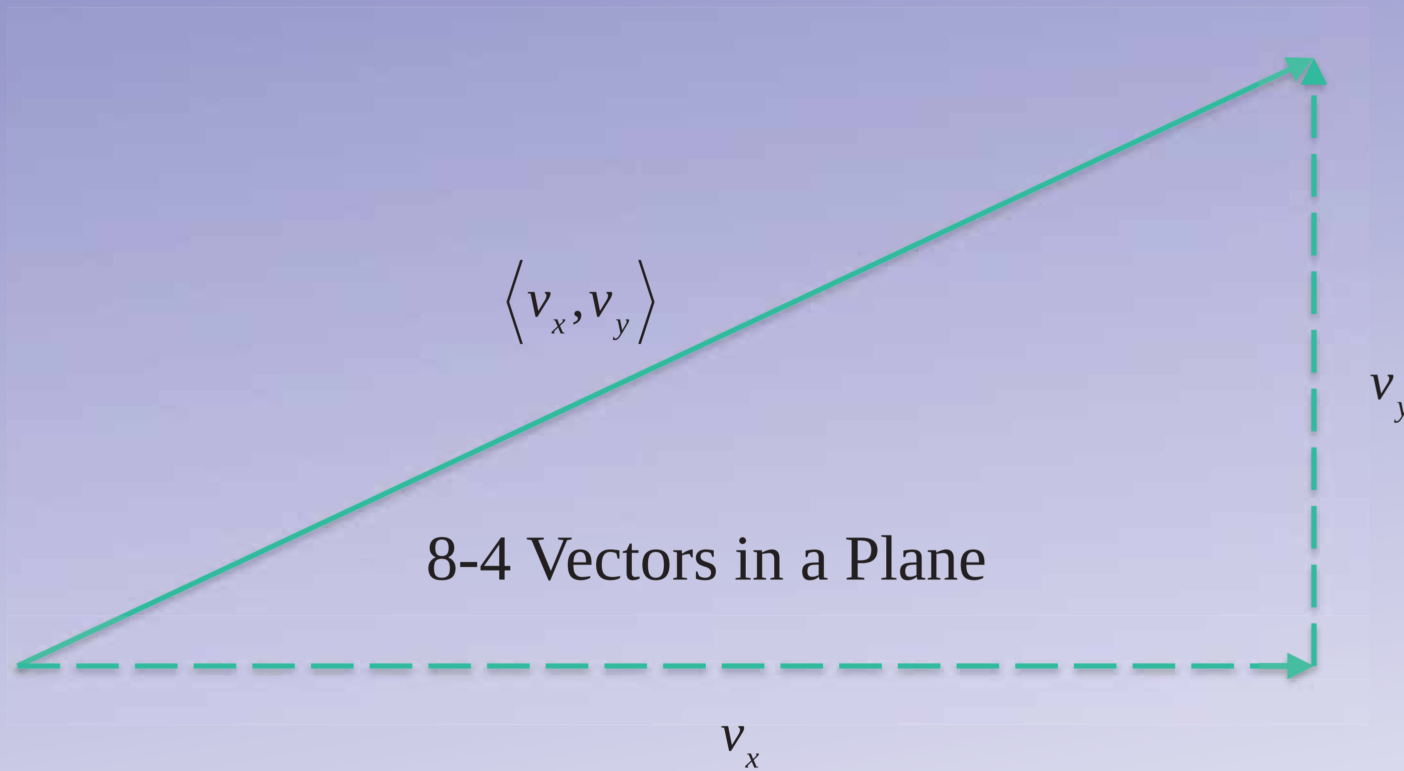
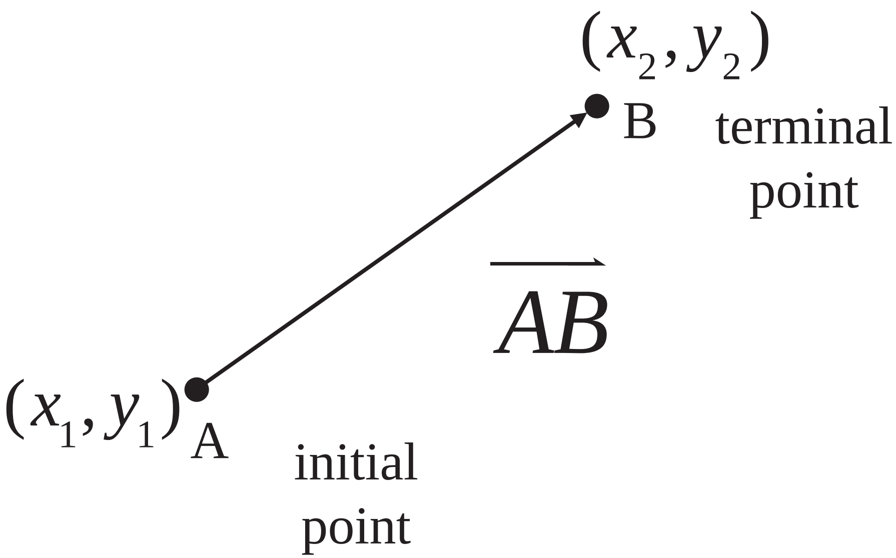
8-4 Vectors in a Plane



In the past we've only worked with lines that have slopes but not necessarily direction.

Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by <u>directed line segments</u>.

