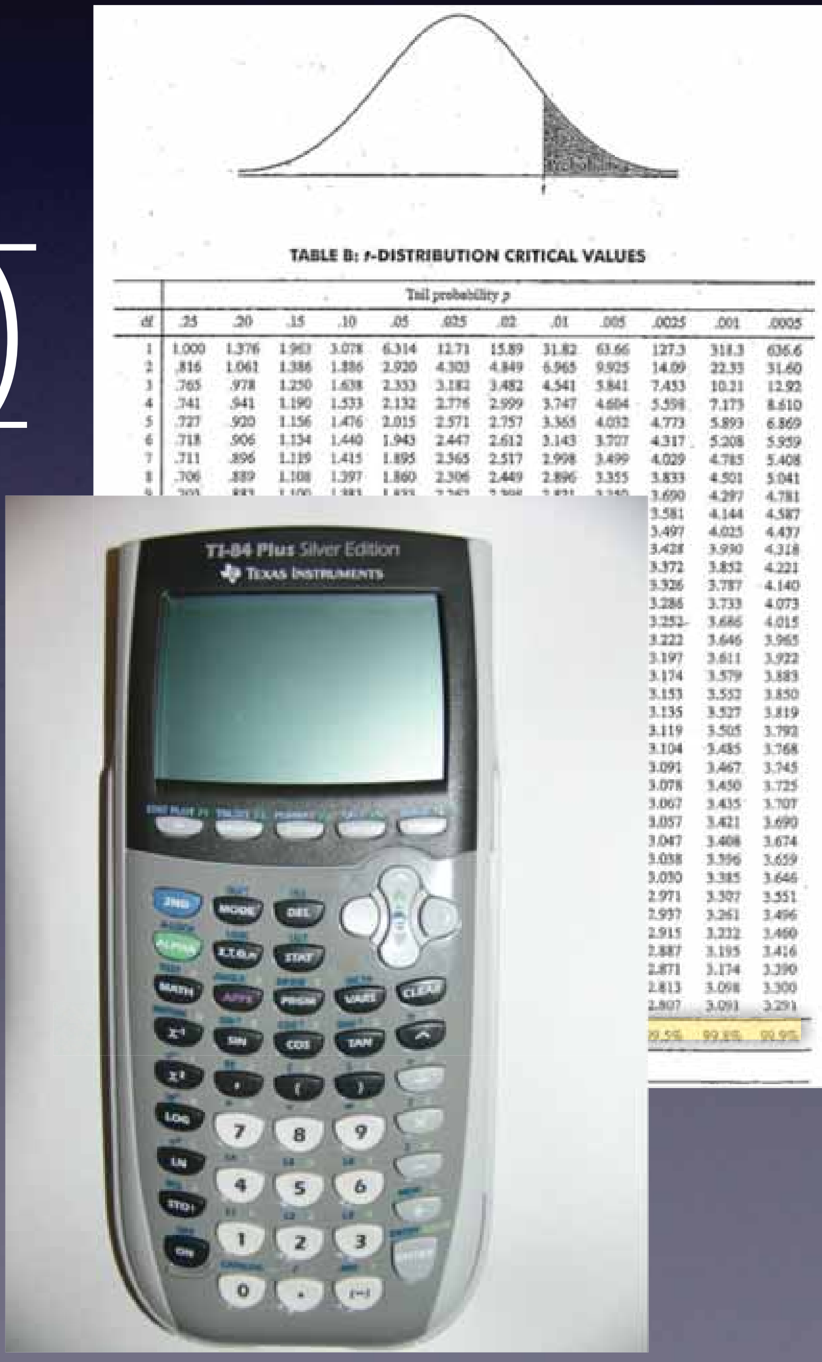
General CI Formula

$$\text{Statistic} \pm \left(\text{Critical Value}\right)\left(\text{Standard Deviation}\right)$$

2 Sample Proportion z CI Formula

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z\sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1} + \frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n_2}}$$

Use Table or Calculator
to get the z critical value



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After trudging through 70 samples of Jacob's matches and 80 samples of Matteo's, they get the following sample proportions:

Does a 95% confidence interval contain the possibility that Jacob is wrong?

$$\hat{p}_J - \hat{p}_M \pm z\sqrt{\frac{\hat{p}_J(1-\hat{p}_J)}{n_J} + \frac{\hat{p}_M(1-\hat{p}_M)}{n_M}}$$

$$\frac{59}{70} - \frac{63}{80} \pm 1.96\sqrt{\frac{\frac{59}{70}\left(1-\frac{59}{70}\right)}{70} + \frac{\frac{63}{80}\left(1-\frac{63}{80}\right)}{80}} = (-0.0684, 0.17907)$$

We are 95% confident that Jacob's win rate is between approximately 7% below and 18% above Matteo's rate. There is a possibility that he is ***wrong***

It's all out war between Jacob and Matteo now!

