When working with Normal Distributions we have a value that is easy to calculate and very helpful in comparing data

z score

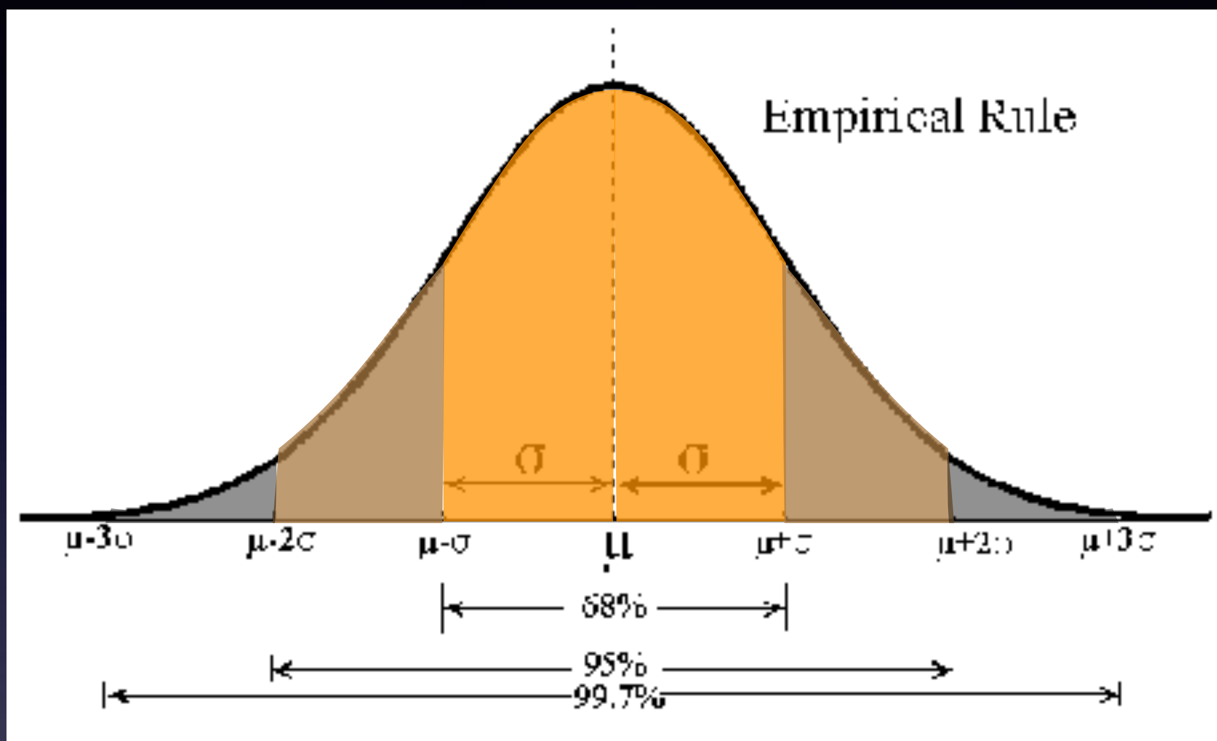
$$= \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

- Provide a common scale to compare data
- Conveys how many standard deviations above/below the mean a data value is
- Positive *z* scores lie above the mean
- Negative *z* scores lie below the mean

$$z_i = \frac{x_i - \mu}{\sigma} \quad \text{or} \quad z_i = \frac{x_i - \bar{x}}{s_x}$$

This value will play an important role in everything we do with normal distributions throughout this class

Empirical Rule - the 68/95/99.7 Rule



Approximately 68% of the observations are within 1 standard deviation of the mean.

Approximately 95% of the observations are within 2 standard deviation of the mean.

