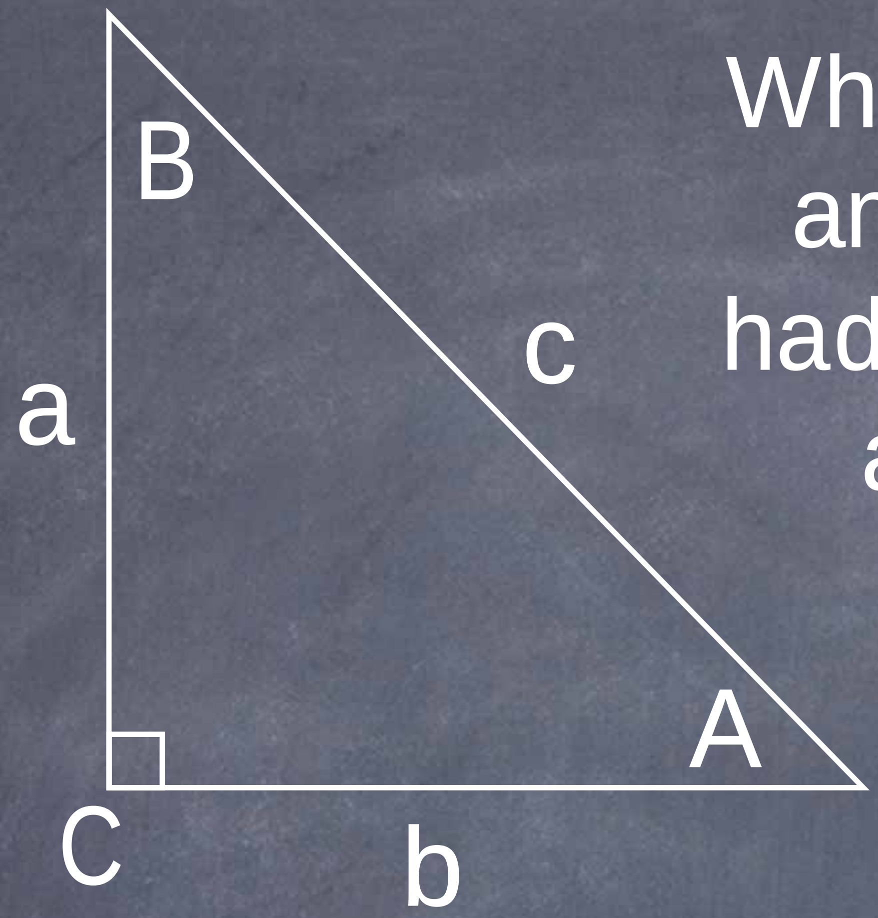
Law of Cosines



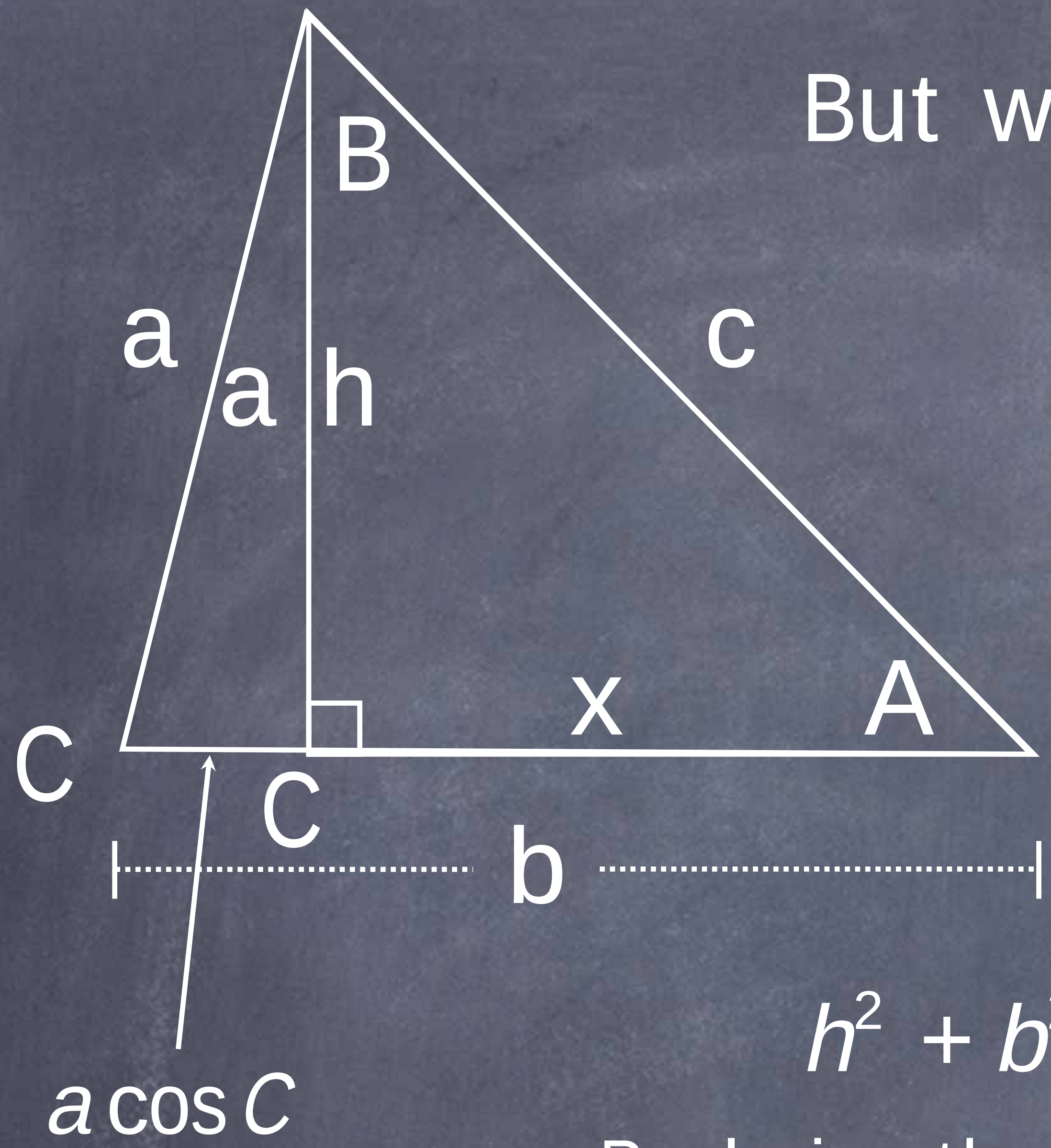
When solving for missing sides and angles with a right triangle, we had the pythagorean theorem plus all the trig functions to use.

$$a^2 + b^2 = c^2$$

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

$$\sin B = \frac{b}{c} \qquad \cos B = \frac{a}{c} \qquad \tan B = \frac{b}{a}$$

But what happens if it is not a right triangle?



