Exponential and Logarithmic Derivatives

The Exponential Rules: There are specific rules for derivatives of logarithmic and exponential functions. The first two basic ones are

$$\frac{d}{dx}a^x = a^x \ln a$$

This implies that

$$\frac{d}{dx}e^x = e^x \ln e$$

Therefore

$$\frac{d}{dx}e^x = e^x$$

Not forgetting of course that:

$$\ln x = \log_e x$$

And we know that

$$\ln e = 1$$

Yes, these derivatives are that simple

$$\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

This implies that

$$\frac{d}{dx}\log_e x = \frac{1}{x \ln e}$$

Therefore

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\left(e^{x}\right)=e^{x} \qquad \frac{d}{dx}\ln x=\frac{1}{x}$$

But what about...

$$\frac{d}{dx}e^{x^2}$$

here we will need the Chain Rule

$$\frac{d}{dx}e^{x^2}=e^{x^2}\cdot 2x=\boxed{2xe^{x^2}}$$

Answer

Derivative of the inside

$$\frac{d}{dx}e^{x^2} = e^{x^2} \cdot 2x = 2xe^{x^2}$$ ← It's Sign Pattern time

$$2xe^{x^2} = 0$$

$2x$ → will determine where the derivative is positive or negative

e^{x^2} → is always a positive number

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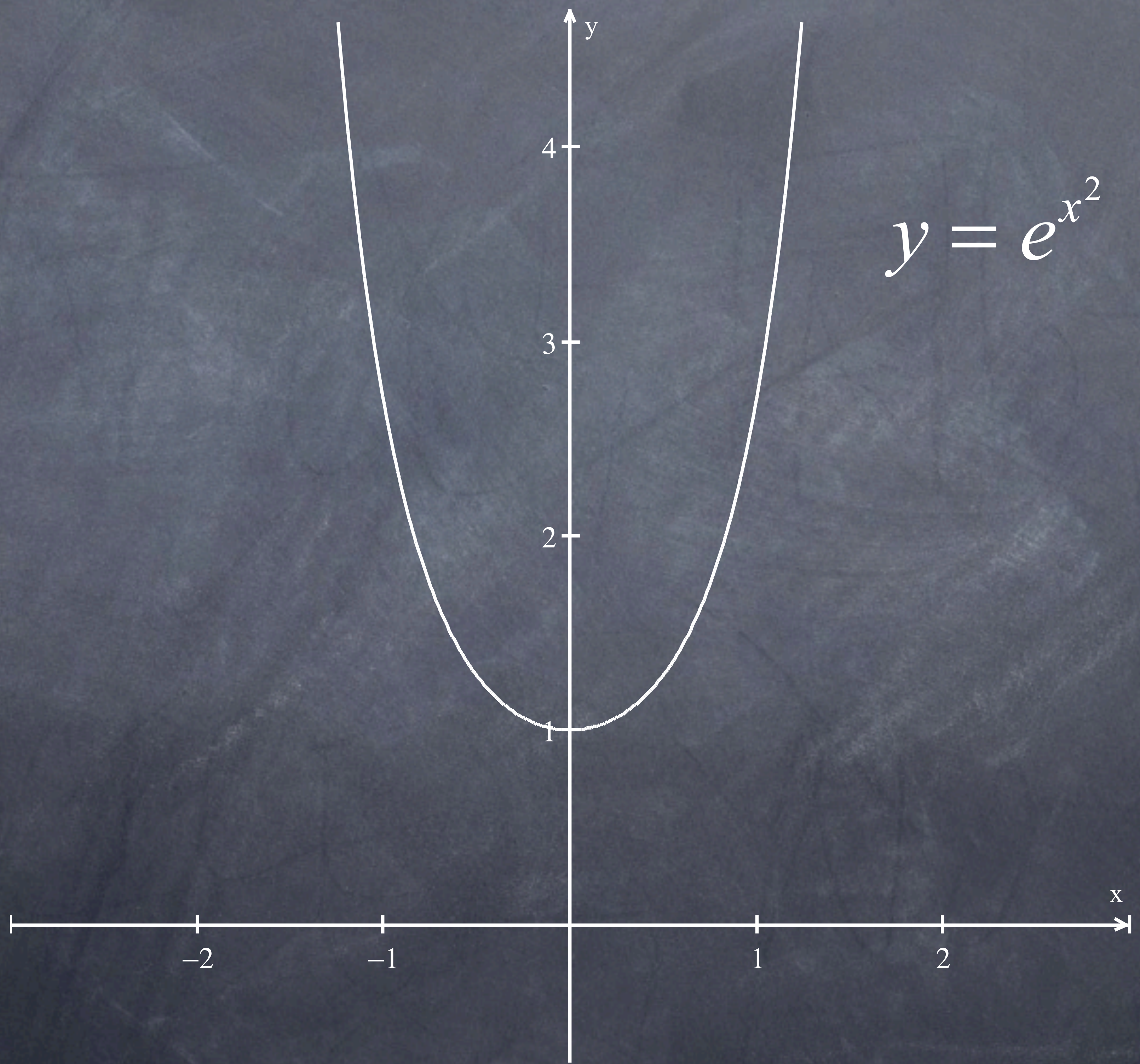
$$\frac{d}{dx}e^{x^2} = e^{x^2} \cdot 2x = 2xe^{x^2}$$