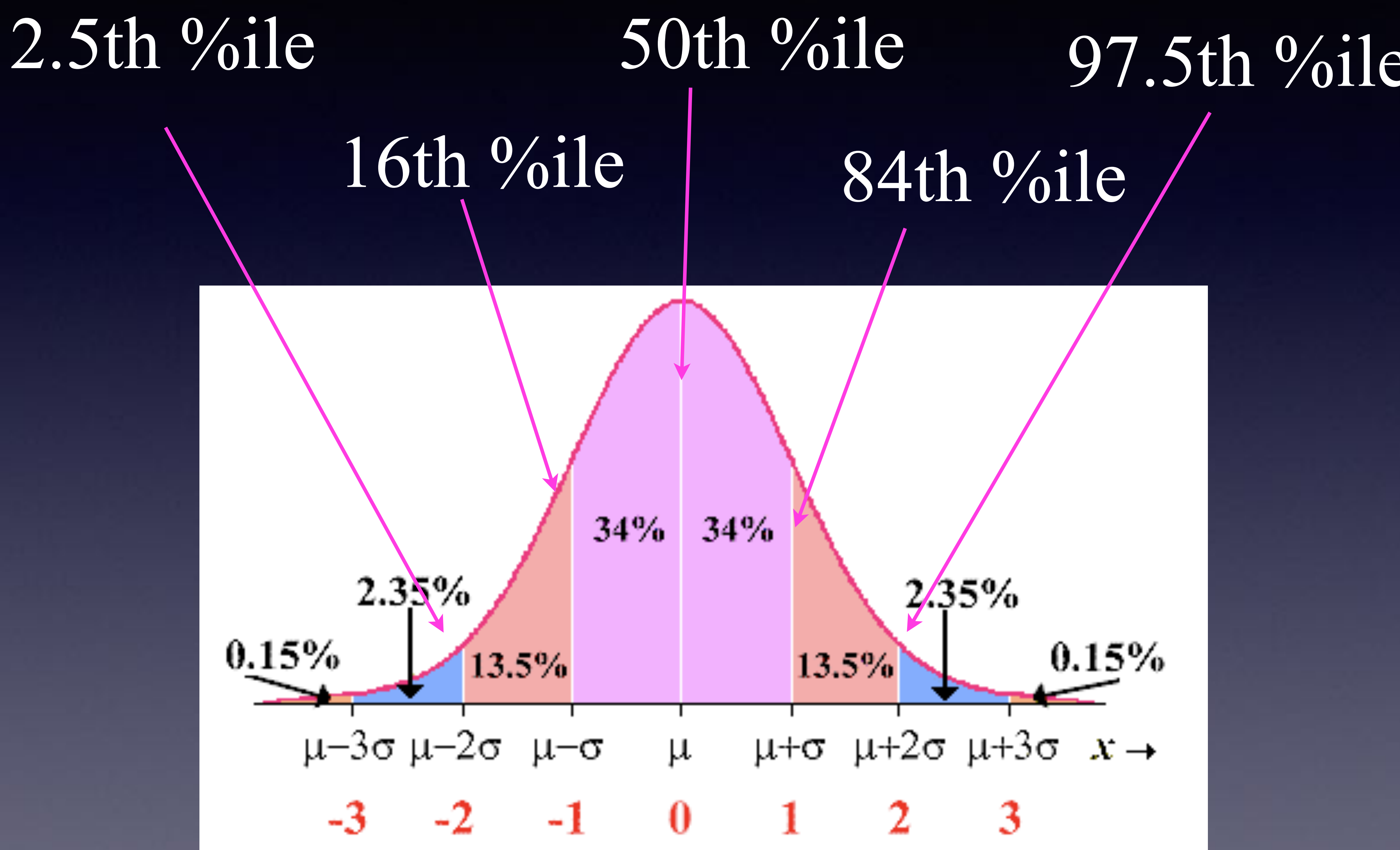
Approximately 99.7% of the observations are within 3 standard deviation of the mean.