Remembering that

$$\cos C = \frac{adj}{hyp}$$

Drawing an altitude here will help

$$h^2 + x^2 = c^2$$

Using simple substitution

$$h^2 + (b - a\cos C)^2 = c^2$$

FOILing the parentheses

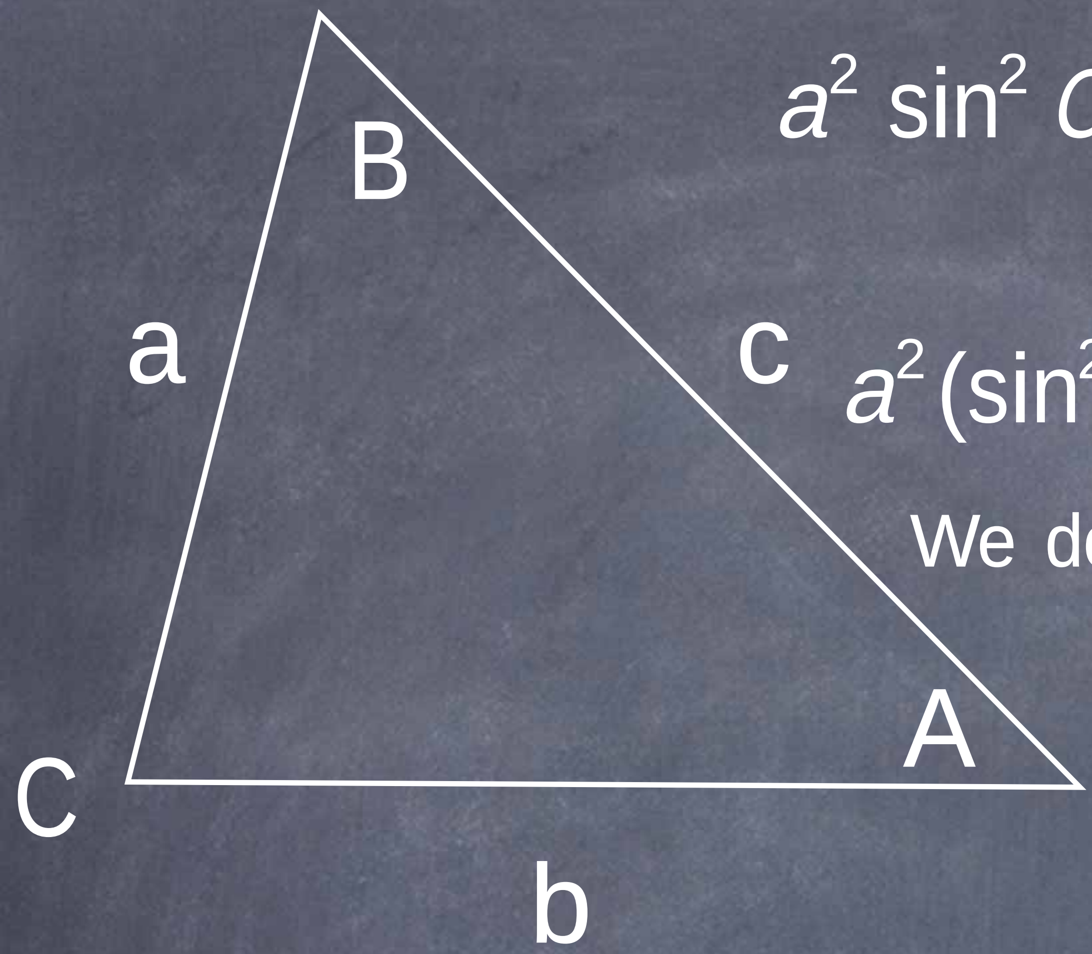
$$h^2 + b^2 - 2ab\cos C + a^2\cos^2 C = c^2$$

Replacing the h^2 with $a^2 \sin^2 C$

$$a^2\sin^2 C + b^2 - 2ab\cos C + a^2\cos^2 C = c^2$$

I'm going to bring together the common a^2 so I can factor

$$a^2\sin^2 C + a^2\cos^2 C + b^2 - 2ab\cos C = c^2$$



$$a^2 \sin^2 C + a^2 \cos^2 C + b^2 - 2ab\cos C = c^2$$

Factoring the a^2 gives us

$$a^2(\sin^2 C + \cos^2 C) + b^2 - 2ab\cos C = c^2$$

We do this because recall that $\sin^2 C + \cos^2 C = 1$

$$a^2(1) + b^2 - 2ab\cos C = c^2$$

And our result is...

