This part of the unit is really about equivalence:

$$12 = 1 \quad ?$$

$$12 \ inches = 1 \ foot$$

$$5280 = 1$$

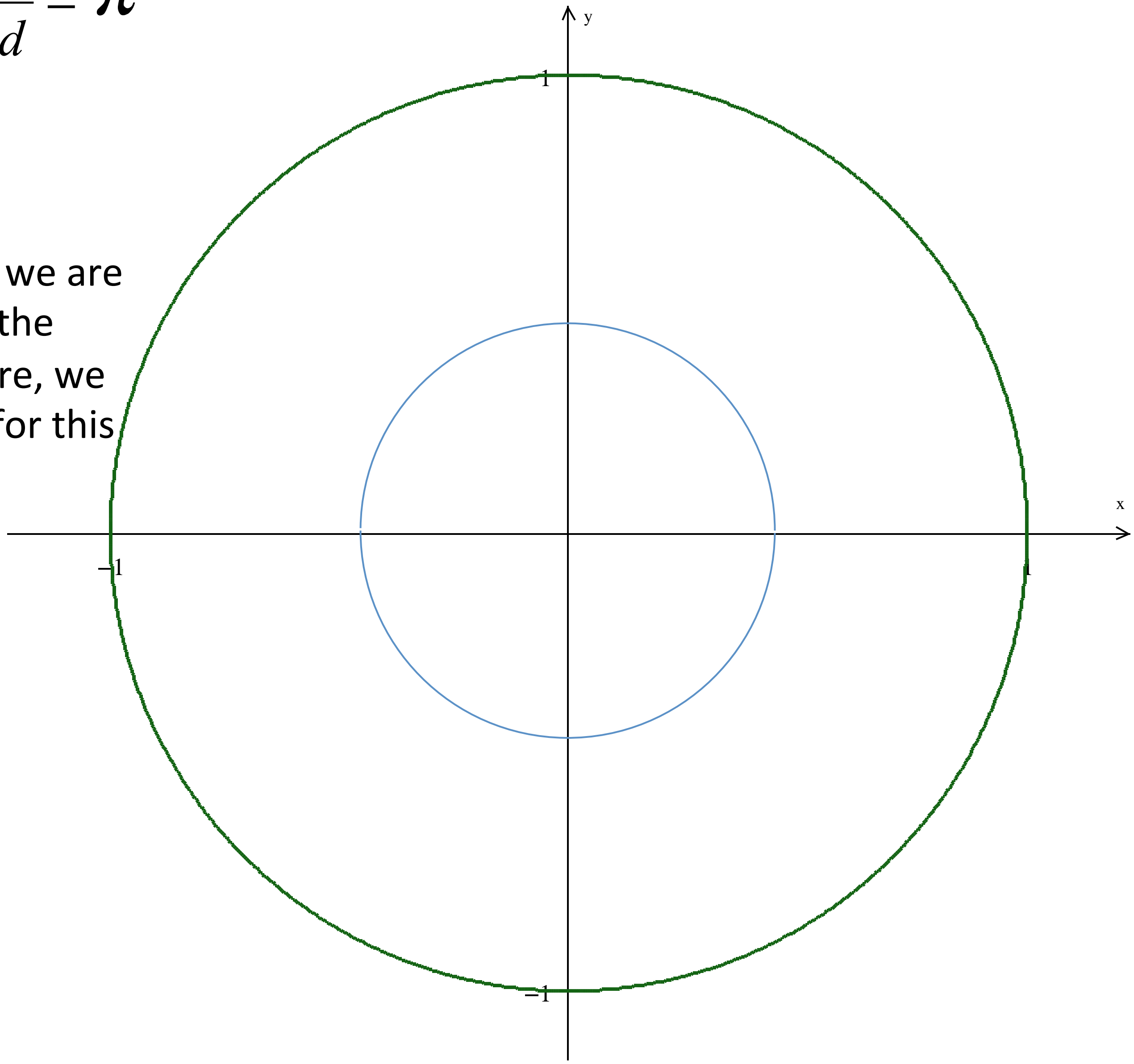
$$5280 \ feet = 1 mile$$

Remember: $\dfrac{C}{d} = \pi$

$$C = 2\pi r$$

And because we are dealing with the unit circle here, we can say that for this special case,

$$C = 2\pi$$



Remember: $\dfrac{C}{d} = \pi$

$$C = 2\pi$$

Since 30° is $\dfrac{30°}{360°} = \dfrac{1}{12}$ of the way around, then the length of this arc is