There's a difference between going 50 mph north and 50 mph south



The length is written as

$$\left|\overrightarrow{AB}\right|$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Look, it's just the distance formula

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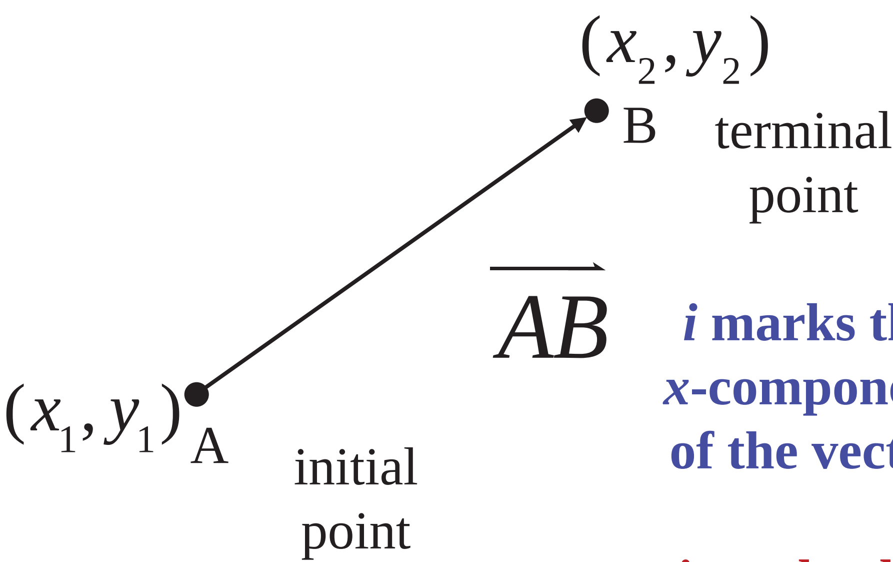

$\overrightarrow{AB}$

The <u>component form</u> of this vector is:

$$\overrightarrow{AB} = (x_2 - x_1)i + (y_2 - y_1)j$$

***i* marks the *x*-component of the vector**

***j* marks the *y*-component of the vector**

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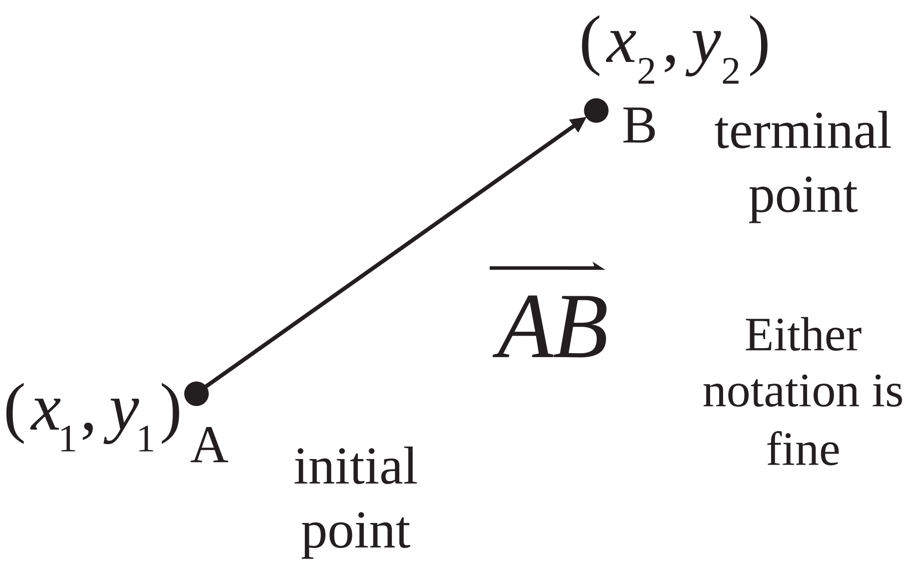

The <u>component form</u> of this vector is:

$$\overrightarrow{AB} = (x_2 - x_1)i + (y_2 - y_1)j$$

or

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

Either notation is fine

Notice here that we always subtract <u>the initial point from the terminal point</u> because we need to establish direction

In the past we've only worked with lines that have slopes but not necessarily direction.

Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by <u>directed line segments</u>.

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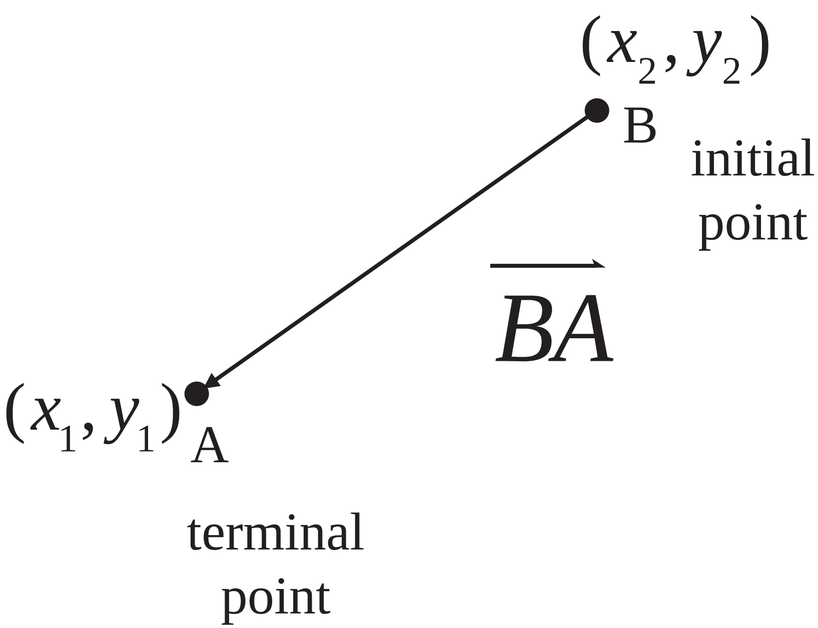

The length is written as

$$\left|\overrightarrow{BA}\right|$$

$$\left|\overrightarrow{BA}\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Notice that the length will be the same