$\mu =$ Mean for the population

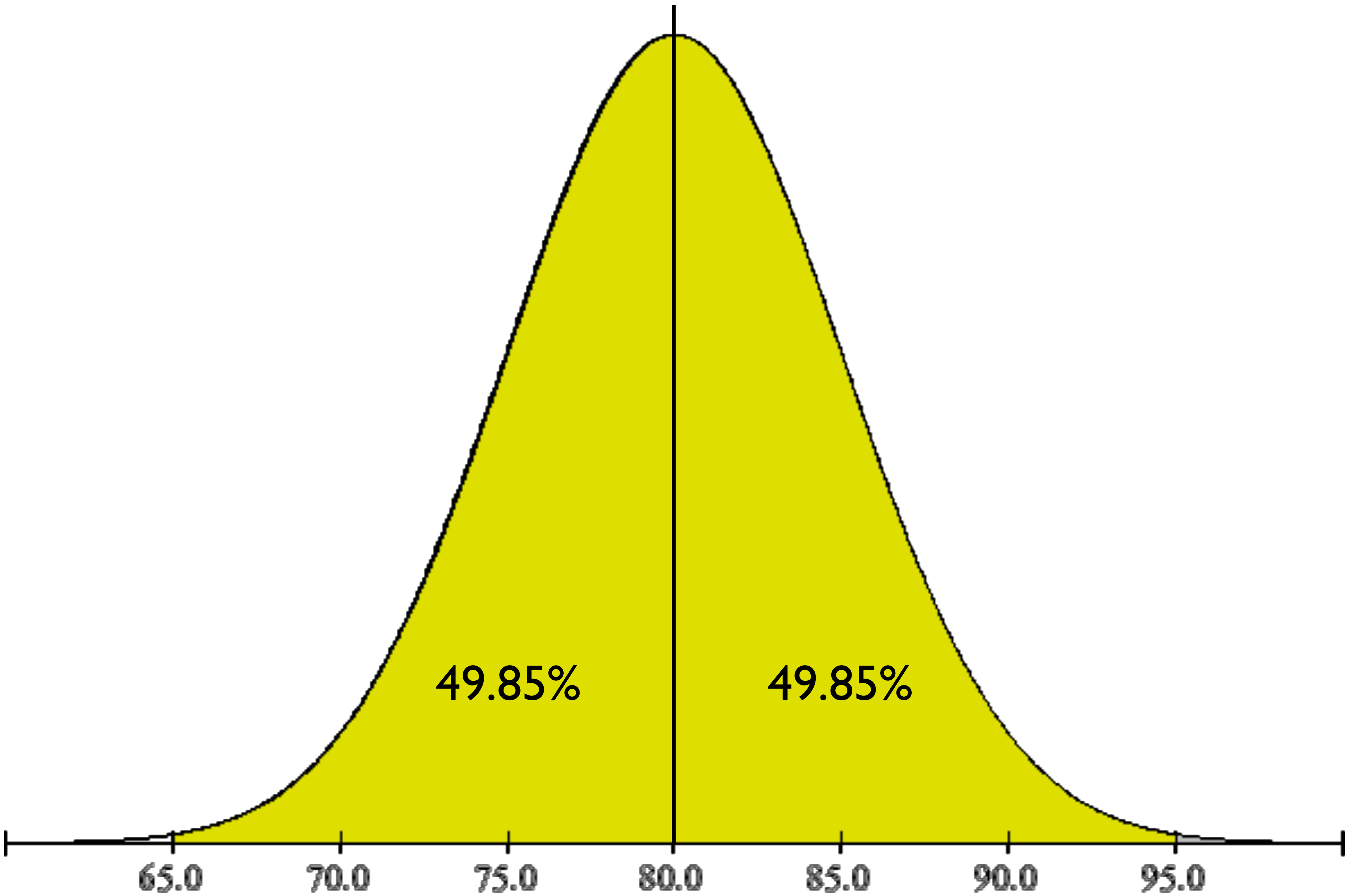
$\sigma =$ Standard Deviation for the population

*Deals with the **middle** ___ % of the data*

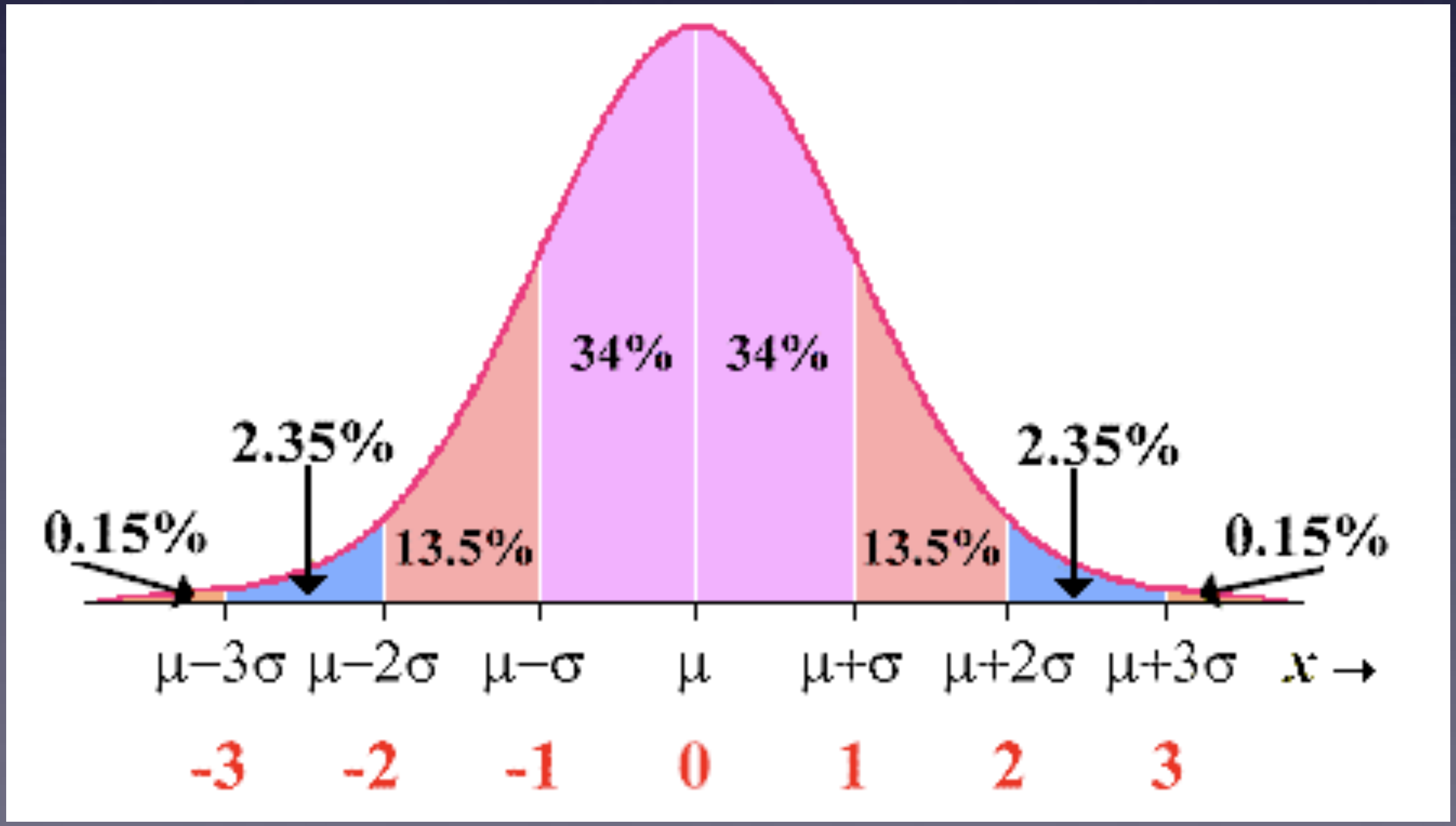
Percentiles - value such that ___ % of the observations in the data set fall ***below*** that value