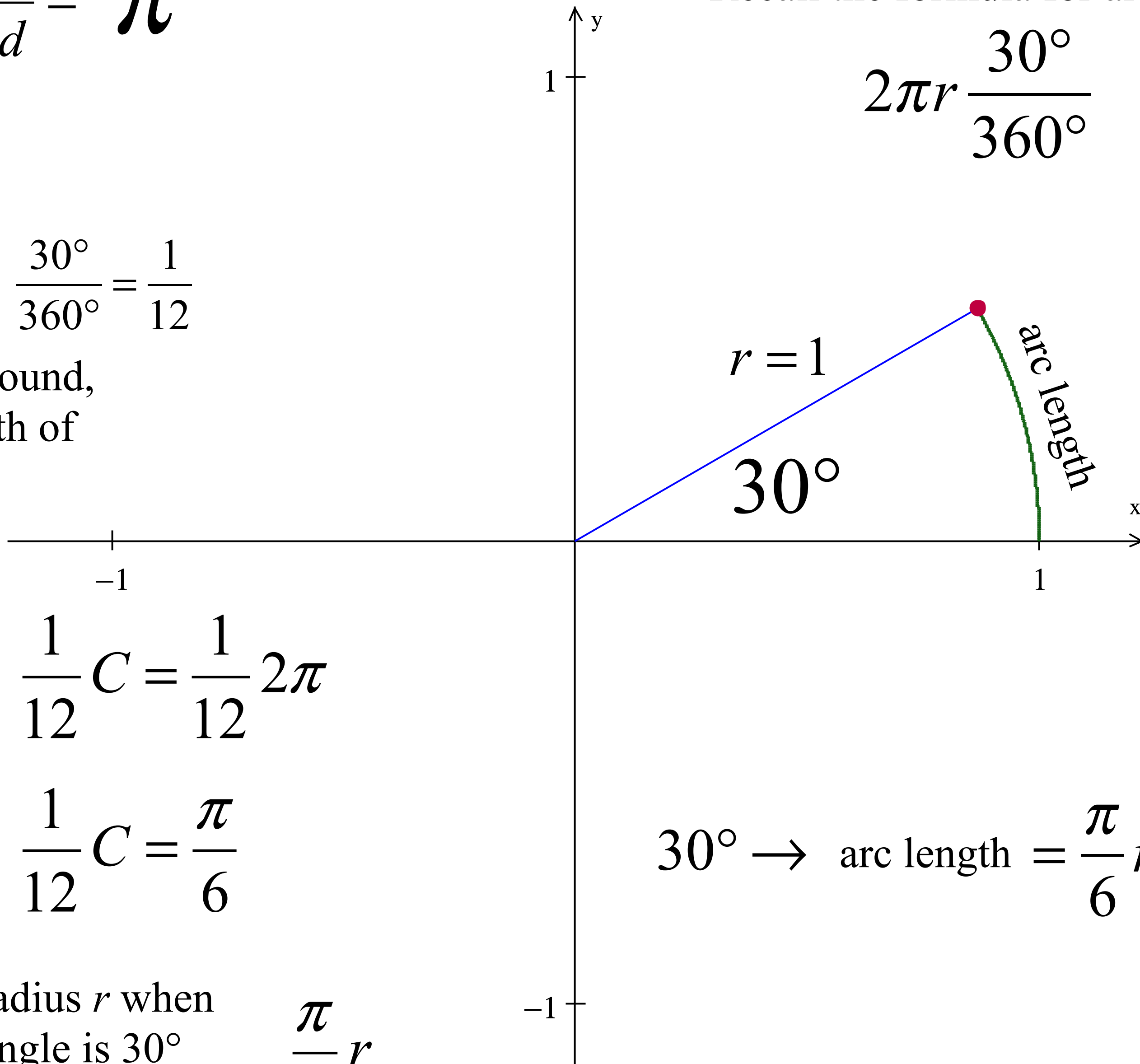
arc length = $\dfrac{1}{12}C = \dfrac{1}{12}2\pi$

$$\frac{1}{12}C = \frac{\pi}{6}$$

So for any radius r when the central angle is 30° the length of the arc is… $\dfrac{\pi}{6}r$