The Law of Cosines

$$a^2 + b^2 \boxed{- 2ab\cos C} = c^2$$

Notice how this is actually the Pythagorean Thm with the added term

The Law of Cosines

$$a^2 + b^2 - 2ab\cos C = c^2$$

$$b^2 + c^2 - 2bc\cos A = a^2$$

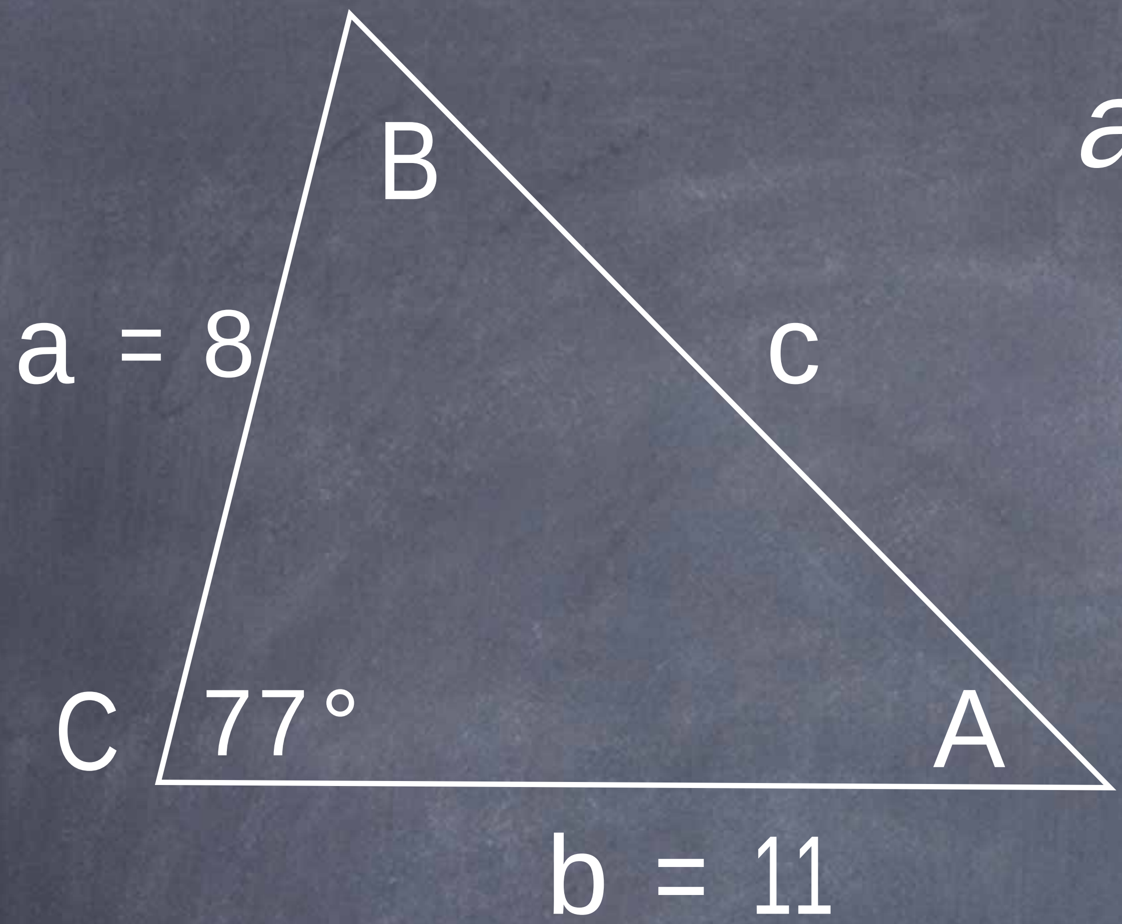
$$a^2 + c^2 - 2ac\cos B = b^2$$

They're all the same. This is just to assure you that the law applies to any labeling you use for a triangle.

And when we need to use it to solve for a missing angle, we have

$$C = \cos^{-1}\frac{a^2 + b^2 - c^2}{2ab}$$

More on this in class...