Suppose we have test scores that are normally distributed with a mean of 80 and a standard deviation of 5

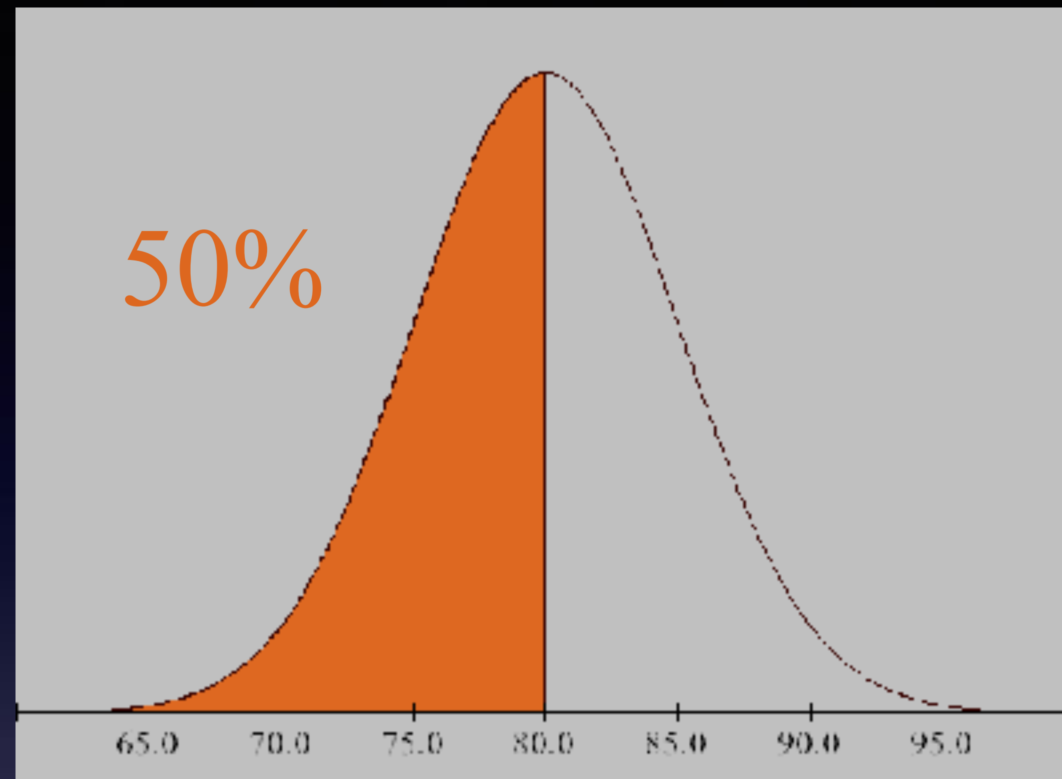


- The Empirical Rule only applies to normal/bell curve distributions
- You must be able to look at a normal distribution through two lenses
  - The middle % of the data - the 68/95/99.7 Rule
  - The % of data from a given value ‘on down’

