Single Sample Hypothesis Tests for Proportions

$$H_0 : p = \#$$

$$H_a : p \; \neq, <, > \; \#$$

Note #1: Use colons

Note #2: Use only PARAMETERS in your hypothesis...although there will be some problems where we'll use words/sentences

Note #3: H_0 ALWAYS gets an = ...even if the wording in the problem sounds like it shouldn't

Note #4: The symbol used in the alternate will come from the context of the problem

$\neq$ - two-sided test, equivalent to a Confidence Interval (CI)

$<$, $>$ } - one-sided test

Errors - We make them, even though we're awesome

	Fail to reject	Reject
true	Hooray!	Type I error
true	Type II error	Hooray!

Type I error - reject H_0 when H_0 is true

Type II error - fail to reject H_0 when H_0 is false

OR

Type I error - 1st equation correct and you pick the 2nd equation

Type II error - 2nd equation correct and you pick the 1st equation

α vs β

$$P(\text{Type I error}) = \alpha$$

$$P(\text{Type II error}) = \beta$$

Also called 'level of significance' or 'significance level'.

If α goes up, then β goes down.

If α goes down, then β goes up.

Game plan - determine which error is worse, then choose the appropriate α and β.

Steps in Hypothesis Testing

1. Define the population characteristic (i.e. parameter) about which hypotheses are to be tested.

2. State the null hypothesis H_0 Light bulb example → $\mu = 1000$ hrs

3. State the alternative hypothesis H_a $\mu < 1000$ hrs

4. State the significance level for the test α What value of α are we going with?

5. Check all assumptions. We've done this before

6. State the name of the test. Sample proportion *z*? Sample mean *z*? Sample mean *t*? etc.

7. State *df* (degrees of freedom) if applicable (not applicable in proportion land).

8. Display the test statistic to be used without any computation at this point. Which formula?

9. Compute the value of the test statistic, showing specific numbers used. Put formula in calculator

10. Calculate the P – value. Is P - value greater than or less than significance level? This determines the outcome, reject or fail to reject.

11. Sketch a picture of the situation.

12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0
    2. Summarize in context discussing H_a

Single Sample Hypothesis Tests for Proportions

Steps in Proportion Hypothesis Testing

1. $p =$ ....

2. $H_0 : p = \#$

3. $H_a : p \; \begin{matrix} \neq \\ < \\ > \end{matrix} \; \#$

4. State the significance level α for the test

5. <u>Assumptions:</u>

   1. Random Sample
   2. $np \geq 10$
      $n(1-p) \geq 10$
   3. SSSRTP

6. 1 Sample Proportion z Test

7. $df = N / A$

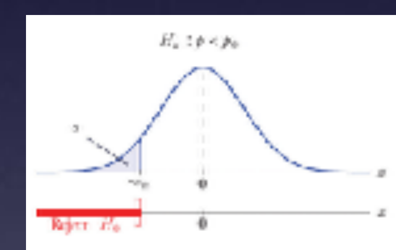
8/9. $$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \#$$

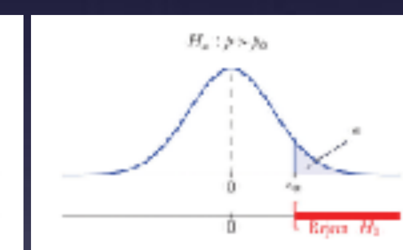
10. $P - value =$

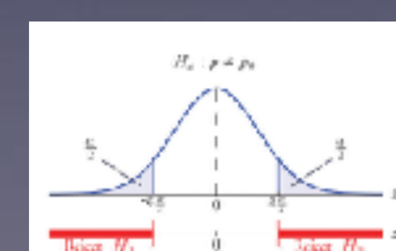
$$P(z > \#) = normalcdf(\#, 1E99, 0, 1)$$
$$P(z < \#) = normalcdf(-1E99, \#, 0, 1)$$
} one-sided tests

$$2P(z > \#) = 2 * normalcdf(\#, 1E99, 0, 1)$$
$$2P(z < \#) = 2 * normalcdf(-1E99, \#, 0, 1)$$
} two-sided tests

11.







12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0 .
    2. Summarize in context discussing H_a .

