# 9-2
# Solving for *x* and *y*

in a sinusoidal function

Standard 9d: Given a sinusoidal equation, find values of *y* from *x* and vice versa.

Find the the radian values of $x$ for which $\sin x = \frac{\sqrt{3}}{2}$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

REMEMBER:

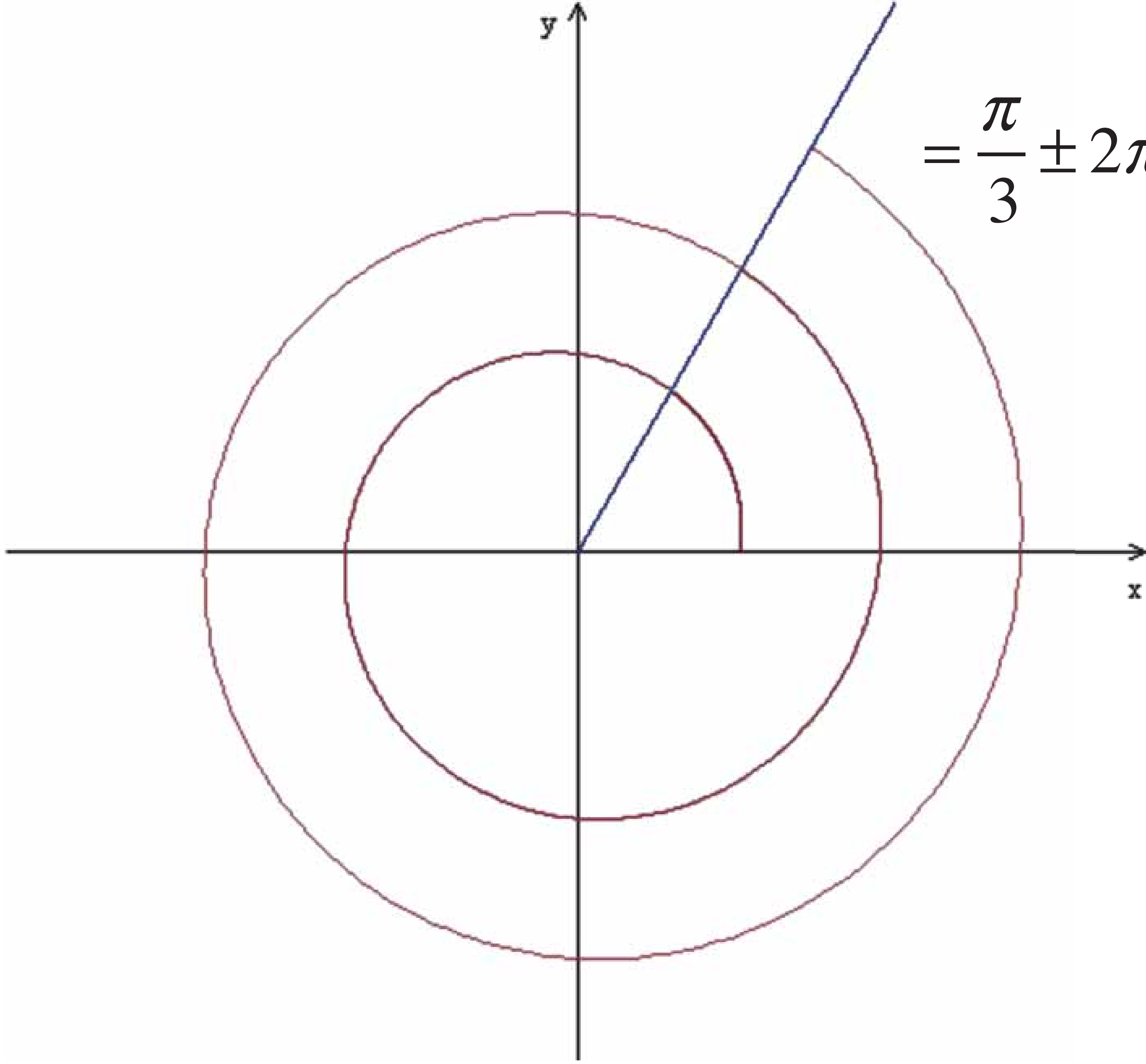
$$\cos^{-1}\frac{x}{r} = \begin{cases} \text{calculator} \pm 2\pi n \\ -\text{calculator} \pm 2\pi n \end{cases} \qquad \sin^{-1}\frac{y}{r} = \begin{cases} \text{calculator} \pm 2\pi n \\ \pi - \text{calculator} \pm 2\pi n \end{cases}$$

$$x = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ \pi - \frac{\pi}{3} \pm 2\pi n \end{cases} = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ \frac{2\pi}{3} \pm 2\pi n \end{cases}$$