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After trudging through 70 samples of Jacob's matches and 80 samples of Matteo's, they get the following sample proportions:

One more thing before we move on: Describe a Type I and Type II error in this scenario and the consequence of each

A Type I error would be rejecting the assumption that their win rates are equal which would overestimate Jacob's talents

A Type II error would be not rejecting the assumption that their win rates are equal which would under estimate Jacob's skills and leave him sad that no one realizes how good a player he is

At one time, Finn could boast scores of over 1000 in Crossy Road in 90% of his games. Lately, getting distracted by his art projects on his iPad, his percentage seems to have dropped or at least Josh thinks so. Finn insists that this is not the case.

They do a comparison of 125 randomly selected recorded scores from before Finn became an artist (BA) to 110 randomly selected scores taken after Finn discovers his artistic talents (AA).

Conduct a 2 proportion z test to determine whether Josh has sufficient evidence to reject Finn's claim that his scores have not dropped off

What's a two proportion *z* test?

Not much different than a single proportion test.

Two Sample Hypothesis Tests for Proportions

Steps in Two Sample Proportion Hypothesis Testing

1. $p_{BA} =$ Proportion before art
   $p_{AA} =$ Proportion after art

5

2. $H_0 : p_{PF} = p_N$

3. $H_a : \begin{cases} p_{BA} < p_{AA} \\ p_{BA} \neq p_{AA} \\ p_{BA} > p_{AA} \end{cases}$

4. State α.

<u>Assumptions:</u>

1. Random **Independent** Samples
2. $n_1\hat{p}_1 \geq 10,\ n_1\left(1-\hat{p}_1\right) \geq 10$
   $n_2\hat{p}_2 \geq 10,\ n_2\left(1-\hat{p}_2\right) \geq 10$
3. SSSTRP

6. 2 Sample Proportion z Test

7. $df = N/A$

only use $\hat{p}_c$ on Hypothesis Tests, not on CIs

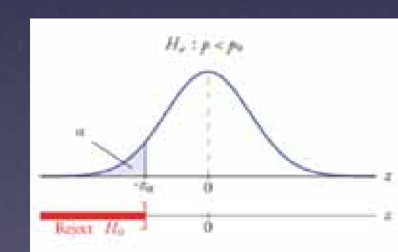
$$\hat{p}_c = \frac{\text{total number of successes}}{\text{total number of trials}}$$

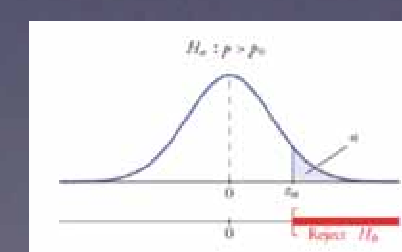
$$\hat{p}_c = \frac{x_{BA} + x_{AA}}{n_{BA} + n_{AA}}$$

8/9. $$z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\hat{p}_c\left(1-\hat{p}_c\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \#$$

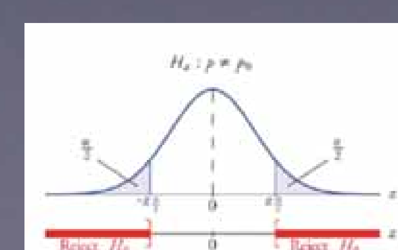
11.

10. $P-value =$
$$P\left(z > \#\right) = normalcdf\left(\#, 1E99, 0, 1\right)$$
$$P\left(z < \#\right) = normalcdf\left(-1E99, \#, 0, 1\right)$$
} one-sided tests
$$2P\left(z > \#\right) = 2 * normalcdf\left(\#, 1E99, 0, 1\right)$$
$$2P\left(z < \#\right) = 2 * normalcdf\left(-1E99, \#, 0, 1\right)$$
} two-sided tests





12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0.
    2. Summarize in context discussing H_a.