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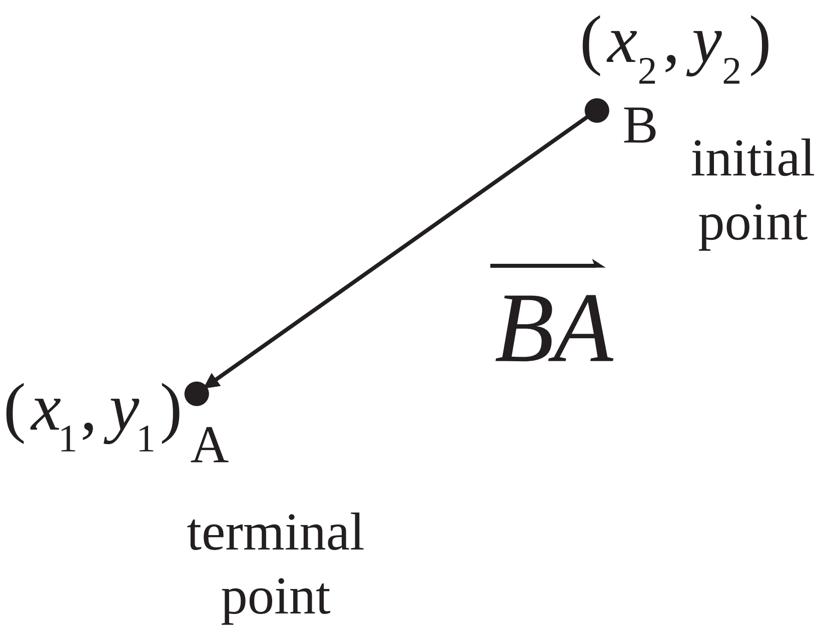
Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by <u>directed line segments</u>.

There's a difference between going 50 mph north and 50 mph south



The <u>component form</u> of this vector is:

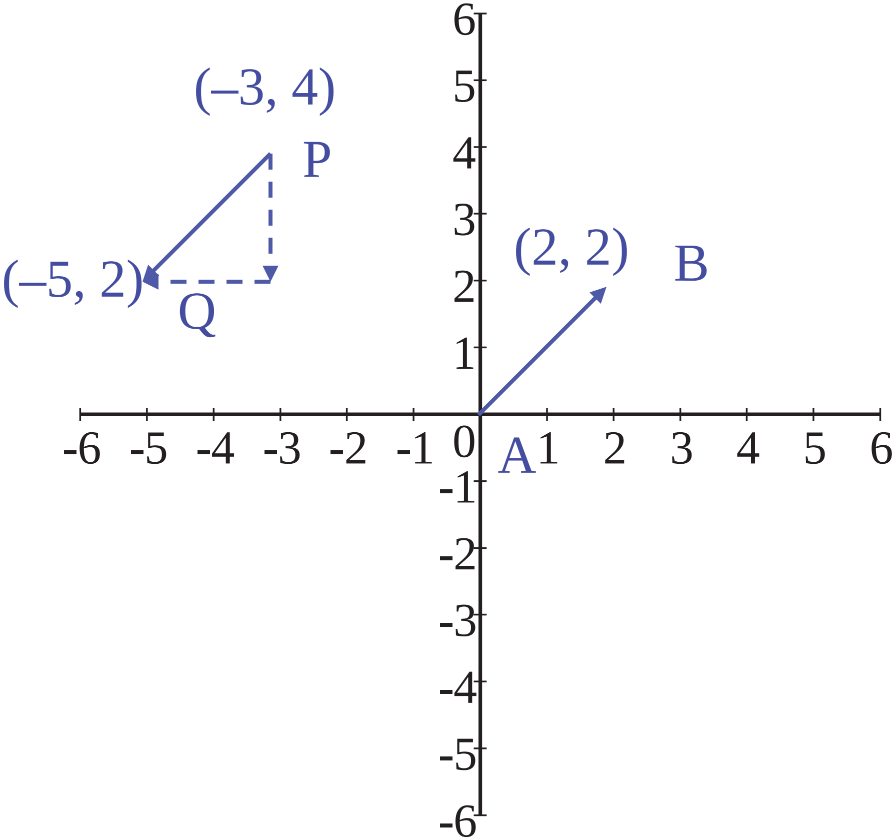
$$\overrightarrow{BA} = (x_1 - x_2)i + (y_1 - y_2)j$$

or

$$\overrightarrow{BA} = \langle x_1 - x_2, y_1 - y_2 \rangle$$

Same vector length (magnitude), same slope (line segment) but opposite vectors.

Find the component form of each vector



$$\overrightarrow{AB} = (2-0)i + (2-0)j$$

$$\overrightarrow{AB} = 2i + 2j \;\; or \; \langle 2, 2 \rangle$$

$$\overrightarrow{PQ} = (-5+3)i + (2-4)j$$

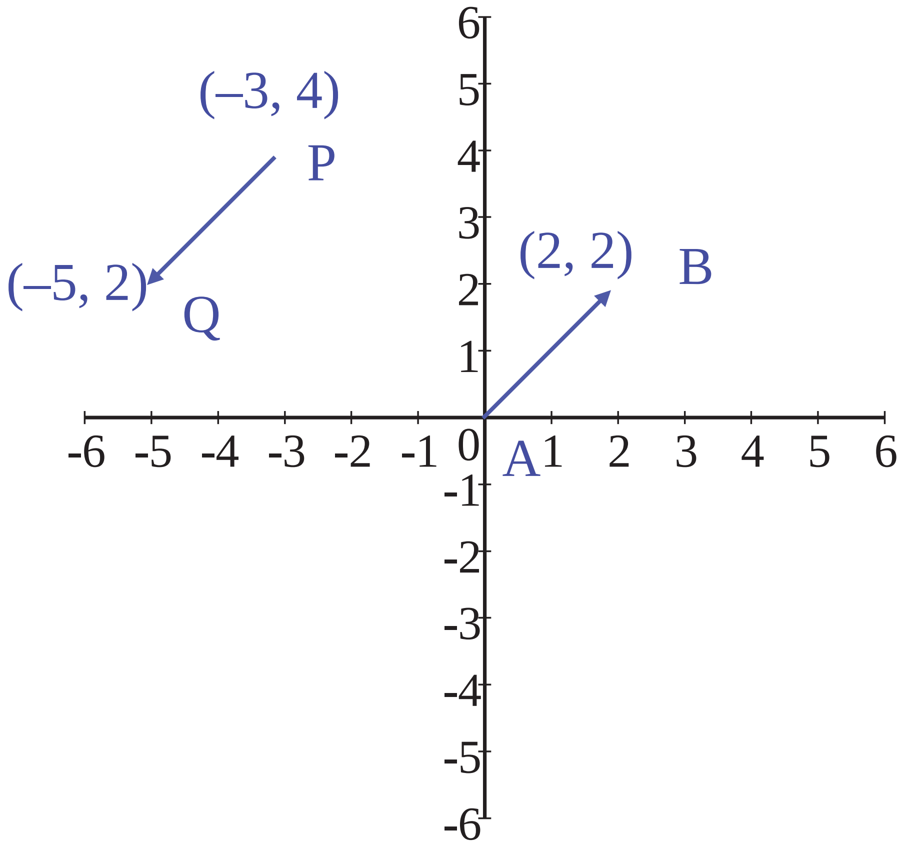
$$\overrightarrow{PQ} = -2i - 2j \; or \; \langle -2, -2 \rangle$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(2)^2 + (2)^2}$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2 + (-2)^2}$$