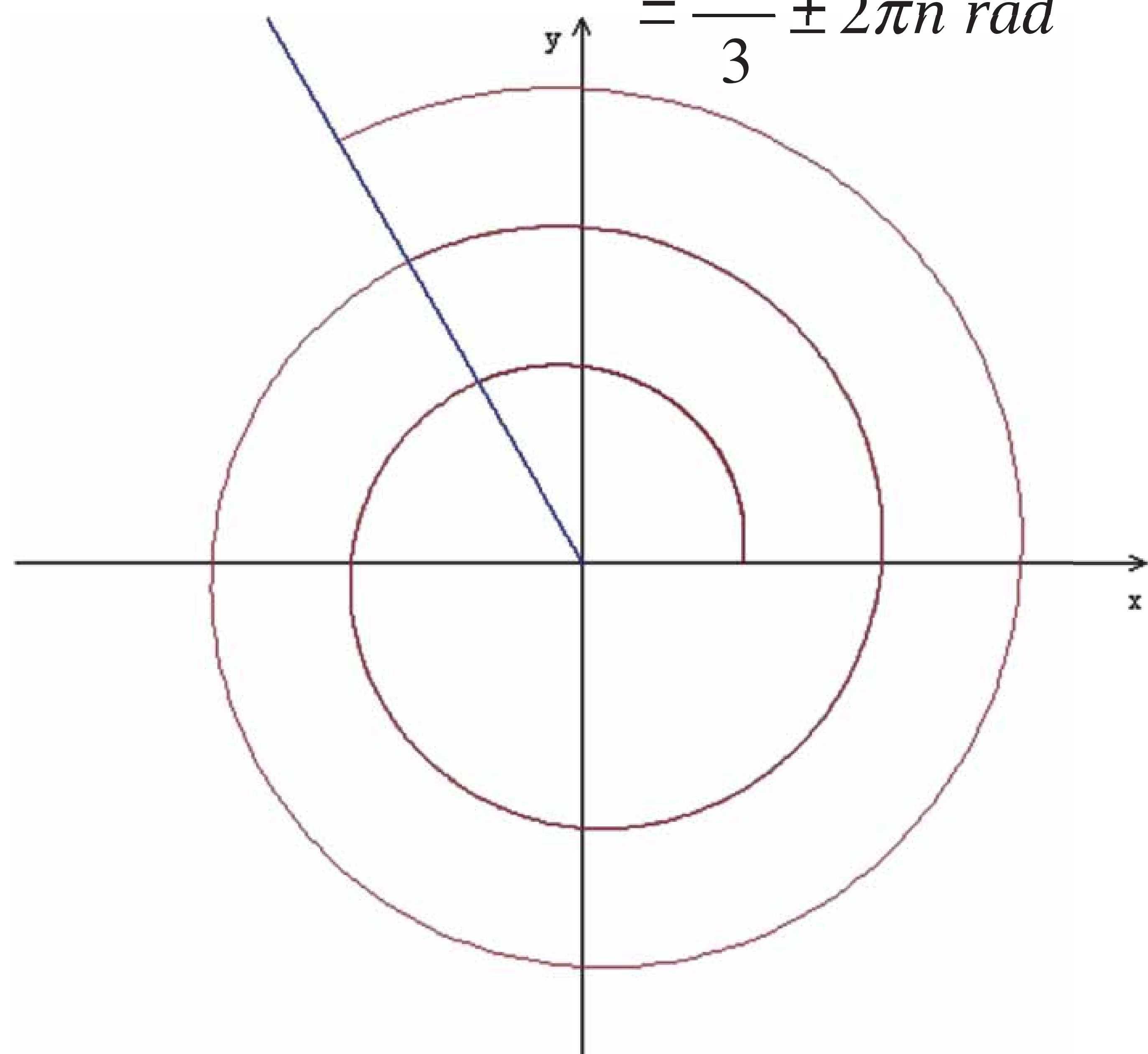
Excuse me?