10. $P - value = \begin{cases} P(z > \#) = normalcdf(\#, 1E99, 0, 1) \\ P(z < \#) = normalcdf(-1E99, \#, 0, 1) \end{cases}$ one-sided tests

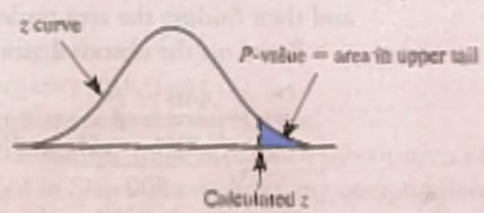
$ \begin{cases} 2P(z > \#) = 2 * normalcdf(\#, 1E99, 0, 1) \\ 2P(z < \#) = 2 * normalcdf(-1E99, \#, 0, 1) \end{cases}$ two-sided tests

Determination of the P-Value When the Test Statistic Is z

1. **Upper-tailed test:**

   H_a: $p >$ hypothesized value

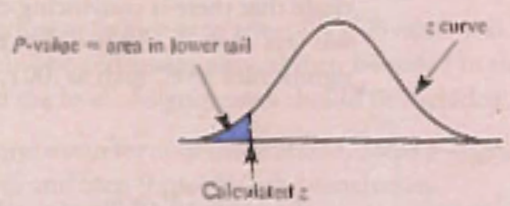
   P-value computed as illustrated:



2. **Lower-tailed test:**

   H_a: $p <$ hypothesized value

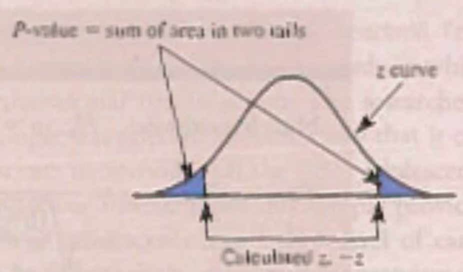
   P-value computed as illustrated:



3. **Two-tailed test:**

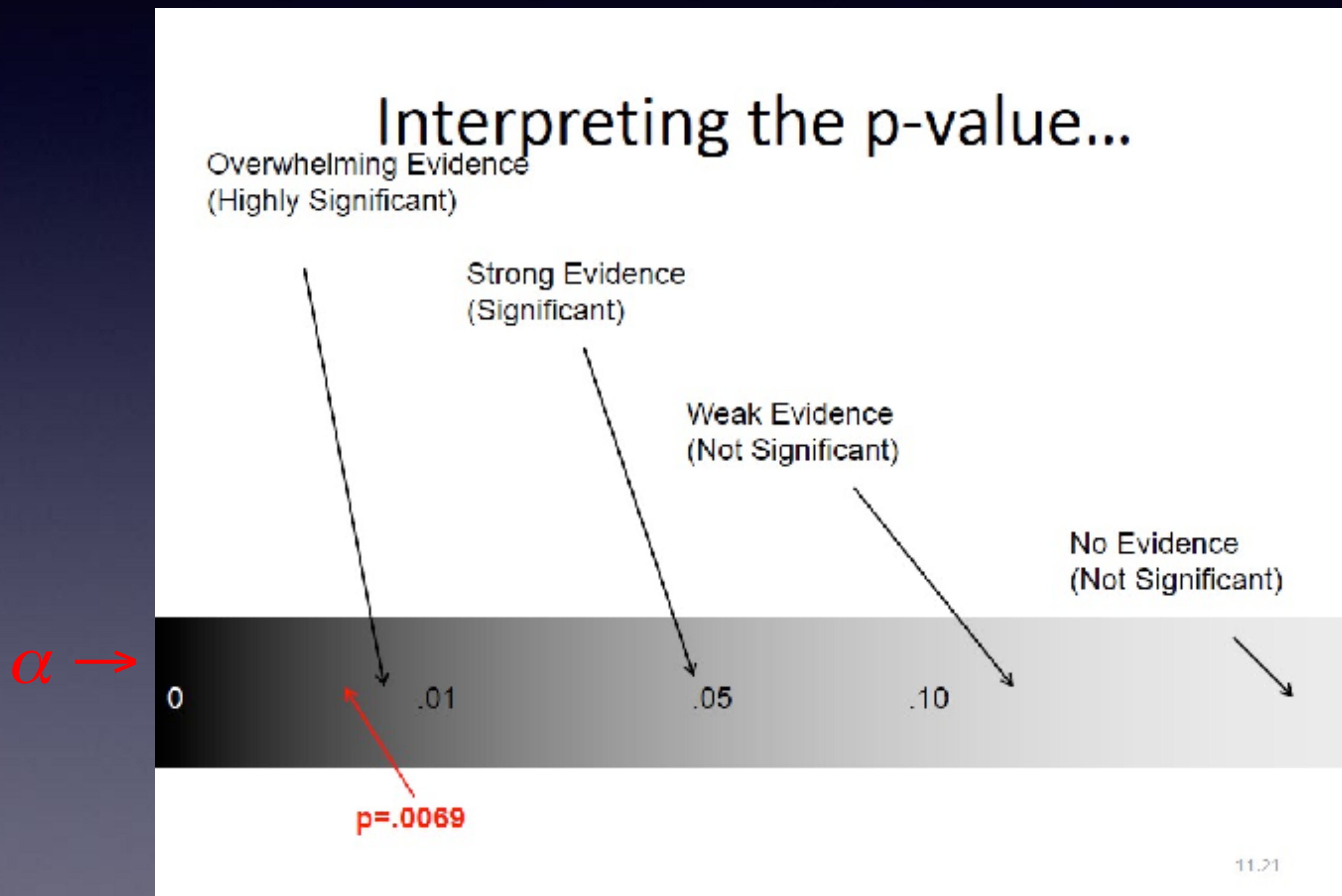
   H_a: $p \neq$ hypothesized value

   P-value computed as illustrated:

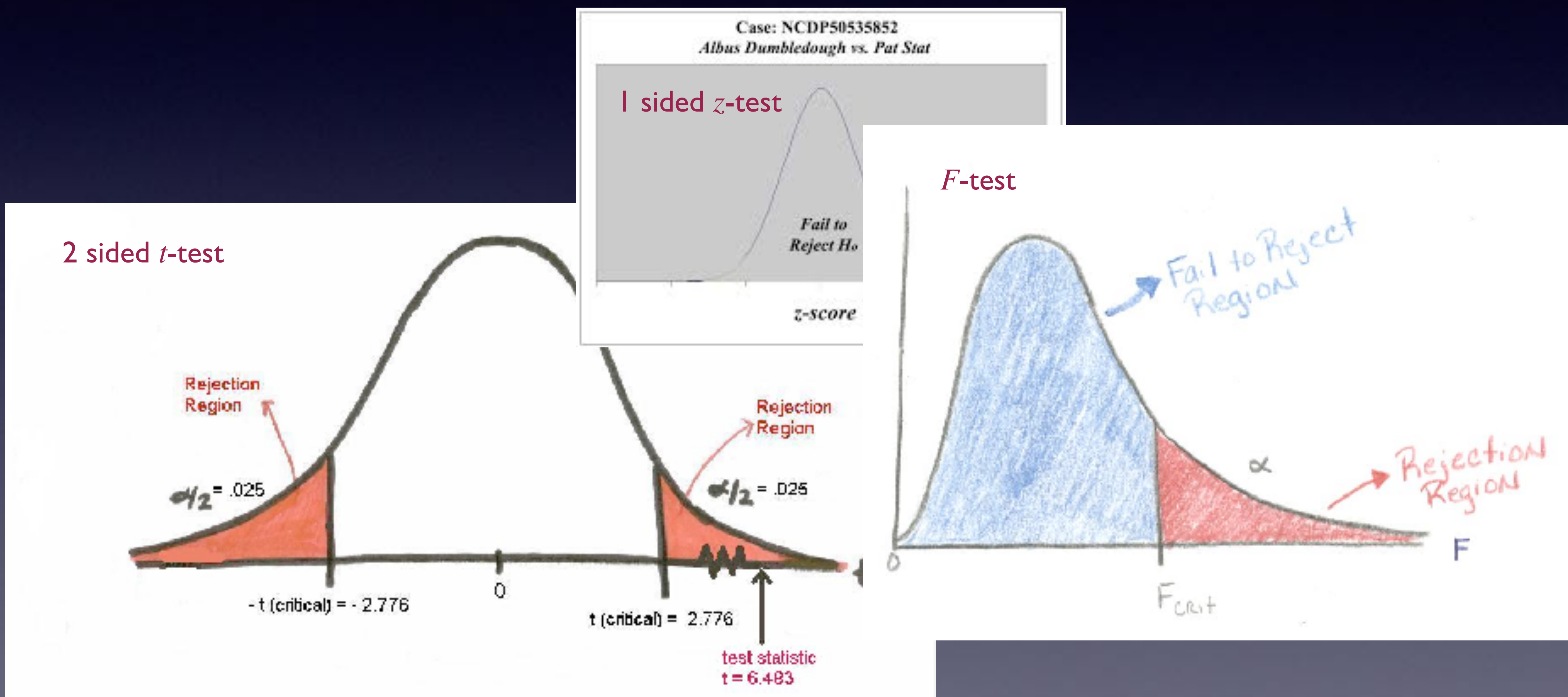


P-Value $< \alpha \Rightarrow$ Reject H_o ; Evidence for H_a

P-Value $> \alpha \Rightarrow$ Fail to Reject H_o ; No Evidence for H_a

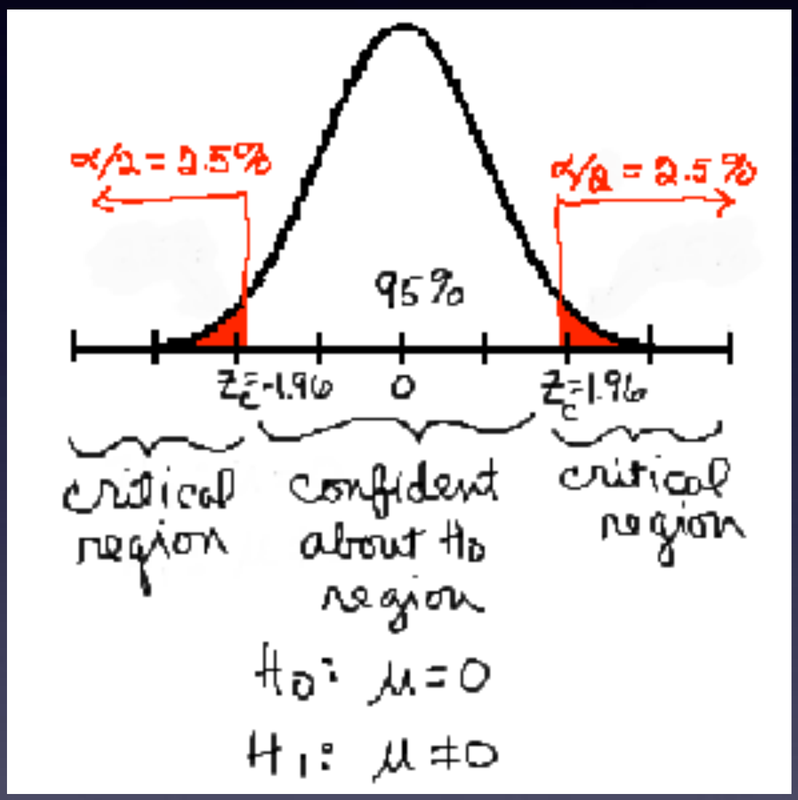


P-Value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true



Confidence Intervals are Related to Two-Sided Tests

In general, for every two-sided test of hypothesis there is an equivalent statement about whether the hypothesized parameter value is included in a confidence interval.



The 95% confidence interval for the mean weight of all the Dole Pineapples grown in the field this year is 31.255 to 32.616 ounces.

$$95\%\ \text{CI:}\ \mu \in (31.255, 32.616)$$

$$H_0 : \mu = 31$$

$$H_a : \mu \neq 31$$

When the two-sided significance test at level α rejects H_0: $\mu = \mu_0$, the 100(1 – α)% confidence interval for μ will not contain the hypothesized value μ_0.

When the two-sided significance test at level α fails to reject the null hypothesis, the confidence interval for μ will contain μ_0.

Justify = hypothesis test

estimate = CI

Statistically significant
= reject null