At one time, Finn could boast scores of over 1000 in Crossy Road in 90% of his games. Lately, getting distracted by his art projects on his iPad, his percentage seems to have dropped or at least Josh thinks so. Finn insists that this is not the case.

They do a comparison of 125 randomly selected recorded scores from before Finn became an artist (BA) to 110 randomly selected scores taken after Finn discovers his artistic talents (AA).

Suppose the sample from "before art" has 92% of his games resulting in a score of over 1000 while a sample drawn "after art" has 80% of his games having a score of over 1000

$$H_0 : p_{BA} = p_{AA} \qquad p_{BA} = 0.92 \qquad p_{AA} = 0.8$$

$$H_a : p_{BA} > p_{AA} \qquad n_{BA} = 125 \qquad n_{AA} = 110$$

$$\hat{p}_c = \frac{x_{BA} + x_{AA}}{n_{BA} + n_{AA}} = \frac{0.92 * 125 + 0.80 * 110}{125 + 110} = \frac{203}{235}$$

At one time, Finn could boast scores of over 1000 in Crossy Road in 90% of his games. Lately, getting distracted by his art projects on his iPad, his percentage seems to have dropped or at least Josh thinks so. Finn insists that this is not the case.

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Suppose the sample from "before art" has 92% of his games resulting in a score of over 1000 while a sample drawn "after art" has 80% of his games having a score of over 1000

Let's check assumptions first

$n\hat{p}_{BA} \geq 10 \longrightarrow 0.92(125) \geq 10$

$n\hat{p}_{AA} \geq 10 \longrightarrow 0.8(110) \geq 10$

$n(1-\hat{p}_{BA}) \geq 10 \longrightarrow 0.08(125) \geq 10$

$n(1-\hat{p}_{AA}) \geq 10 \longrightarrow 0.2(110) \geq 10$

Since we know he plays Crossy Road ***a lot***, this is easily less than 10%

At one time, Finn could boast scores of over 1000 in Crossy Road in 90% of his games. Lately, getting distracted by his art projects on his iPad, his percentage seems to have dropped or at least Josh thinks so. Finn insists that this is not the case.

$$z = \frac{0.92 - 0.8}{\sqrt{\frac{203}{235}\left(1 - \frac{203}{235}\right)\left(\frac{1}{125} + \frac{1}{110}\right)}} = 2.676$$

normalcdf(2.676, 1E99) = 0.0037

This is much smaller than any level of significance, even 0.01

Since our *p-value* is below even a small level of significance such as 0.01, we reject the null hypothesis. Josh has significant statistical evidence that taking up art projects on his iPad has weakened Finn's Crossy Road game

Does a 95% confidence interval contain the possibility that Josh is wrong?

Confidence Intervals

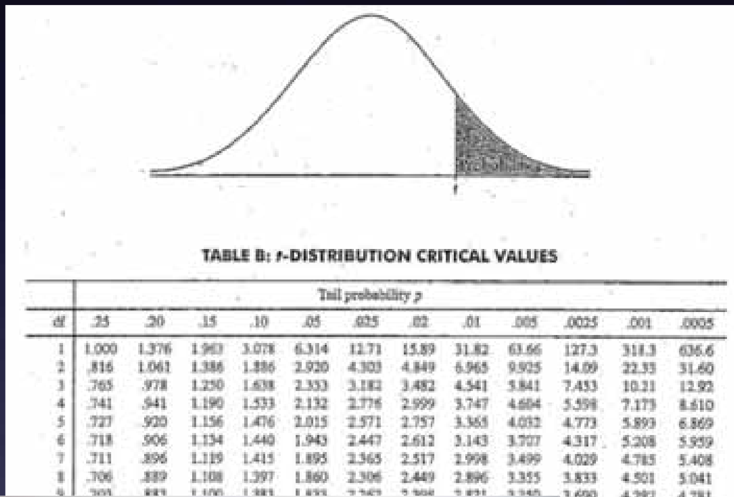
General CI Formula

$$\text{Statistic} \pm (\text{Critical Value})(\text{Standard Deviation})$$

2 Sample Proportion z CI Formula

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z\sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1} + \frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n_2}}$$

Use Table or Calculator
to get the z critical value



At one time, Finn could boast scores of over 1000 in Crossy Road in 90% of his games. Lately, getting distracted by his art projects on his iPad, his percentage seems to have dropped or at least Josh thinks so. Finn insists that this is not the case.