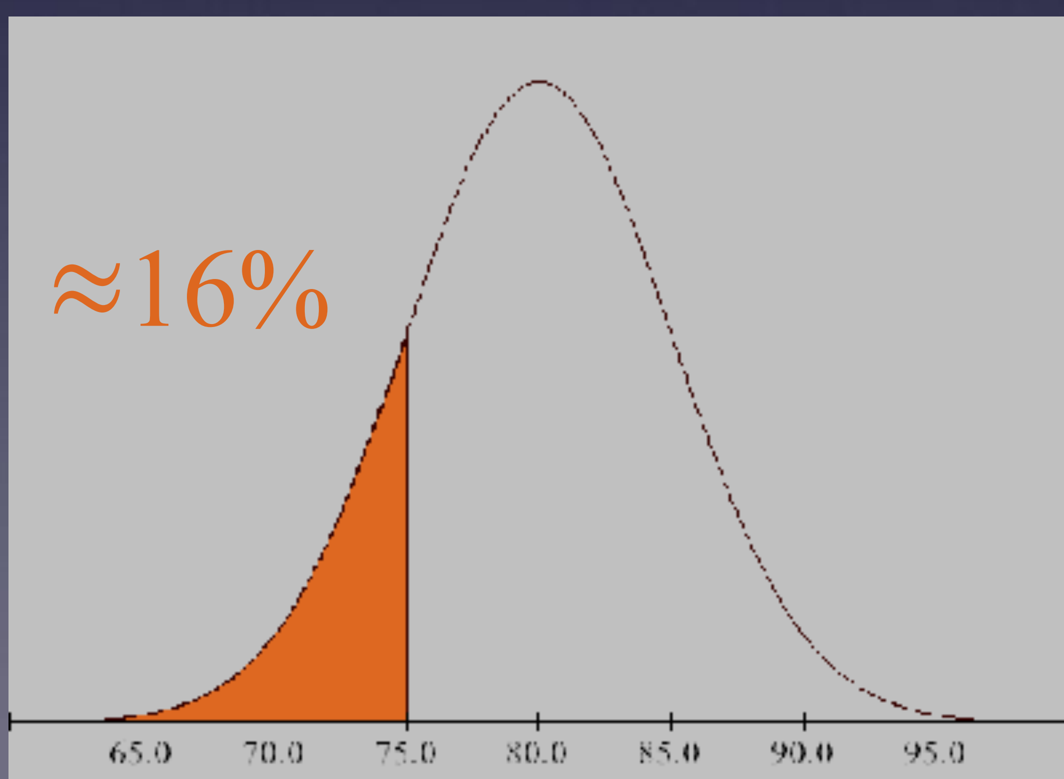


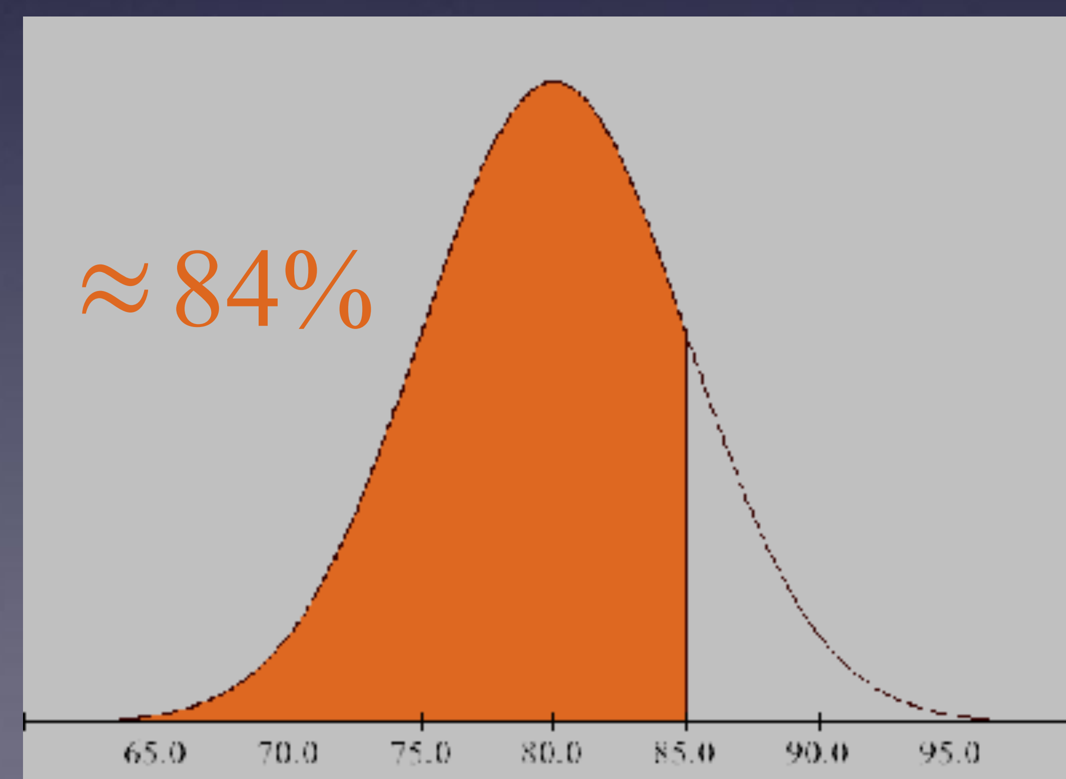
•The % of data from a given value 'on down'