$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{PQ}\right| = \sqrt{8} \;\; = 2\sqrt{2}$$

Find the component form of each vector



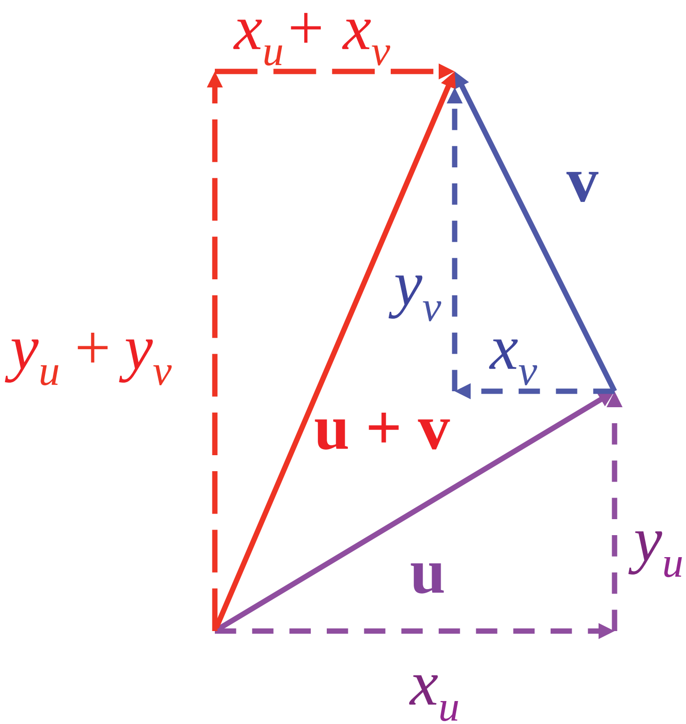
$$\overrightarrow{AB} = 2i + 2j \ \ or \ \left\langle 2, 2 \right\rangle$$

$$\overrightarrow{PQ} = -2i - 2j \ or \ \left\langle -2, -2 \right\rangle$$

$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{PQ}\right| = \sqrt{8} \ = 2\sqrt{2}$$

Notice that the vectors are pointed in opposite directions

but have the same length.



Vector Addition:

u + **v**

u + **v** is the resultant vector.

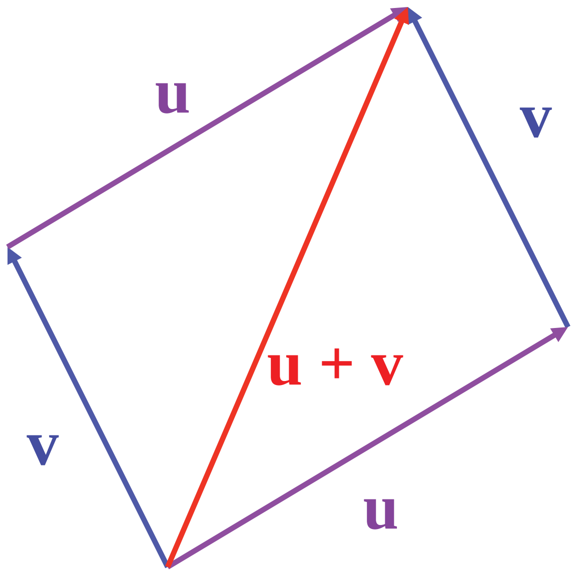
$$\mathbf{u} = x_u i + y_u j$$

$$\mathbf{v} = x_v i + y_v j$$

$$\mathbf{u} + \mathbf{v} = (x_u + x_v) i + (y_u + y_v) j$$

(Add the components.)

Vector Addition:



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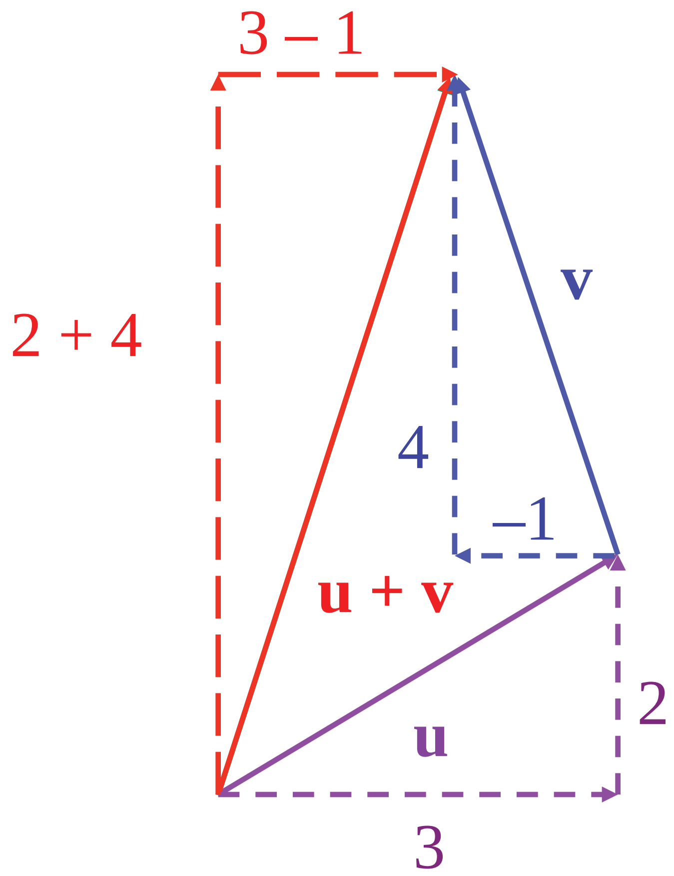