Let’s sketch these angles

$$= \frac{\pi}{3} \pm 2\pi n \ rad$$




$$= \frac{2\pi}{3} \pm 2\pi n \ rad$$




$$x = \begin{cases} \frac{\pi}{3} \pm 2\pi n \\ \frac{2\pi}{3} \pm 2\pi n \end{cases}$$

Find the $x$ intercepts of $y = 2 + 3\cos\left(\frac{\pi}{3}(x-1)\right)$

$$0 = 2 + 3\cos\left(\frac{\pi}{3}(x-1)\right)$$

$$-2 = 3\cos\left(\frac{\pi}{3}(x-1)\right)$$

$$-\frac{2}{3} = \cos\left(\frac{\pi}{3}(x-1)\right)$$

$$\cos^{-1}\left(-\frac{2}{3}\right) = \frac{\pi}{3}(x-1)$$

$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) = (x-1)$$

$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) = (x-1)$$

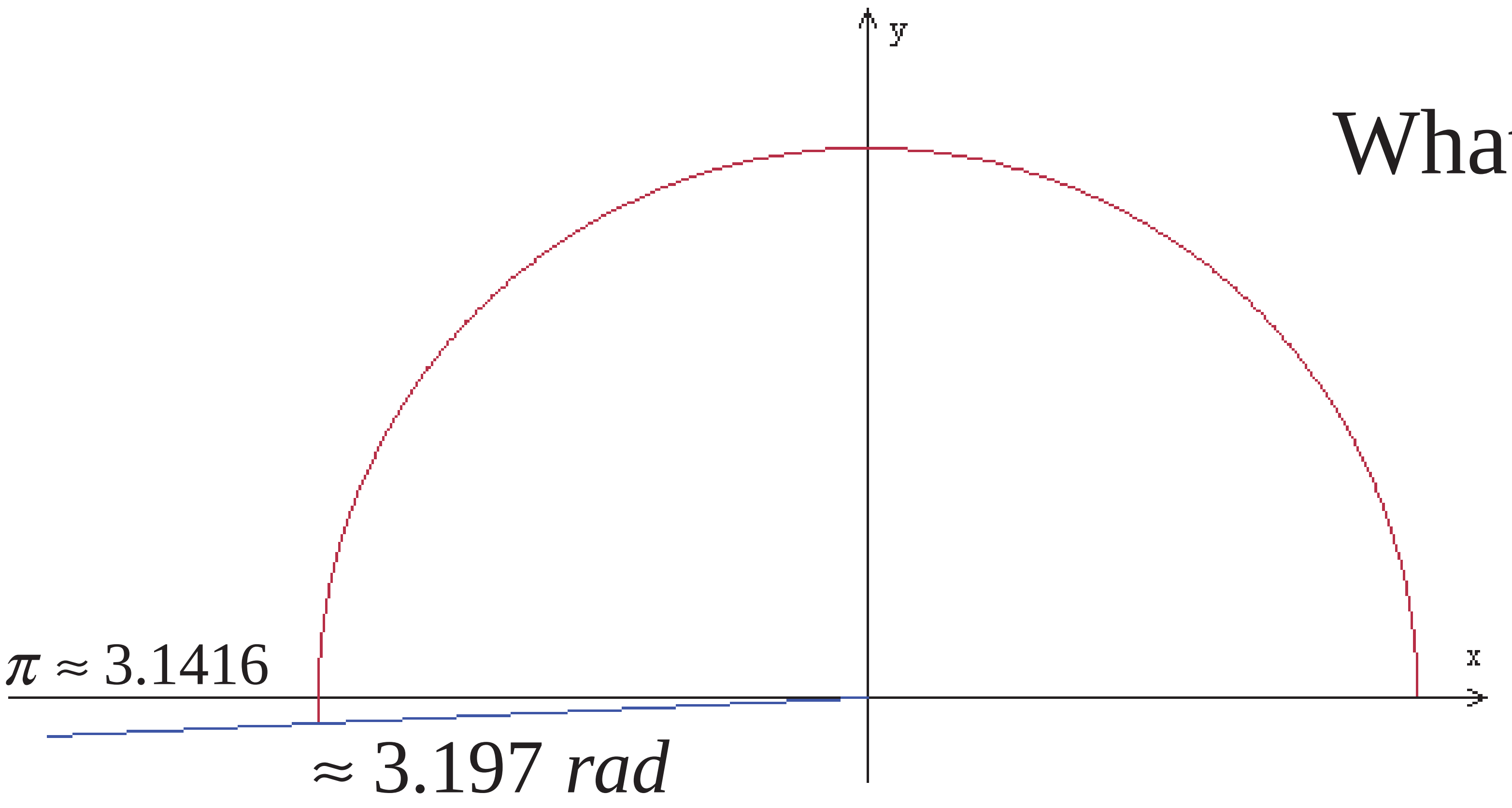
$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) + 1 = x$$