$$y = e^{x^2}$$

$$\frac{d}{dx}\left(e^{x}\right)=e^{x} \qquad \frac{d}{dx}\ln x=\frac{1}{x}$$

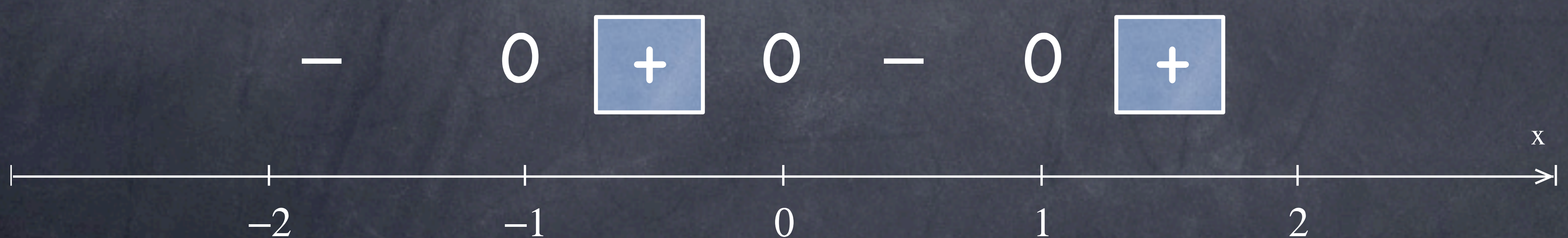
But what about...

$$\frac{d}{dx}\ln(x^{3}-x)$$

First we will need to find the domain

Remember that we can only take the log of a number > 0

$$x^{3}-x>0 \longrightarrow x(x^{2}-1)>0 \longrightarrow x(x-1)(x+1)>0$$



So the domain here is $x \in (-1,0) \cup (1,\infty)$

But what about... $\frac{d}{dx}\ln(x^3 - x)$

again we will need the Chain Rule

Derivative of the outside $\left(\frac{d}{dx}\ln x = \frac{1}{x}\right)$

$$\frac{d}{dx}\ln(x^3 - x) = \frac{1}{x^3 - x}\cdot(3x^2 - 1) = \frac{3x^2 - 1}{x^3 - x}$$

Answer

Derivative of the inside

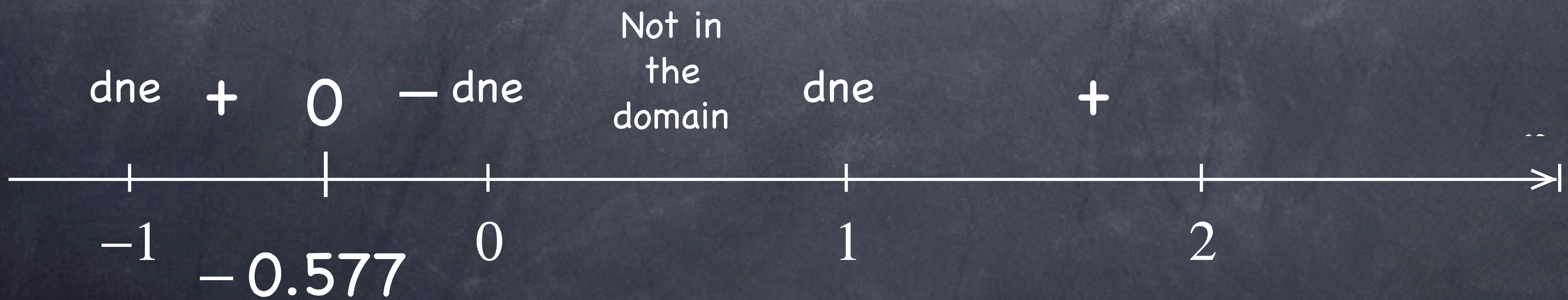
So the domain here is $x \in (-1,0) \cup (1,\infty)$

