Traits & Graphs of Radical Functions

Standard 8c: Find the critical values and extreme points of radical functions

Standard 8d: Find all the traits and sketch a radical curve algebraically

8-4: General Radical Curve Sketching

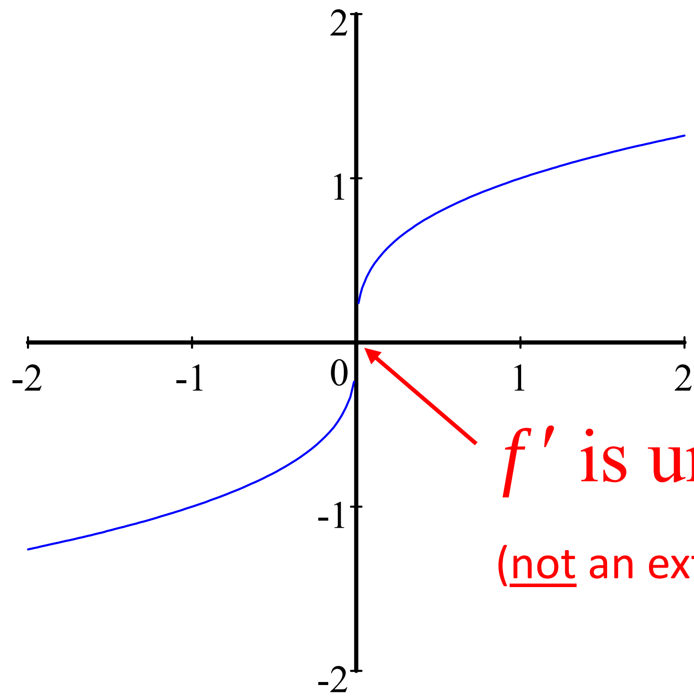
REMEMBER: Radical Traits

1. Domain
2. Zeros
3. *y*-intercept
4. Extreme Points
5. Range
6. End Behavior (EB)

And, if there is a rational function within the radical:

7. POE
8. Vertical Asymptotes

$$y = x^{1/3}$$



f' is undefined.

(<u>not</u> an extreme)

If a value is a critical value, then either

i) $\frac{dy}{dx} = 0$ at that value;

ii) $\frac{dy}{dx}$ does not exist at that value;

or iii) a value at an endpoint of an arbitrarily stated domain.

FINDING ABSOLUTE EXTREMA

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

$$f(x) = x^{2/3}$$

There are no values of x that will make the first derivative equal to zero.

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$$

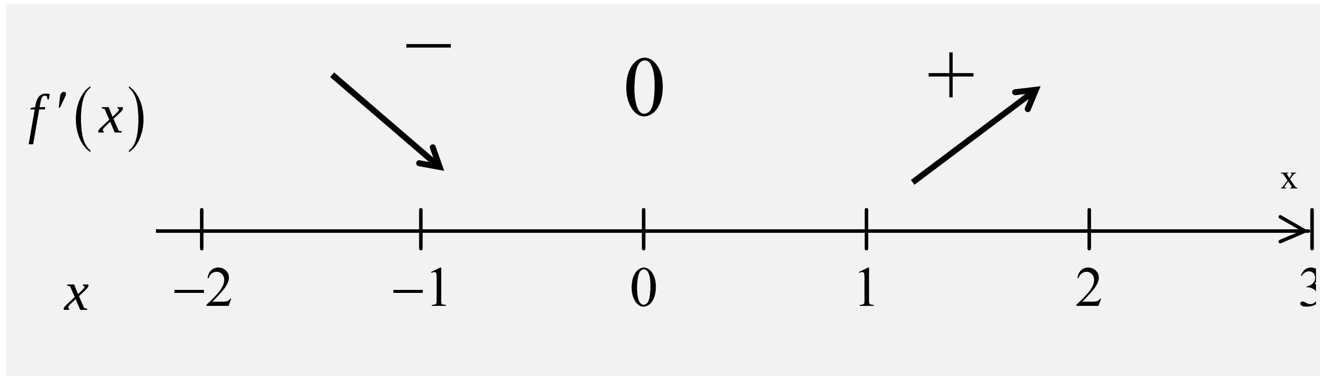
The first derivative is undefined at $x = 0$, so (0,0) is a critical point.

$$f'(x) = \frac{2}{3\left(\sqrt[3]{x}\right)}$$

Because the function is defined over a closed interval, we also must check the endpoints.