Does a 95% confidence interval contain the possibility that Josh is wrong?

$$0.92 - 0.8 \pm z\sqrt{\frac{0.92(1-0.92)}{125} + \frac{0.8(1-0.8)}{110}} = (0.0314,\ 0.2086)$$

Since we have 95% confidence that the difference between the two proportions is at least 0.0314 > 0, we are confident that Finn's proportion before art is greater than after art which means we are pretty sure that Josh is right.

Interpretation for Two Sample Proportion Confidence Intervals

We are __% confident that $p_1 - p_2$, the true difference in proportions of _____, is between ___ and ___.

Interpretation for the Confidence <u>Level</u> of Two Sample Proportion Confidence Intervals

We used a method to construct this estimate that in the long run will successfully capture the true value of $p_1 - p_2$ ___% of the time.

ALWAYS check your assumptions and interpret your interval, even you are not specifically asked to in the problem. Just do it. Seriously.

General Work Flow -

1. Assumptions
2. Construction of Interval
3. Interpretation(s)

Free Response

1. Even though landlords participating in a telephone survey indicated that they would generally be willing to rent to persons with AIDS, it was wondered whether this was true in actual practice. To investigate, researchers independently selected two random samples of 80 advertisements for rooms for rent from newspaper advertisements in three large cities. An adult male caller responded to each ad in the first sample of 80 and inquired about the availability of the room and was told that the room was still available in 61 of these calls. The same caller also responded to each ad in the second sample. In these calls, the caller responded to each ad in the second sample. In these calls, the caller indicated that he was currently receiving some treatment for AIDS and was about to be released from the hospital and would require a place to live. The caller was told that a room was available in 32 of these calls. Based on this information, the study concluded that "reference to AIDS substantially decreased the likelihood of a room being described as available." Do the data support this conclusion? Carry out a hypothesis test with $\alpha = 0.01$

1. p_1= true proportion of rooms described available when there is NO AIDS reference
   p_2= true proportion of rooms described available when there is an AIDS reference

2. $H_0 : p_1 = p_2$

3. $H_a : p_1 > p_2$

4. $\alpha = 0.01$

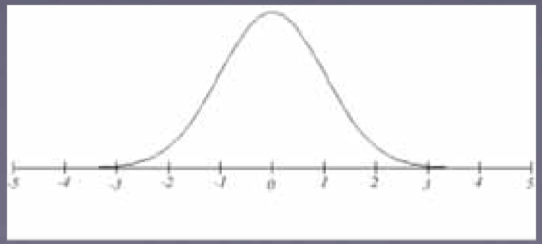
5. Assumptions:
   1. Independent Random Samples ✓
   2. $n_1 p_1 = 80(0.76) = 61 \geq 10$ $\quad n_1(1-p_1) = 80(1-0.76) = 19 \geq 10$ ✓
      $n_2 p_2 = 80(0.4) = 32 \geq 10$ $\quad n_2(1-p_2) = 80(1-0.4) = 48 \geq 10$ ✓
   3. SSSRTP✓

6. 2 Sample Proportion z Test

8/9. $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.76 - 0.4) - (0)}{\sqrt{0.582(1-0.582)\left(\frac{1}{80} + \frac{1}{80}\right)}} = 4.647$$

10. $P-value = P(z > 4.647) = normalcdf(4.647, 1E99, 0, 1) = 0$

11.



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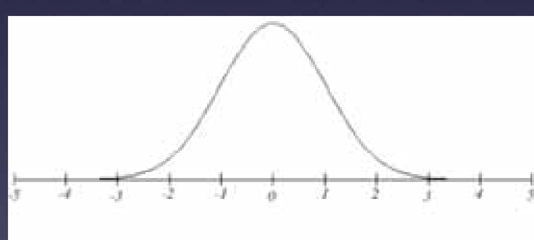
2 Sample Proportion *z* Test

10. $P - value = P(z > 4.647) = normalcdf(4.647, 1E99, 0, 1) = 0$

11.



12. Because our *P* - *value* = 0 < 0.01 = α, we reject H_0 at the 0.01 level of significance. We have evidence that the true proportion of rooms described available with no AIDS reference is larger than with an AIDS reference.