Recall the formula for arc length

$$2\pi r\frac{30°}{360°}$$



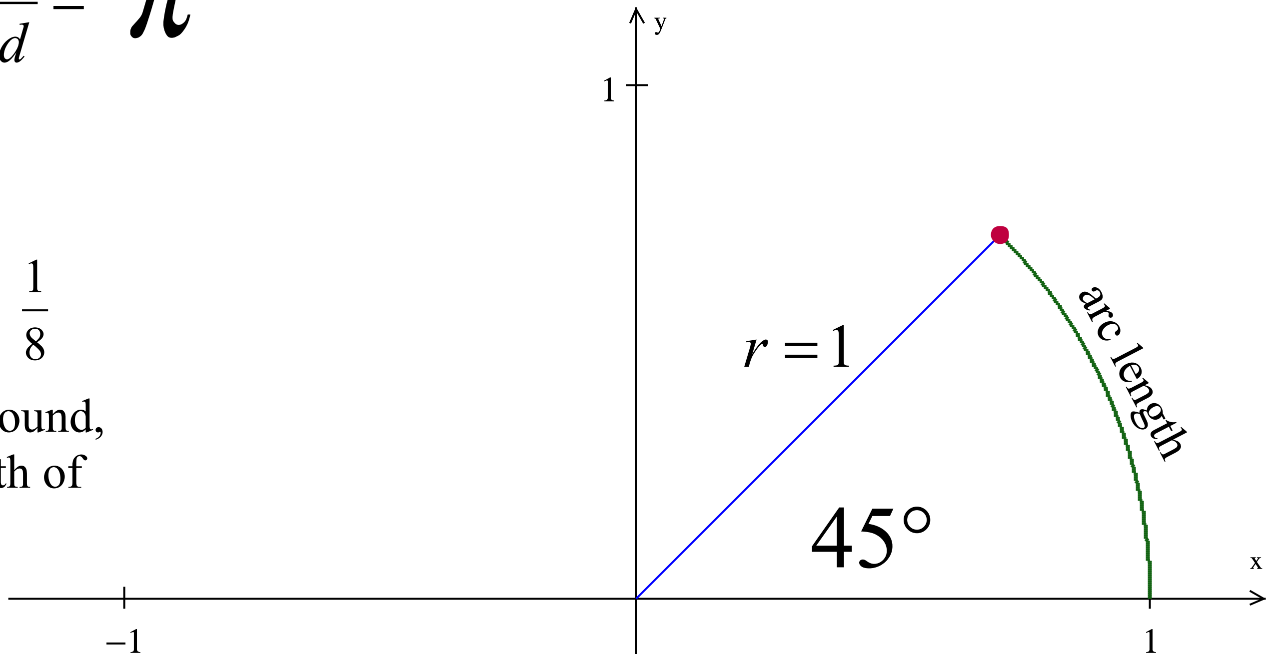
$$30° \rightarrow \text{arc length} = \frac{\pi}{6}r$$

Remember: $\dfrac{C}{d} = \pi$

$$C = 2\pi$$

Since 45 is $\dfrac{1}{8}$

of the way around, then the length of this arc is



arc length = $\dfrac{1}{8}C = \dfrac{1}{8}2\pi$

$$\frac{1}{8}C = \frac{\pi}{4}$$

$$45° \rightarrow \text{ arc length } = \frac{\pi}{4}r$$

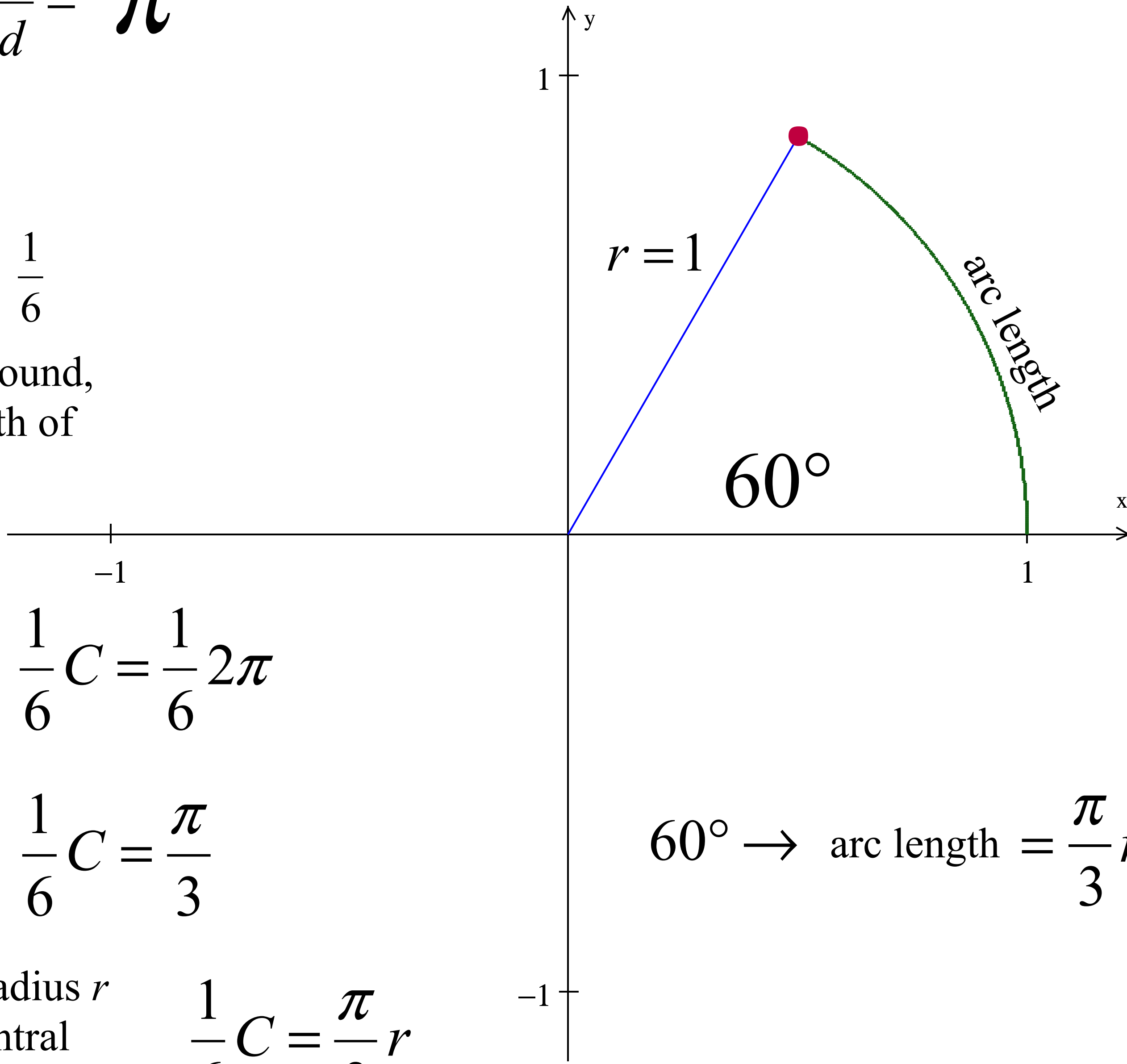
So for any radius *r* when the central angle is 45 $\quad \dfrac{1}{8}C = \dfrac{\pi}{4}r$