But what about... $\dfrac{d}{dx}\ln(x^3 - x) = \dfrac{\boxed{3x^2 - 1}}{x^3 - x}$

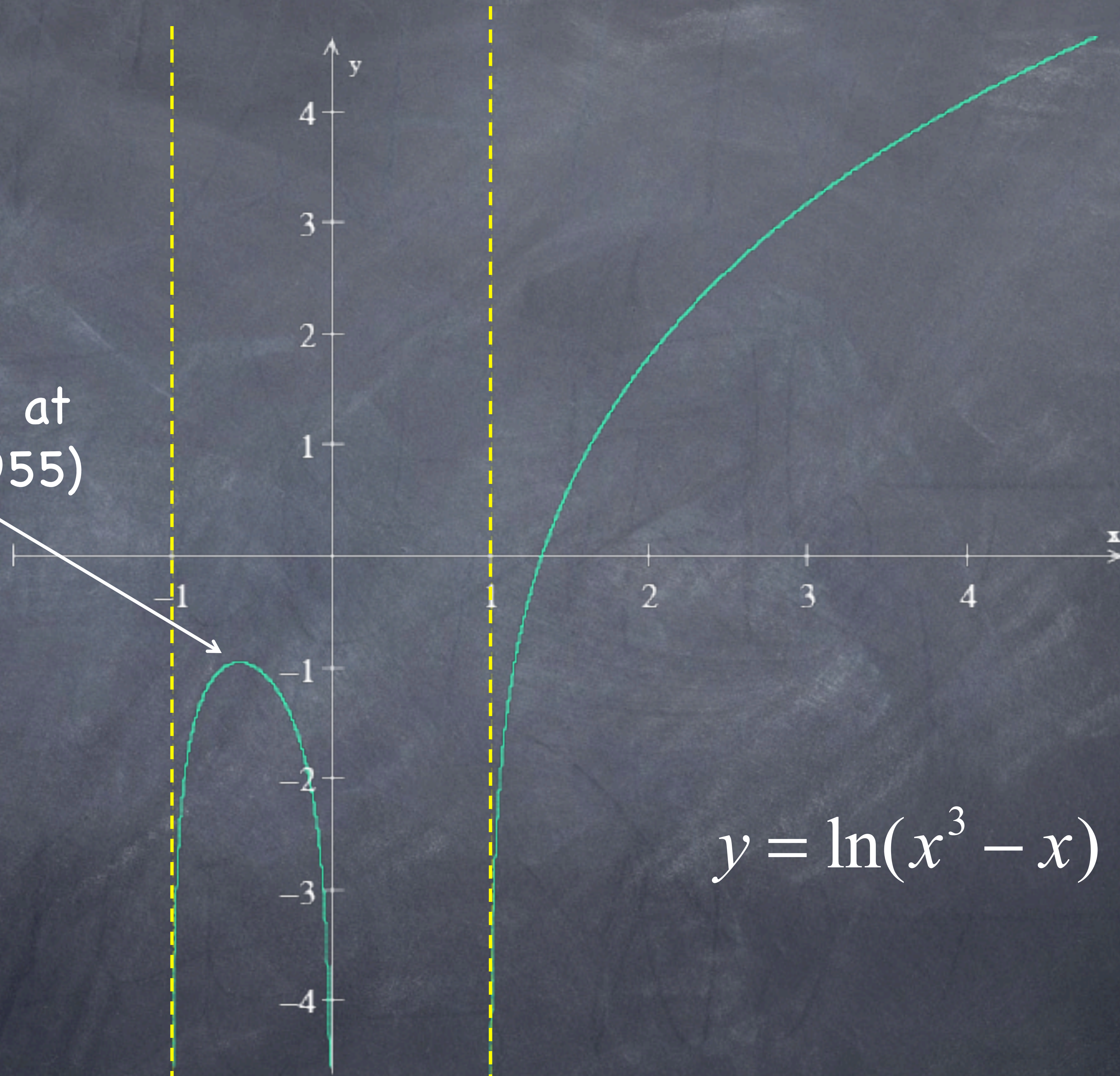
$3x^2 - 1 = 0$ at $x = \pm\dfrac{1}{\sqrt{3}} \approx \pm 0.577$

Note that 0.577 is not in the domain

Now we do another sign pattern to find the extreme values



Relative max at
(-0.577, −0.955)



$$y = \ln(x^3 - x)$$