$$f(x) = x^{2/3} \qquad D = [-2, 3]$$

$$f'(x) = \frac{2}{3\left(\sqrt[3]{x}\right)}$$



At: $x = 0 \qquad f(0) = 0$

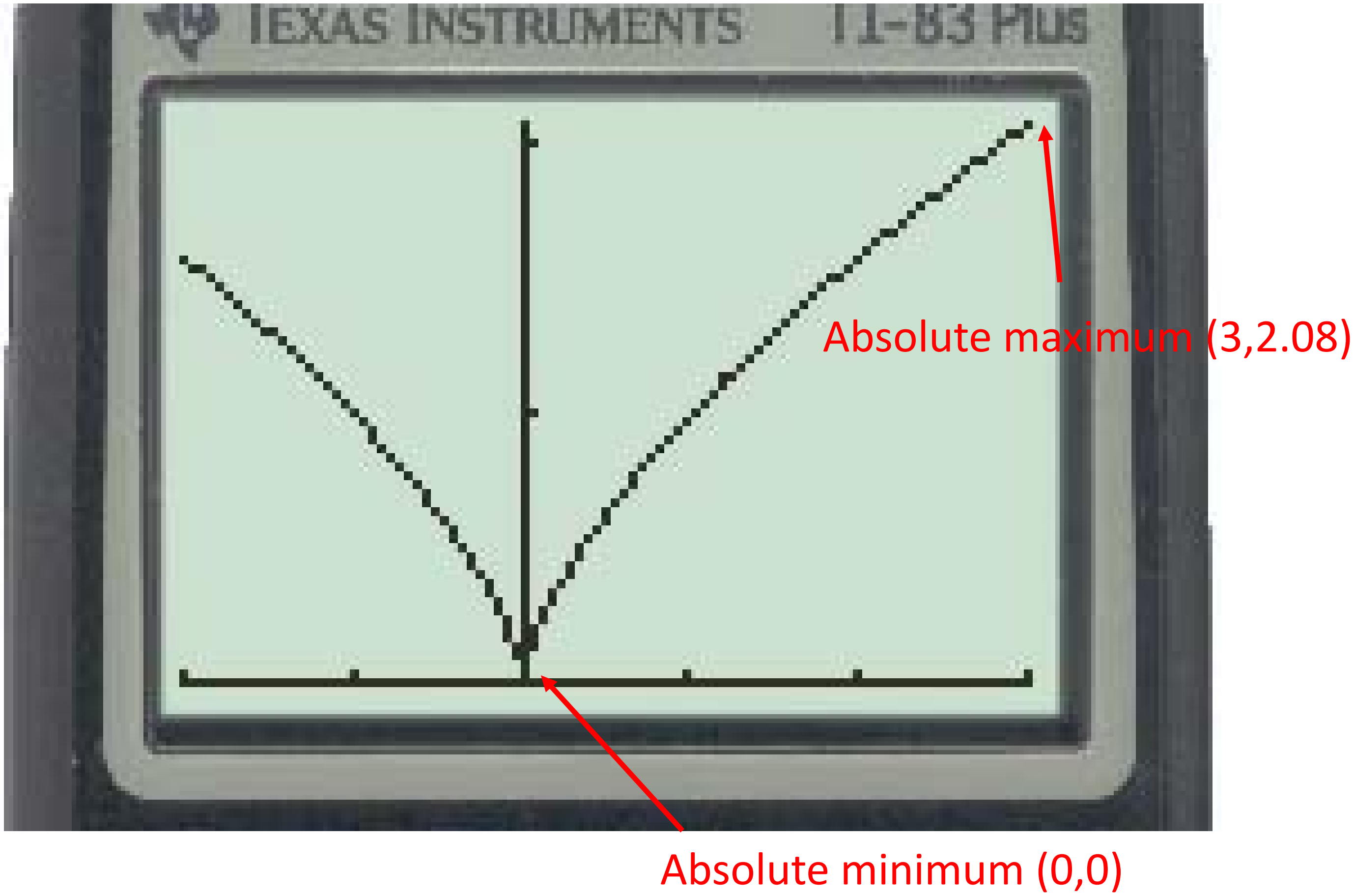
At: $x = -2$

$$f(-2) = (-2)^{\frac{2}{3}} \approx 1.5874$$

At: $x = 3 \qquad f(3) = (3)^{\frac{2}{3}} \approx 2.08008$

Absolute minimum: $(0, 0)$

Absolute maximum: $(3, 2.08)$


TEXAS INSTRUMENTS
TI-83 Plus
Absolute maximum (3,2.08)
Absolute minimum (0,0)

$$f(x) = x^{2/3}$$

1. Domain $x \in [-2, 3]$
2. Zeros $x = 0$
3. *y*-intercept $y = 0$
4. Extreme Points

Absolute minimum:

$$(0, 0)$$

Absolute maximum:

$$(3, 2.08)$$

5. Range $y \in [0, 2.08]$
6. End Behavior (EB)

Since our domain is restricted we won't worry about end behavior

Finding Maximums and Minimums Analytically:

1. Find the derivative of the function, and determine where the derivative is zero or undefined. These are the critical points.