$$x = \frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) + 1 \approx 3.197$$

According to the calculator

But this is not the final answer…

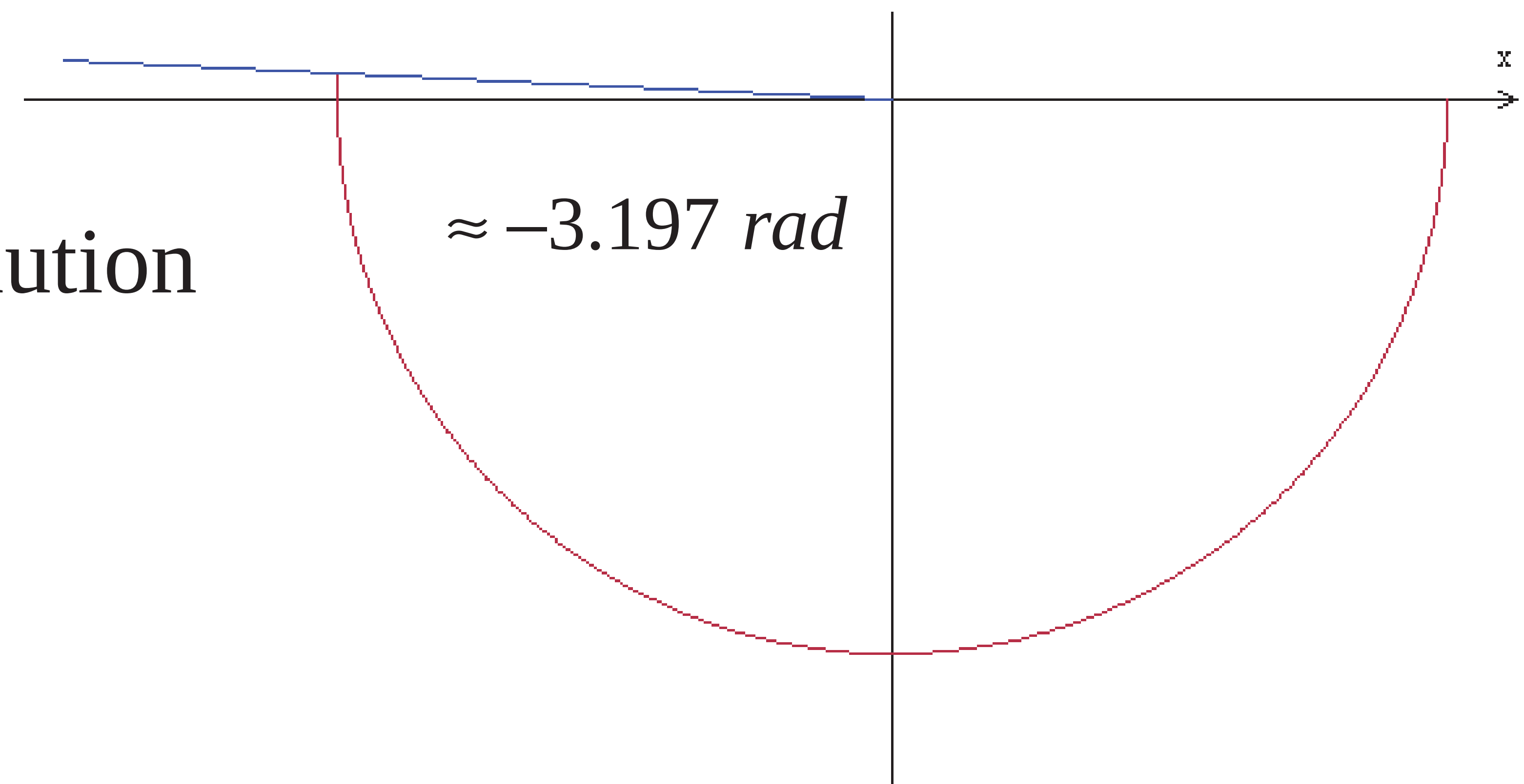
$$x = \frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) + 1 \approx 3.197$$