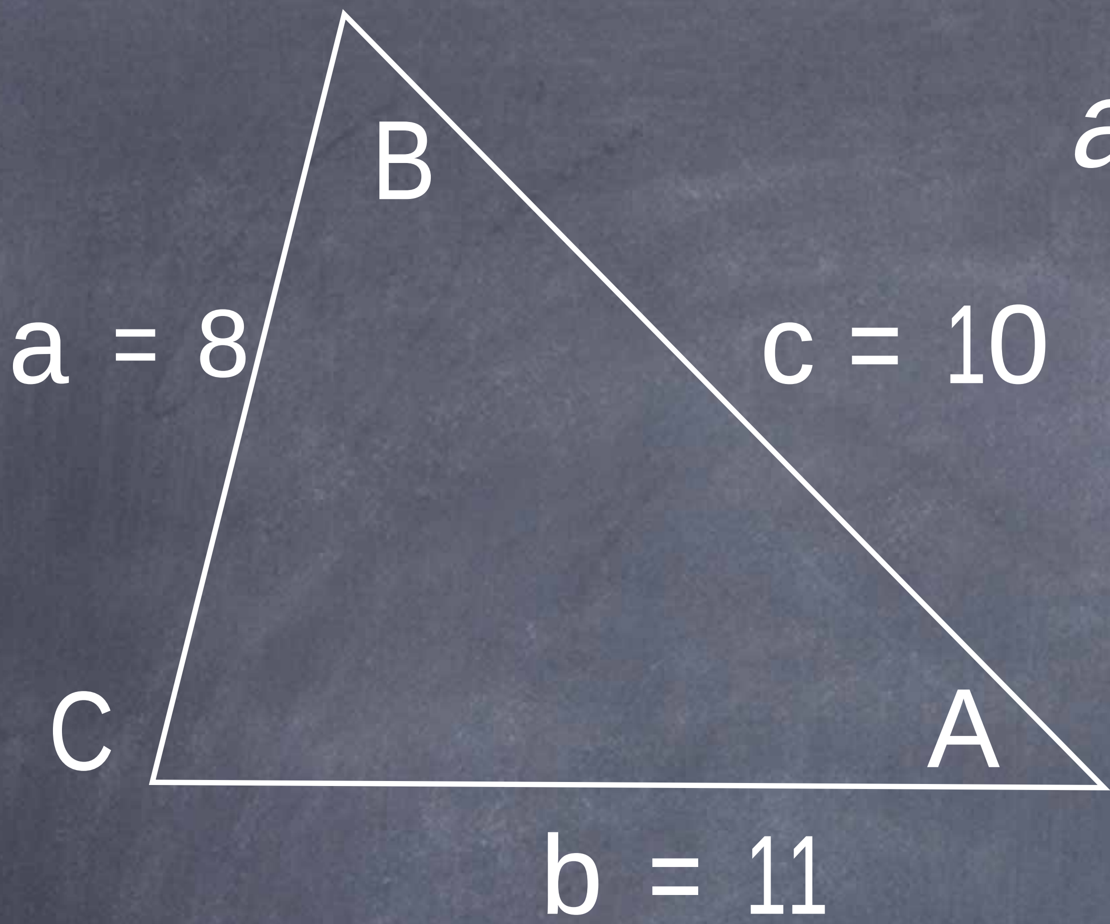
$$a^2 + b^2 - 2ab\cos C = c^2$$

Find the missing side

$$8^2 + 11^2 - 2(8)(11)\cos 77 = c^2$$

$$c^2 \approx 145.409$$

$$c \approx 12.059$$



$$a^2 + b^2 - 2ab\cos C = c^2$$

Find the missing angle

$$8^2 + 11^2 - 2(8)(11)\cos C = 10^2$$

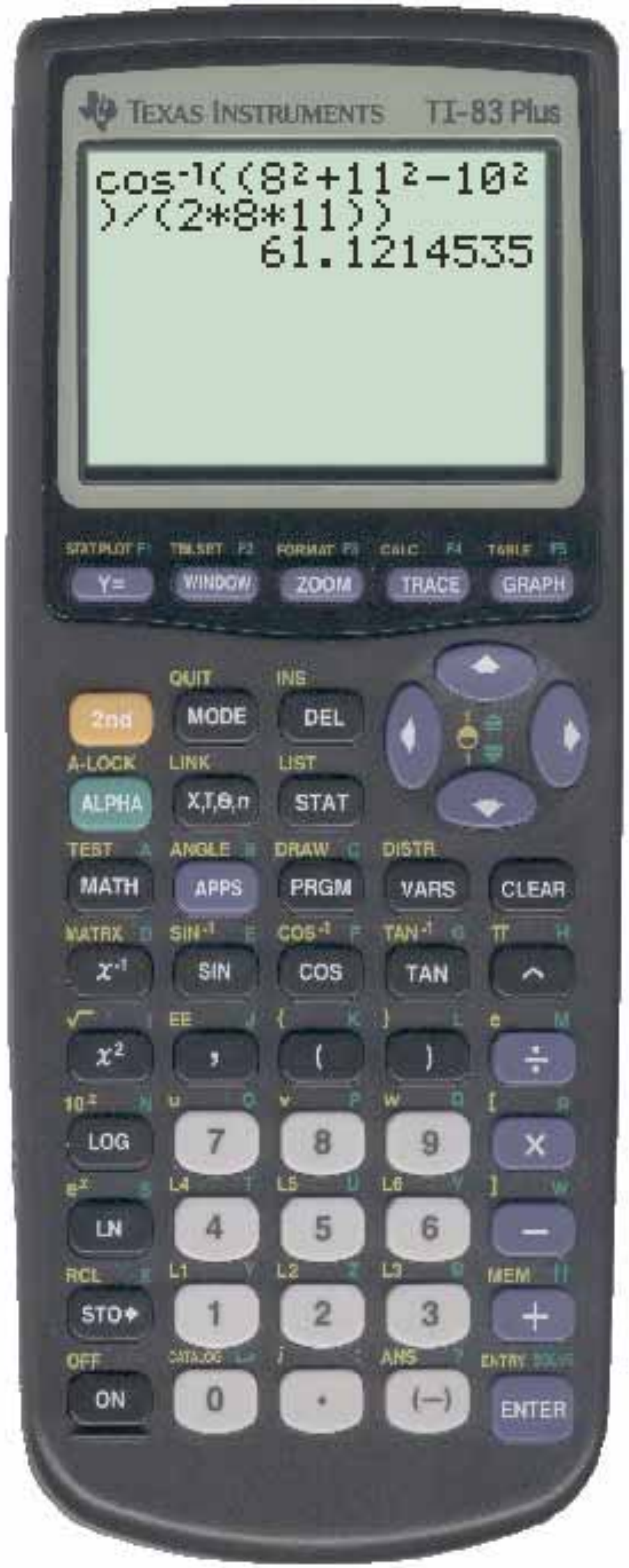
$$-2(8)(11)\cos C = 10^2 - 8^2 - 11^2$$

$$\cos C = \frac{10^2 - 8^2 - 11^2}{-2(8)(11)}$$

Be careful entering this into the calculator

$$\cos C = \frac{8^2 + 11^2 - 10^2}{2(8)(11)}$$

$$C = \cos^{-1}\left(\frac{8^2 + 11^2 - 10^2}{2(8)(11)}\right) \approx 61.121°$$



$$C = \cos^{-1}\left(\frac{(8^2 + 11^2 - 10^2)}{(2(8)(11))}\right)$$

$$C = \cos^{-1}\left(8^2 + 11^2 - \frac{10^2}{2} * 8 * 11\right)$$