Remember: $\dfrac{C}{d} = \pi$

$$C = 2\pi$$

Since 60 is $\dfrac{1}{6}$

of the way around, then the length of this arc is



arc length = $\dfrac{1}{6}C = \dfrac{1}{6}2\pi$

$$\frac{1}{6}C = \frac{\pi}{3}$$

$$60^\circ \rightarrow \text{ arc length } = \frac{\pi}{3}r$$

So for any radius r when the central angle is 60

$$\frac{1}{6}C = \frac{\pi}{3}r$$

$$30^\circ \to \frac{\pi}{6} \qquad 45^\circ \to \frac{\pi}{4} \qquad 60^\circ \to \frac{\pi}{3}$$

$$360^\circ \to 2\pi$$

We now can measure angles with a new unit of measurement: Radians

$$360^\circ = 2\pi \ radians$$

Recall how to cancel units from science classes?

$$12 \ inches = 1 \ foot \qquad 5280 \ feet = 1 \, mile$$

$$80 \ inches = 80 \ \cancel{inches}\left(\frac{1 \ ft}{12 \, \cancel{inches}}\right) \qquad 2.3 \ miles = 2.3 \ \cancel{miles}\left(\frac{5280 \ ft}{1 \, \cancel{mile(s)}}\right)$$

$$= 6.75 \ ft \qquad = 12{,}144 \ ft$$

We now have this ratio: $\dfrac{2\pi \ radians}{360°} = \dfrac{\pi \ radians}{180°}$

Converting degrees to radians

Multiply by $\dfrac{\pi \ rad}{180°}$

Converting radians to degrees

Multiply by $\dfrac{180°}{\pi \ rad}$

To sum this up, in PreCalc and beyond you'll see how important radian measurements are in math. The first direct application we're seeing here is how they are the direct way to translate angle measurements into arc lengths.

Convert to radians

$$60° \frac{\pi \ rad}{180°} = \frac{\pi}{3} \ rad$$

$$90° \frac{\pi}{180} = \frac{\pi}{2} \ rad$$

$$135° \frac{\pi}{180} = \frac{3\pi}{4} \ rad$$

$$200° \frac{\pi}{180} = \frac{10\pi}{9} \ rad$$

Convert to degrees

$$\frac{\pi}{6} \ \frac{180}{\pi} = 30°$$

$$\frac{2\pi}{3} \ \frac{180}{\pi} = 120°$$

$$\frac{5\pi}{4} \ \frac{180}{\pi} = 225°$$

$$\frac{7\pi}{6} \ \frac{180}{\pi} = 210°$$

Notice that the radian problems on the right don't have units. For reasons we won't discuss here, angle measurements without units are assumed to be radians.