What other angles are solutions?

Don’t forget about

$$\cos^{-1}\frac{x}{r} = \begin{cases} \text{calculator} \pm 2\pi n \\ -\text{calculator} \pm 2\pi n \end{cases}$$




Now let’s go back to our solution

Find the $x$ intercepts of $y = 2 + 3\cos\left(\frac{\pi}{3}(x-1)\right)$

$$0 = 2 + 3\cos\left(\frac{\pi}{3}(x-1)\right)$$

$$-2 = 3\cos\left(\frac{\pi}{3}(x-1)\right)$$

$$-\frac{2}{3} = \cos\left(\frac{\pi}{3}(x-1)\right)$$

$$\cos^{-1}\left(-\frac{2}{3}\right) \pm 2\pi n = \frac{\pi}{3}(x-1)$$

Let's not forget all the coterminals

$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) \pm \frac{3}{\pi}2\pi n = (x-1)$$

Nor that the distributive property requires that we multiply $\frac{3}{\pi}$ by the $2\pi$ as well

$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) \pm 6n = (x-1)$$

$$\frac{3}{\pi}\cos^{-1}\left(-\frac{2}{3}\right) + 1 \pm 6n = x$$

$$x \approx \pm 3.197 \pm 6n$$

Given $y = 4 - 6\sin\left(\pi(x+1)\right)$ find the first three positive values of $x$ for which

$$y = 2$$

$$2 = 4 - 6\sin\left(\pi(x+1)\right)$$

$$-2 = -6\sin\left(\pi(x+1)\right)$$

$$\frac{1}{3} = \sin\left(\pi(x+1)\right)$$

$$\sin^{-1}\left(\frac{1}{3}\right) = \pi(x+1)$$

$$\frac{1}{\pi}\sin^{-1}\left(\frac{1}{3}\right) = x+1$$

$$\frac{1}{\pi}\sin^{-1}\left(\frac{1}{3}\right) - 1 = x$$

$$x = \frac{1}{\pi}\left\{\sin^{-1}\left(\frac{1}{3}\right)\right\} - 1$$

$$x \approx \begin{cases} \frac{1}{\pi}\left(0.33983 \pm 2\pi n\right) + 1 \\ \frac{1}{\pi}\left(\pi - 0.33983 \pm 2\pi n\right) + 1 \end{cases}$$

Let's distribute the $\frac{1}{\pi}$

$$x \approx \begin{cases} \left(0.108173 \pm 2n\right) + 1 \\ \left(0.891829 \pm 2n\right) + 1 \end{cases}$$

Notice that we also multiplied the $\frac{1}{\pi}$ by $2\pi$

Given $y = 4 - 6\sin\left(\pi(x+1)\right)$ find the first three positive values of $x$ for which $y = 2$