Remembering the 68/95/99.7 Rule



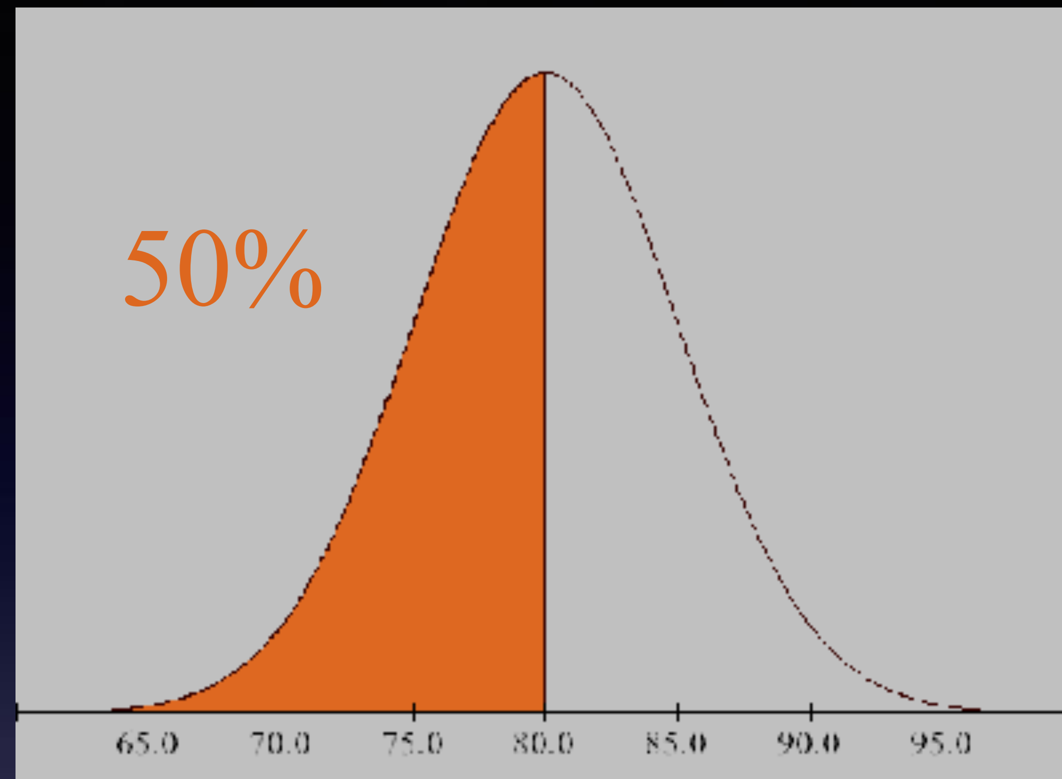
Note that both shaded regions end one standard deviation from the mean so we determine our % using the 68% marker