Vector Addition:

u + **v**

u + **v** is the resultant vector.

$$\mathbf{u} = 3i + 2j$$

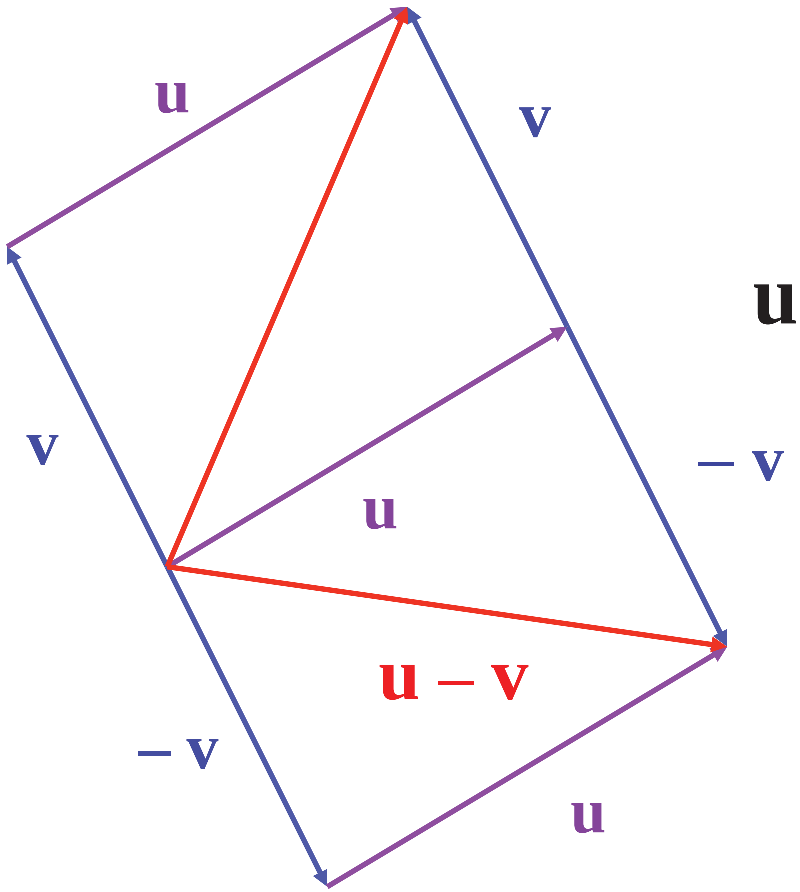
$$\mathbf{v} = -i + 4j$$

$$\mathbf{u} + \mathbf{v} = 2i + 6j$$

See? Just add the components

By the way, what's the length of this new vector?

$$|\mathbf{u} + \mathbf{v}| = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

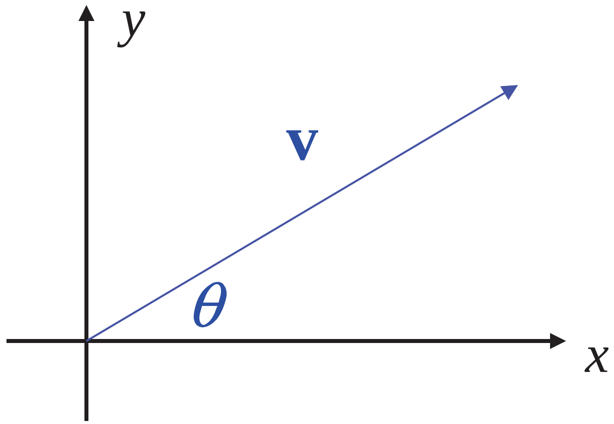


u – v

(Subtract the components.)

$$\mathbf{u} - \mathbf{v} = (x_u - x_v)i + (y_u - y_v)j$$

A vector is in standard position if the initial point is at the origin.



What if we only knew the length of the vector and the angle?

$|\mathbf{v}|$ θ

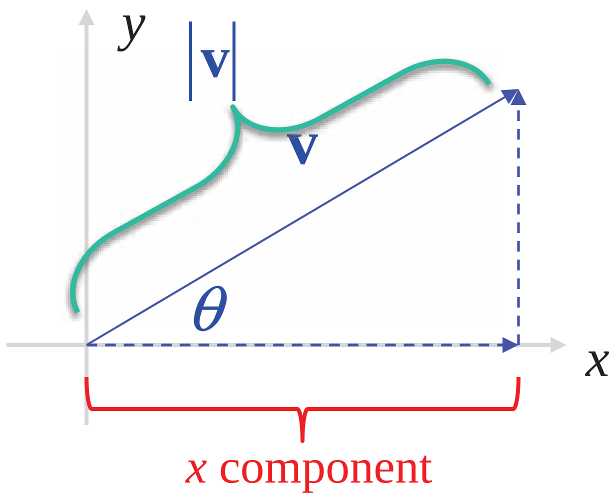
The component form of this vector is:

$$\mathbf{v} = \left(|\mathbf{v}|\cos\theta\right)i + \left(|\mathbf{v}|\sin\theta\right)j \quad \text{or} \quad \mathbf{v} = \left\langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \right\rangle$$

Before anyone panics, this is just SOHCAHTOA…

Just watch…

A vector is in standard position if the initial point is at the origin.