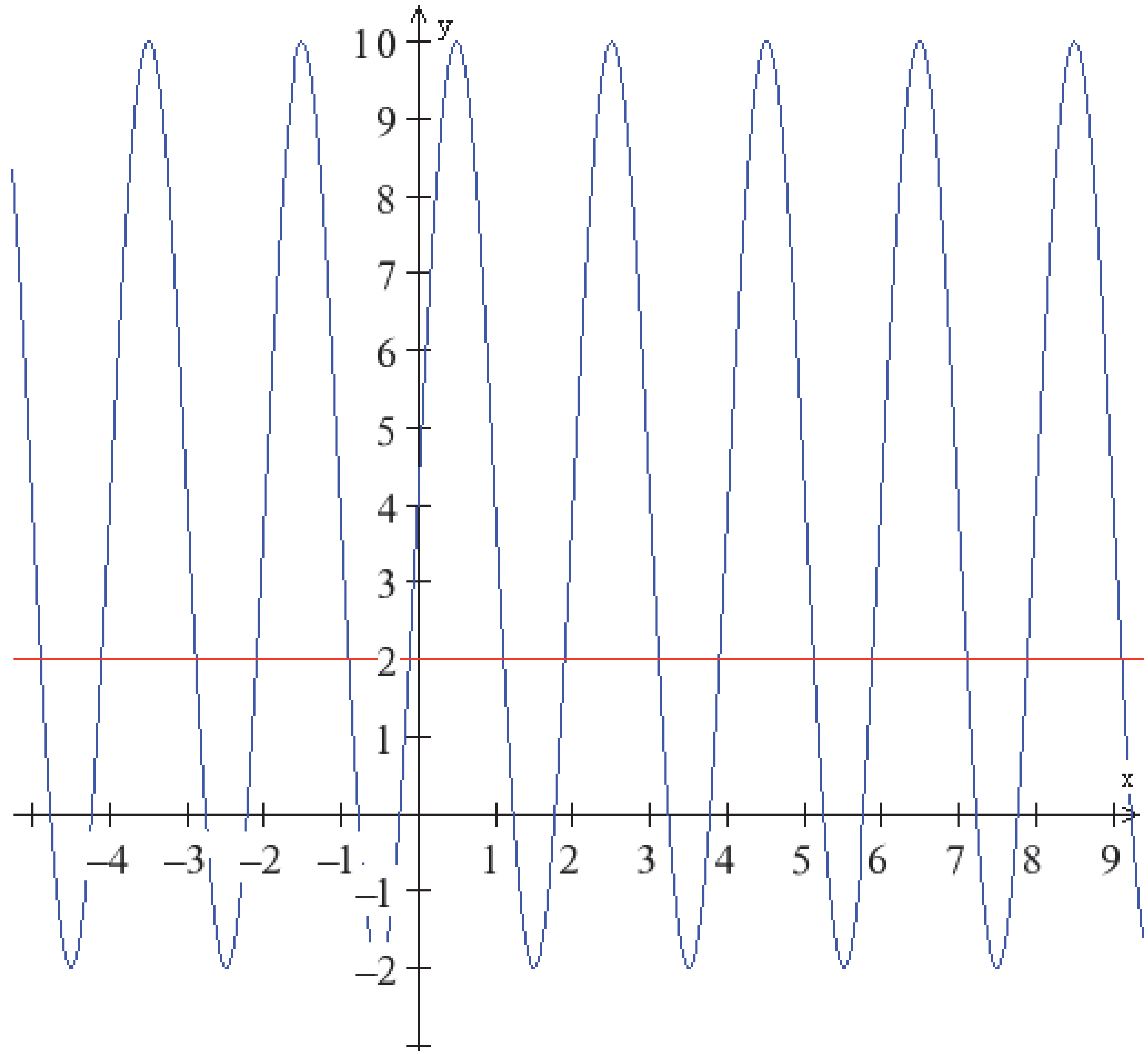
$$x \approx \begin{Bmatrix} (0.108173 \pm 2n) + 1 \\ (0.891829 \pm 2n) + 1 \end{Bmatrix}$$

Add the 1 to both solutions

$$x \approx \begin{Bmatrix} 1.108173 \pm 2n \\ 1.891829 \pm 2n \end{Bmatrix}$$

Now let's see this on the graph

And we'll focus on the three points at which the graph intersects the line $y = 2$



Given $y = 4 - 6\sin\left(\pi(x+1)\right)$ find the first three positive values of $x$ for which

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And we'll focus on the three points at which the graph intersects the line $y = 2$

Notice that these two values differ by 2

$x = 1.89183$

$x = 1.108173$

$x = 3.108173$