2. Find the value of the function at each critical point.

3. Find values or slopes for points between the critical points to determine if the critical points are maximums or minimums.

4. For closed intervals, check the end points as well.

Find the domain of $y = \sqrt{16x - x^3}$

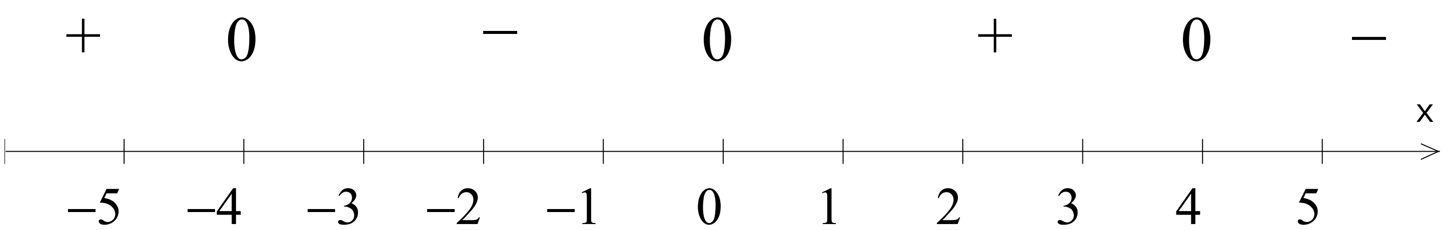
$16x - x^3 \geq 0$

What values of x give us something with a real number square root?

$x(16 - x^2) \geq 0$

$x(4 - x)(4 + x) \geq 0$

Now make a sign pattern number line



$x \leq -4$

$0 \leq x \leq 4$

Differentiate: $y=\sqrt{16x-x^3}$

$$y=\left(16x-x^3\right)^{\frac{1}{2}}$$

Re-write with an exponent

$$y'=\frac{1}{2}\left(16x-x^3\right)^{-\frac{1}{2}}\left(16-3x^2\right)$$

Don't forget the inside

Now find the critical points (Where y' is 0 or undefined)

$$y'=\frac{\left(16-3x^2\right)}{2\sqrt{16x-x^3}}=0$$

or where

$$16-3x^2=0$$

$$16=3x^2$$

$$\frac{16}{3}=x^2$$

$$x=\pm\frac{4}{\sqrt{3}}$$

$$x = \pm\frac{4}{\sqrt{3}} \approx \pm 2.309$$

But the only value that works here is…

Because the other value is not in the domain

$$x = \frac{4}{\sqrt{3}}$$

Why?

Recall that the domain is $x \le -4$, $0 \le x \le 4$

Now find the critical points (Where y' is 0 or undefined)

$$y' = \frac{\left(16 - 3x^2\right)}{2\sqrt{16x - x^3}} = \textit{undefined} \text{ or where}$$

$$16x - x^3 = 0$$

$$x(16 - x^2) = 0$$

$$x = 0, \pm 4$$

So the critical values here are

$$x = -4, 0, \frac{4}{\sqrt{3}}, 4$$

So how do we do this again?

- Find the domain of the radical function
- Differentiate (don't forget the Chain Rule)
- Find the critical points
  - where the derivative is 0 (numerator)
  - where the derivative is undefined (denominator)
- Check the critical points against the domain
- Make a sign pattern to locate the minima and maxima