$$\frac{adj}{hyp} = \frac{x \text{ component}}{|\mathbf{v}|} = \cos\theta$$

$$x \text{ component} = |\mathbf{v}|\cos\theta$$

Remember what this really means: $|\mathbf{v}|$ ← Think of it as a hypotenuse of the right triangle above because it's the length of the vector

$$\mathbf{v} = \underbrace{\left(|\mathbf{v}|\cos\theta\right)}_{x \text{ component}} i + \left(|\mathbf{v}|\sin\theta\right) j \quad \text{or} \quad \mathbf{v} = \left\langle \underbrace{|\mathbf{v}|\cos\theta}_{x \text{ component}}, |\mathbf{v}|\sin\theta \right\rangle$$

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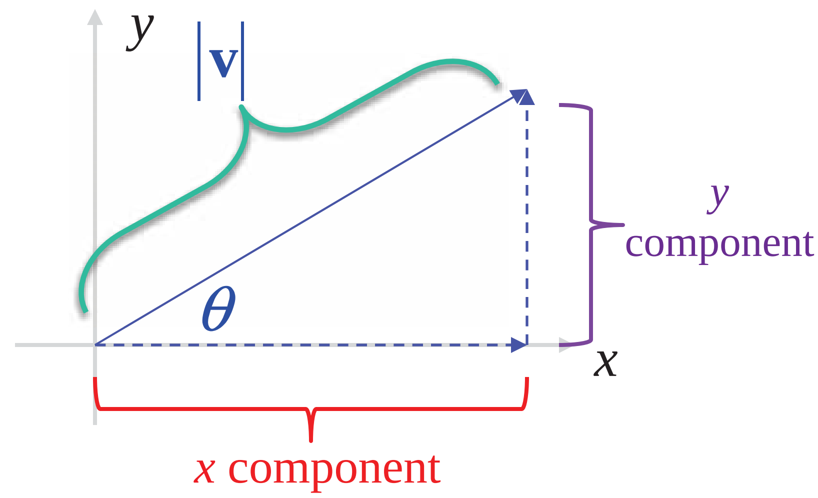


$$\frac{opp}{hyp} = \frac{y \text{ component}}{|\mathbf{v}|} = \sin\theta$$

$$y \text{ component} = |\mathbf{v}|\cos\theta$$

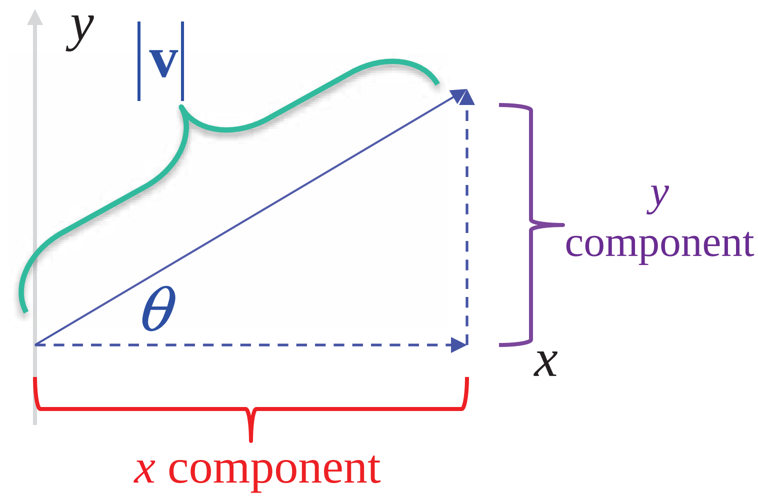
See? Just SOHCAHTOA

Remember what this really means: $|\mathbf{v}|$ ← Think of it as a hypotenuse of the right triangle above because it's the length of the vector

$$\mathbf{v} = \underbrace{\left(|\mathbf{v}|\cos\theta\right)}_{x \text{ component}} i + \underbrace{\left(|\mathbf{v}|\sin\theta\right)}_{y \text{ component}} j \quad \text{or} \quad \mathbf{v} = \left\langle \underbrace{|\mathbf{v}|\cos\theta}_{x \text{ component}}, \underbrace{|\mathbf{v}|\sin\theta}_{y \text{ component}} \right\rangle$$

If it's the angle that you need to find, then you need to know this:

Remember that the magnitude and components form a right triangle



The direction of a vector **v** is found this way:

$$\cos\theta = \frac{adj}{hyp} = \frac{x\,\text{component}}{|\mathbf{v}|} = \frac{x_{\mathbf{v}}}{|\mathbf{v}|}$$

The direction **v** is the angle θ

$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{v}}}{|\mathbf{v}|}\right)$$

How would we determine which one?

So this is just the *x* component divided by the magnitude

If it's the angle that you need to find, then you need to know this:

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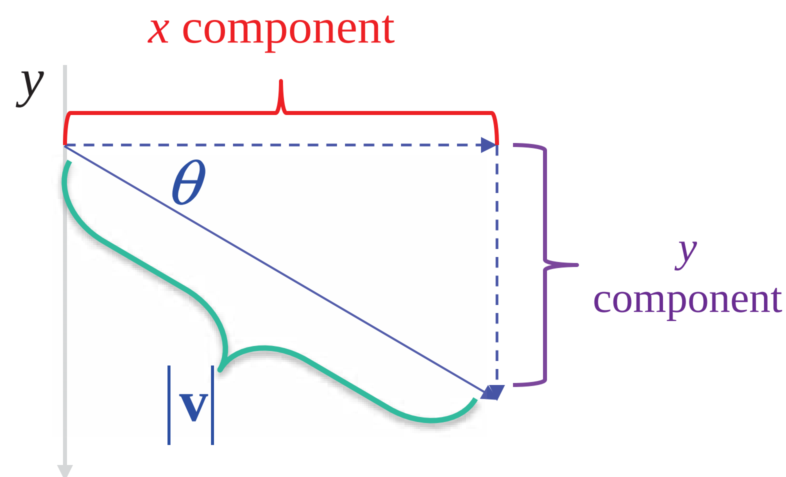

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The direction angle here is negative because the *y* component is in a lower quadrant