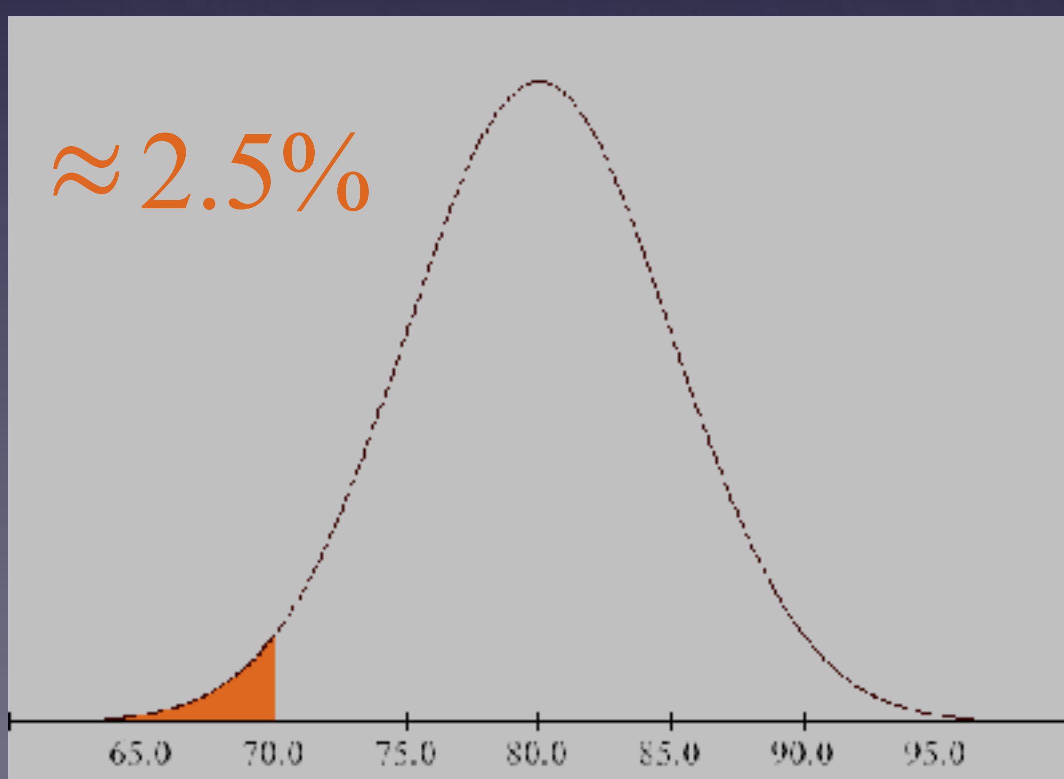


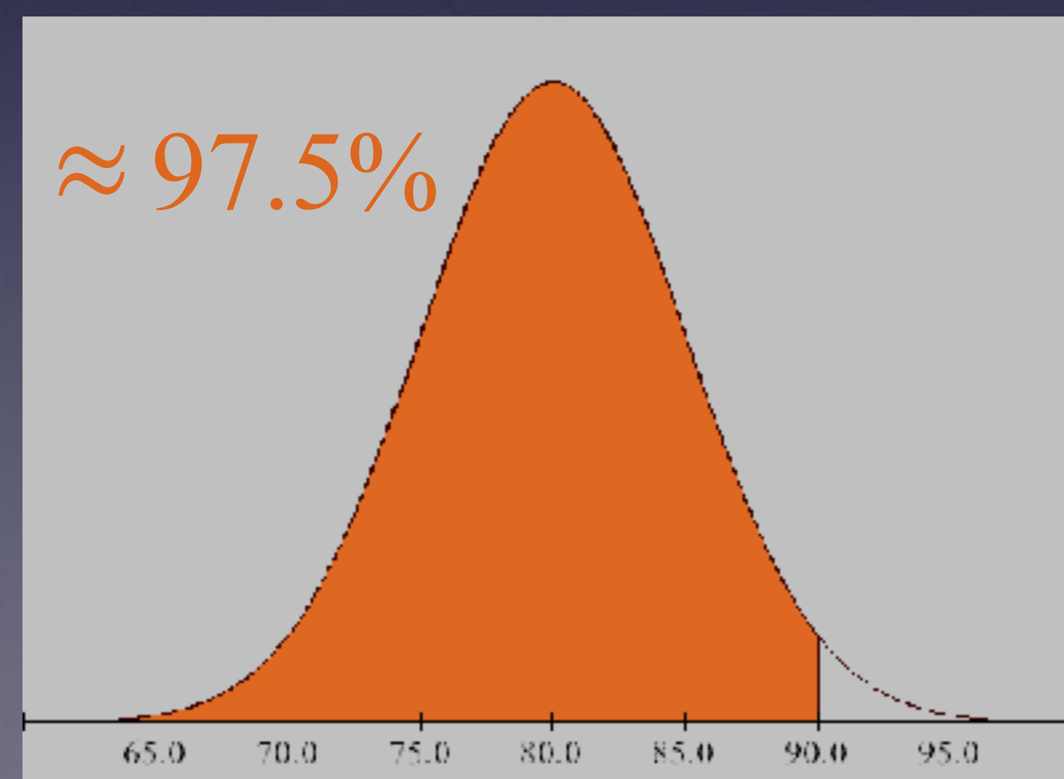
•The % of data from a given value 'on down'