And one more thing…

So the critical values here are

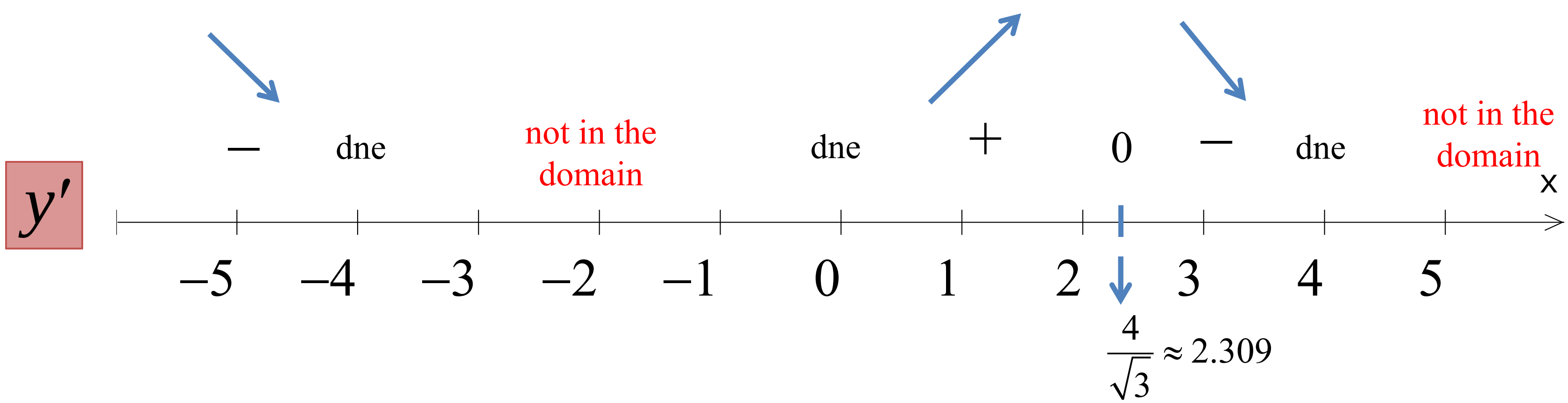
$$x = -4, 0, \frac{4}{\sqrt{3}}, 4$$

So where are the maxima and minima?

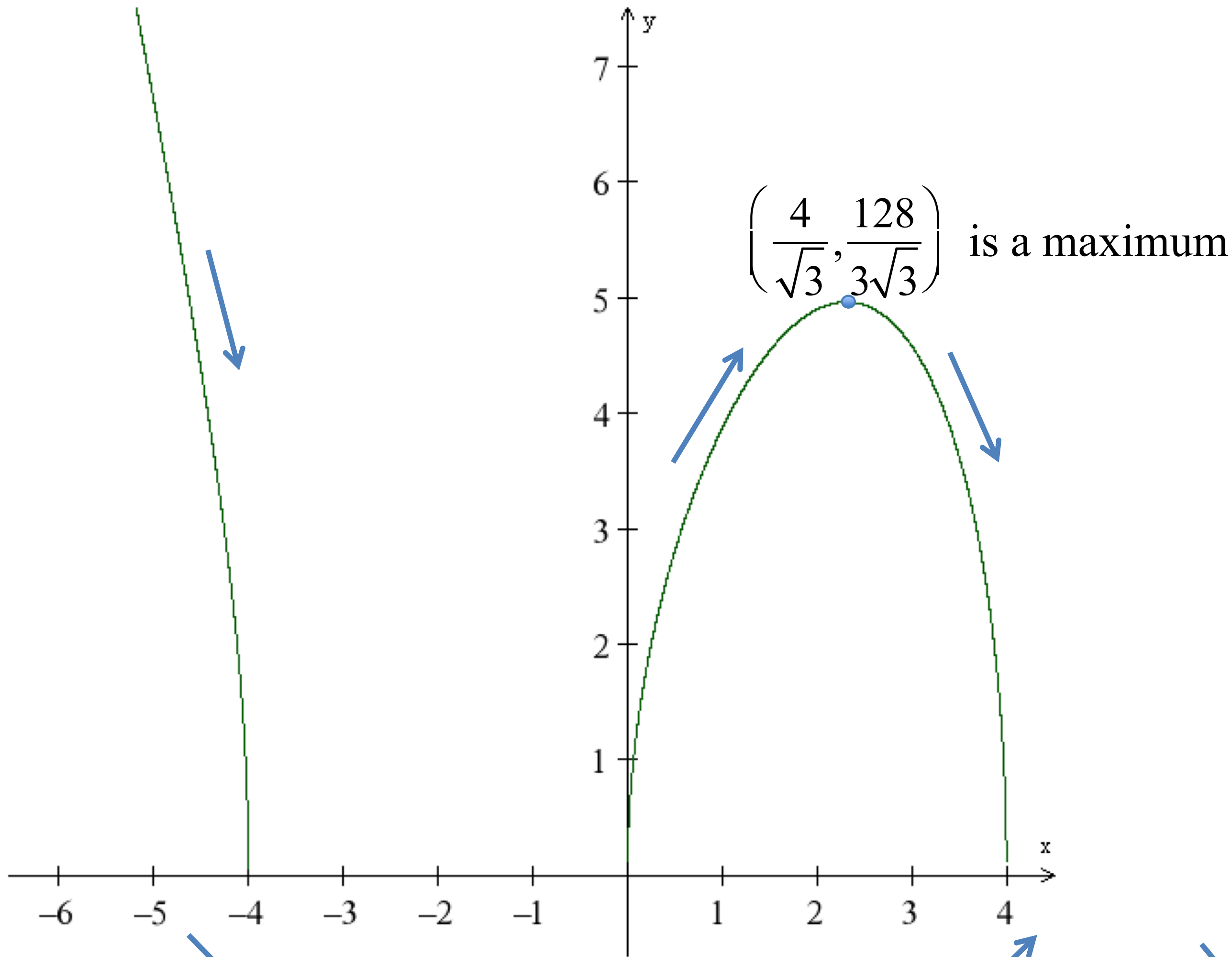
Here is where we will need to make the sign pattern