The direction **v** is the angle θ

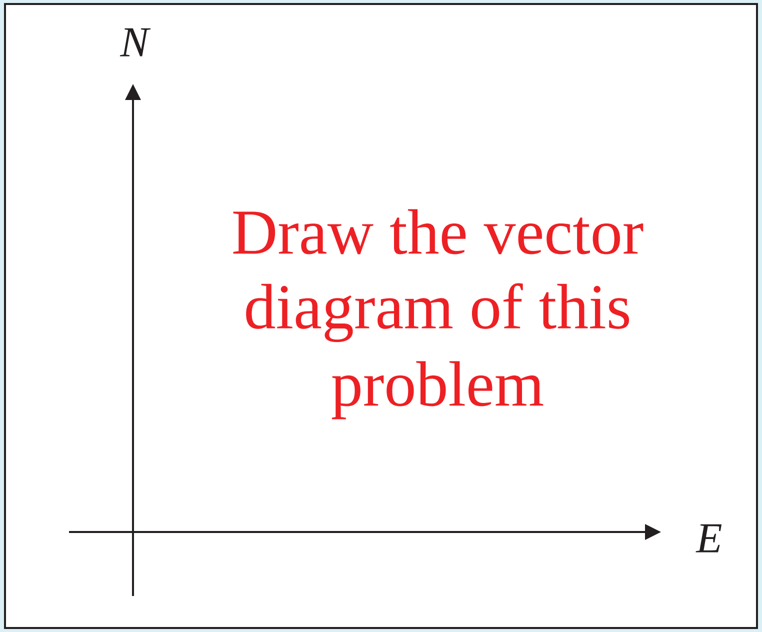
$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{v}}}{|\mathbf{v}|}\right)$$

So the sign is determined by the sign of the *y* component

So this is just the *x* component divided by the magnitude

Application:

A Boeing 727 airplane, flying due east at 500mph in still air, encounters a 70-mph tail wind acting in the direction of 60^{o} north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?



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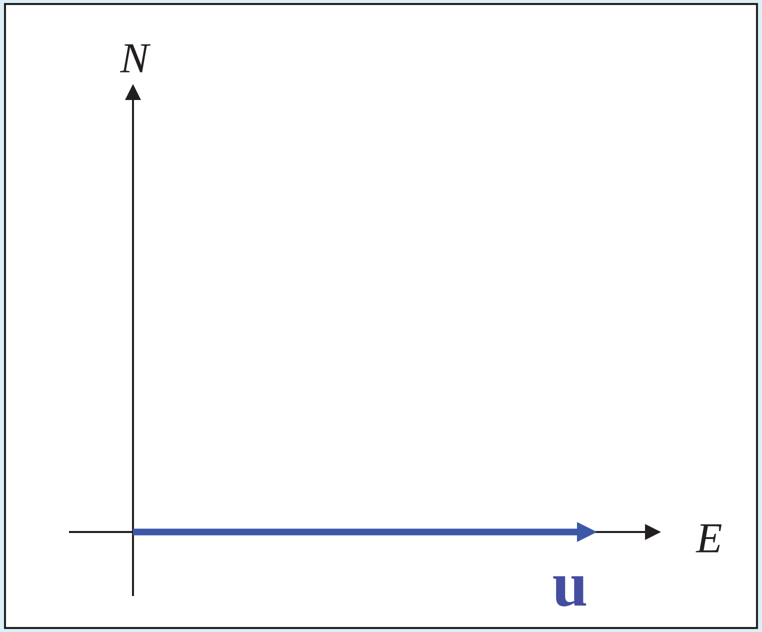

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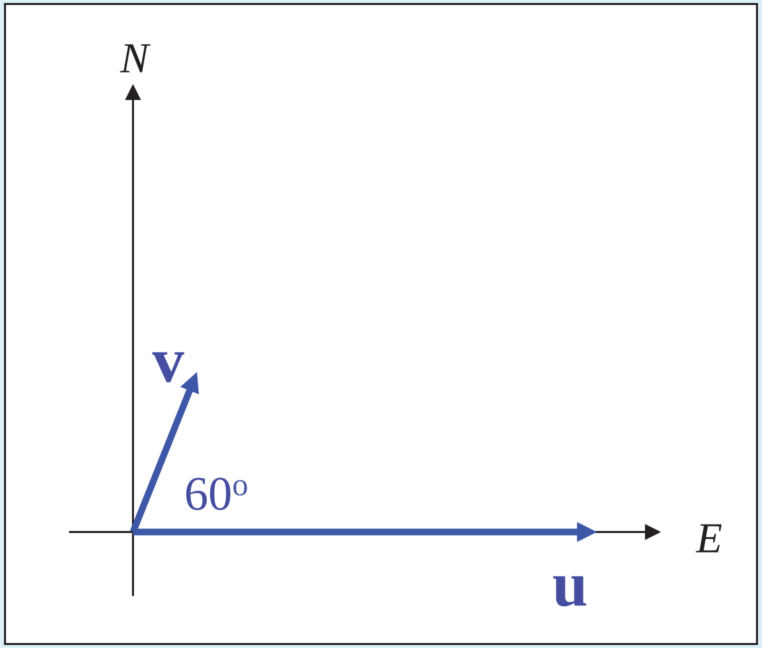
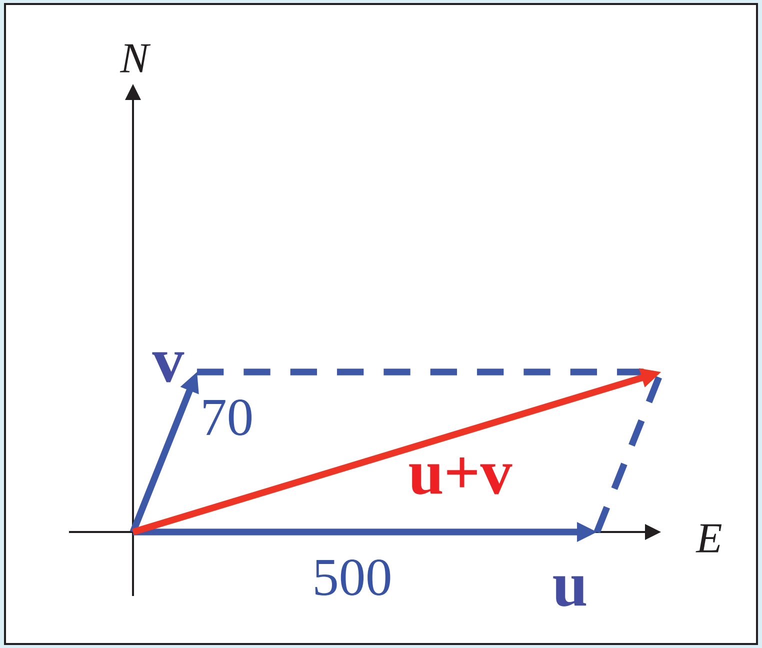