Remembering the 68/95/99.7 Rule



Now we have both shaded regions ending two standard deviations from the mean so we determine our % using the 95% marker





So where do z-scores fit into all of this?

z scores

$$= \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

If you had a score of 85 on your test, your z-score would be

$$\frac{85-80}{5} = 1$$

If you had a score of 75 on your test, your z-score would be

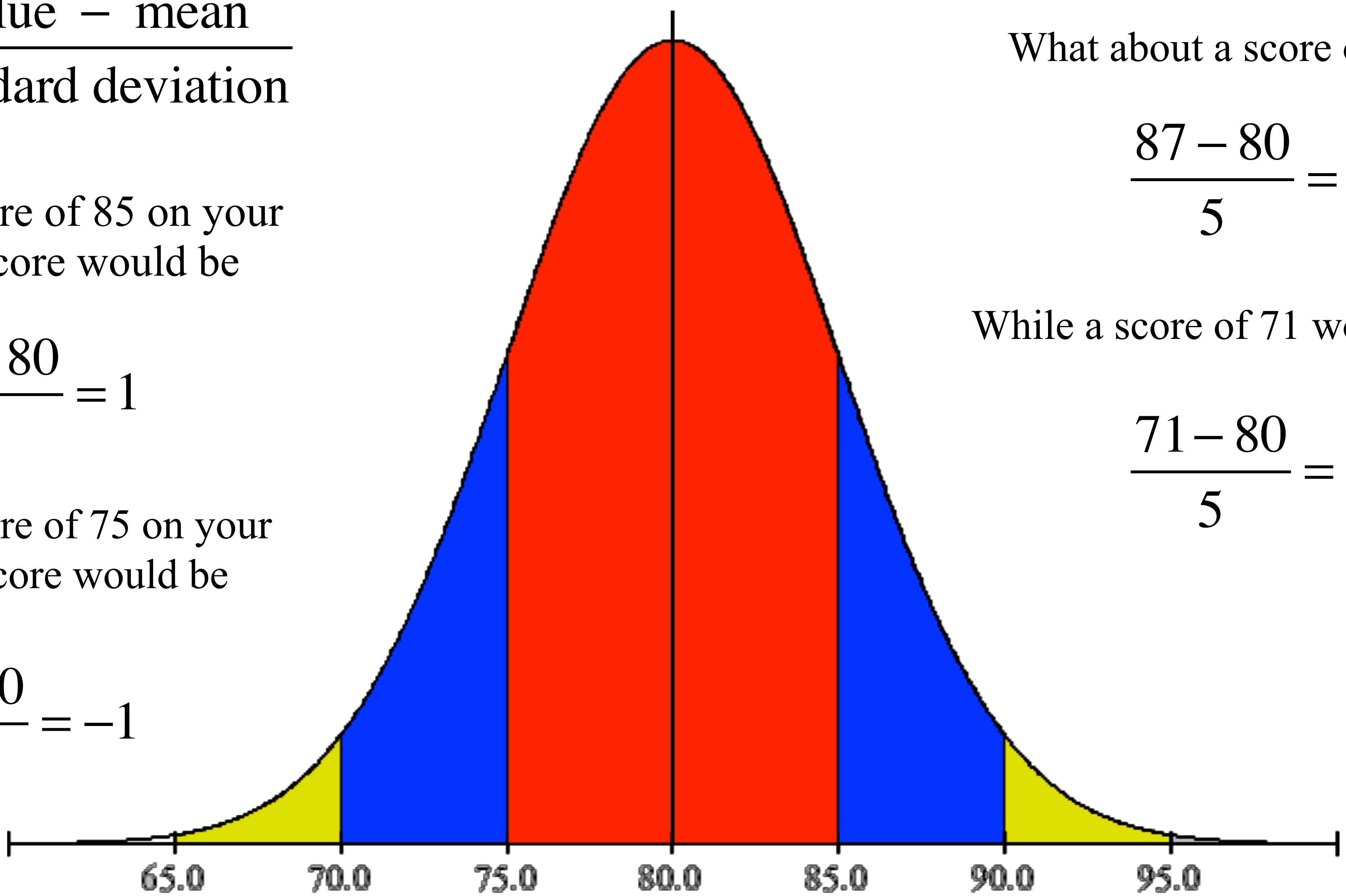
$$\frac{75-80}{5} = -1$$

What about a score of 87?

$$\frac{87-80}{5} = 1.4$$

While a score of 71 would give you

$$\frac{71-80}{5} = -1.8$$



So the z-score tells you both how many standard deviations you are from the mean and in which direction