$$y' = \frac{(16-3x^2)}{2\sqrt{16x-x^3}}$$

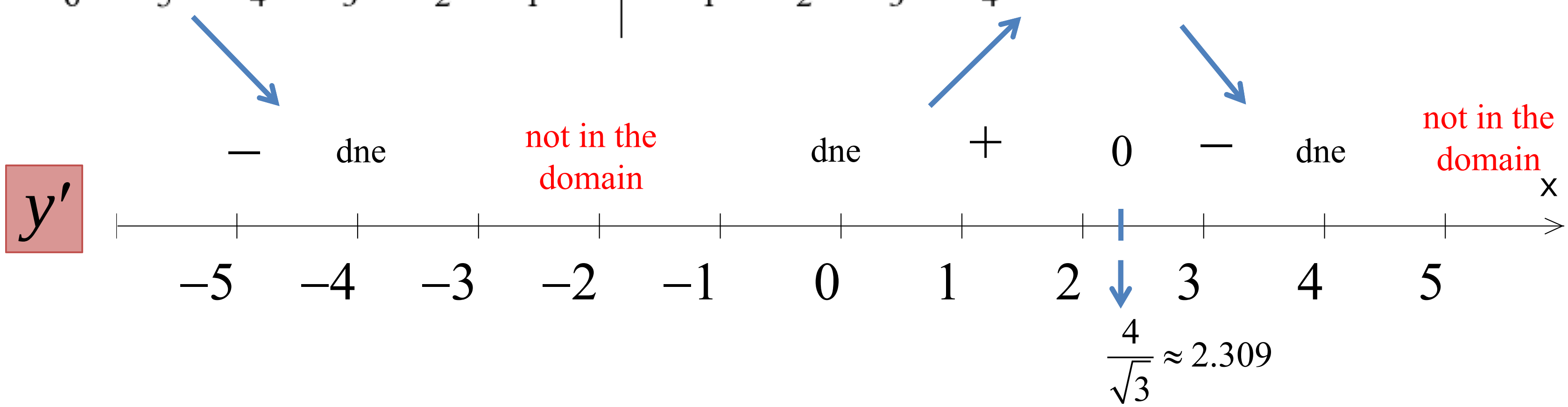
$\left(\frac{4}{\sqrt{3}}, \frac{128}{3\sqrt{3}}\right)$ is a maximum



$$y = \sqrt{16x - x^3}$$



y
7
6
5
4
3
2
1
x
–6
–5
–4
–3
–2
–1
1
2
3
4
$\left(\frac{4}{\sqrt{3}}, \frac{128}{3\sqrt{3}}\right)$ is a maximum



y'
–
dne
not in the domain
dne
+
0
–
dne
not in the domain
x
–5
–4
–3
–2
–1
0
1
2
3
4
5
$\frac{4}{\sqrt{3}} \approx 2.309$

1. Domain $x \in (-\infty, -4] \cup [0, 4]$
2. Zeros $x = 0, \pm 4$
3. y-intercept $y = 0$
4. Extreme Points

Recall we can write it either way

$x \le -4$

$0 \le x \le 4$

Absolute minimums:

$$(-4,0)\ (0,0)\ (4,0)$$

Relative maximum:

$$\left(\frac{4}{\sqrt{3}}, \frac{128}{3\sqrt{3}}\right)$$

5. Range $y \in [0, \infty)$
6. End Behavior (EB)

Since our domain is restricted we only need to worry about the left side which has the function coming down from infinity

[redacted]