Application:

A Boeing 727 airplane, flying due east at 500mph in still air, encounters a 70-mph tail wind acting in the direction of 60º north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

We need to find the magnitude and direction of the resultant vector **u** + **v**.



$$\mathbf{u} = 500i + 0j = \langle 500, 0 \rangle$$

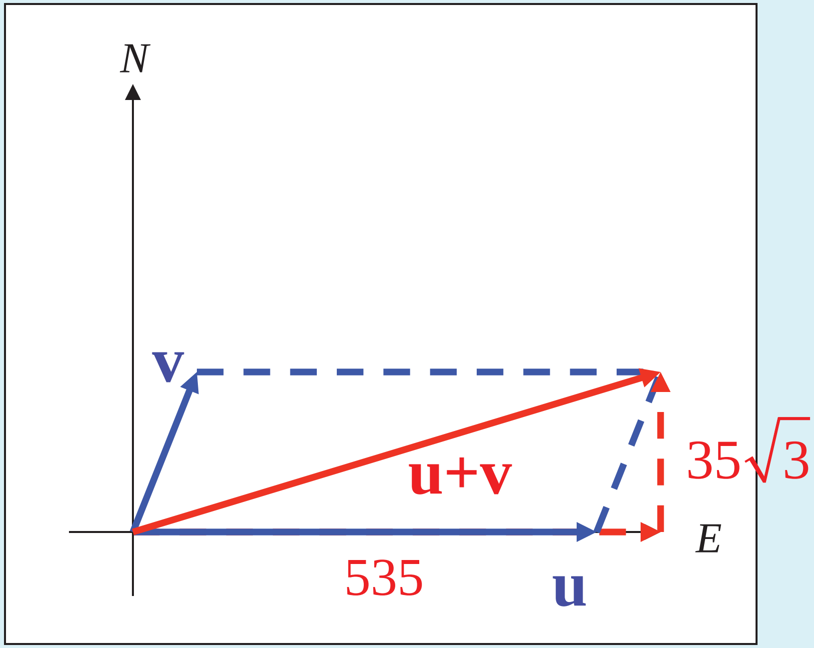
$$\mathbf{v} = (70\cos 60^\circ)i + (70\sin 60^\circ)j$$

$$\mathbf{v} = \langle 70\cos 60^\circ, 70\sin 60^\circ \rangle$$

The component forms of **u** and **v**

$$\mathbf{u} = \left\langle 500, 0 \right\rangle$$

$$\mathbf{v} = \left\langle 35, 35\sqrt{3} \right\rangle$$



Adding them gives us:

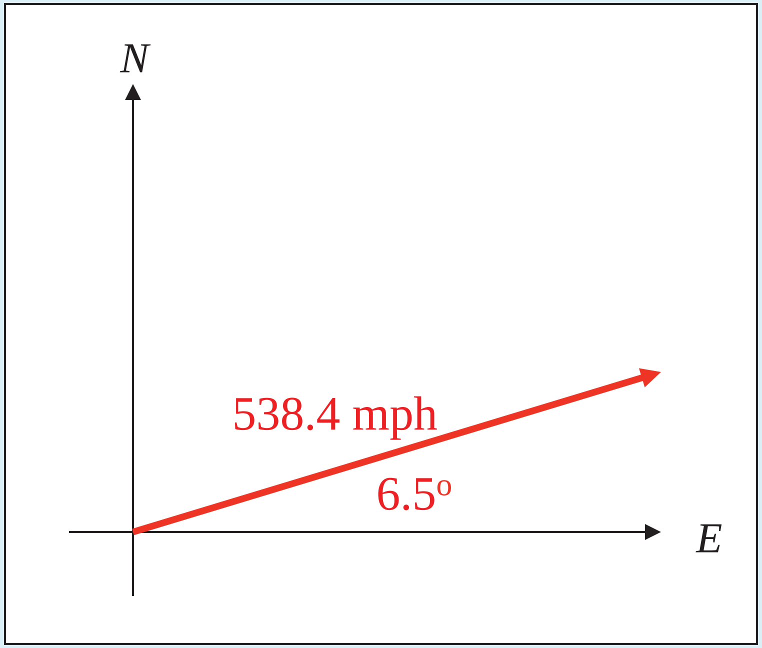
$$\mathbf{u} + \mathbf{v} = \left\langle 535, 35\sqrt{3} \right\rangle$$

The new ground speed is the magnitude of the new vector:

$$|\mathbf{u} + \mathbf{v}| = \sqrt{535^2 + \left(35\sqrt{3}\right)^2} \approx 538.423625$$

And the direction is the new angle:

$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{u}+\mathbf{v}}}{|\mathbf{u} + \mathbf{v}|}\right) \qquad \theta = \cos^{-1}\left(\frac{535}{538.423625}\right) \approx 6.5^{\circ}$$



The new ground speed of the airplane is about 538.4 mph, and its new direction is about 6.5° north of east.

$$|\mathbf{u}+\mathbf{v}| = 538.424\ mph \qquad \theta \approx 6.5^\circ$$