The Quotient Rule

Easy from top to bottom

Remember the Power Rule? Of course you do

And not just because it's so easy to remember but also...

...because it's 2d: Find the derivative using the Power Rule

But now we want to build on this standard to master two more:

Standard 4c: Find the derivative of a rational function using the Quotient Rule

Standard 4d: Apply sign patterns to the first derivative

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$$

Huh?

Take the derivative of the quotient of these two separate functions

Derivative of the top times the bottom minus the derivative of the bottom times the top...

...all divided by the bottom squared

...or as some have said to help them remember...

"Low D-High, High D-Low, square the bottom and off you go"

Of course, nothing tells the story better than examples:

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{f'g-g'f}{g^2}$$

Huh?

$$\frac{d}{dx}\left(\underbrace{\overbrace{x^2+5x-1}^{f}}_{\substack{3x^2-4\\ g}}\right)$$

$$\frac{d}{dx}\left(\frac{\overbrace{x^2+5x-1}^{f}}{\underbrace{3x^2-4}_{g}}\right)=\frac{\overbrace{(2x+5)}^{f'}\overbrace{(3x^2-4)}^{g}-\overbrace{(6x)}^{g'}\overbrace{(x^2+5x-1)}^{f}}{\underbrace{\left(3x^2-4\right)^2}_{g^2}}=$$

$$=\frac{6x^3+15x^2-8x-20-6x^3-30x^2+6x}{\left(3x^2-4\right)^2}=\frac{-15x^2-2x-20}{\left(3x^2-4\right)^2}$$

Let's try another one…

The Quotient Rule

Huh?

$$\frac{d}{dx}\left(\frac{x^2+4}{x}\right) = \frac{\overbrace{(2x)}^{f'}\overbrace{(x)}^{g} - \overbrace{(1)}^{g'}\overbrace{(x^2+4)}^{f}}{\underbrace{x^2}_{g^2}} = \frac{2x^2-(x^2+4)}{x^2} =$$

$$\frac{x^2-4}{x^2}$$

It's that simple

The Quotient Rule

So now that we know that

$$\frac{d}{dx}\left(\frac{x^2+4}{x}\right)=\frac{x^2-4}{x^2}$$

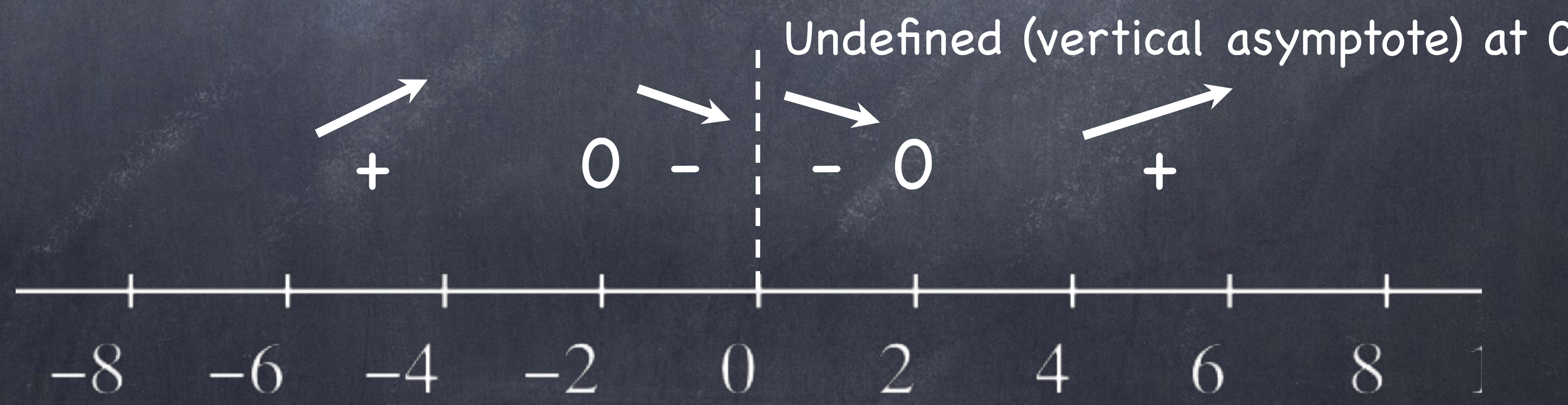
Find the critical points for $y=\left(\frac{x^2+4}{x}\right)$

We already know it's derivative

so...

$$\frac{x^2-4}{x^2}=0 \longrightarrow x^2-4=0 \longrightarrow (x-2)(x+2)=0$$

$x=\pm 2$



Max at x = -2

Min at x = 2

It really is that simple