# Independent Samples Hypothesis Tests for Means

$$H_0: \mu_1 = \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 = 0$$

Note #3: $H_0$ ALWAYS gets an = ...even if the wording in the problem sounds like it shouldn't

Note #1: Use colons

$$H_a: \mu_1 \; \begin{matrix} \neq \\ < \\ > \end{matrix} \; \mu_2 \quad \text{or} \quad \mu_1 - \mu_2 \; \begin{matrix} \neq \\ < \\ > \end{matrix} \; 0$$

Note #2: Use only PARAMETERS in your hypothesis...although there will be some problems where we'll use words/sentences

Note #4: The symbol used in the alternate will come from the context of the problem

$\neq$ - two-sided test, equivalent to a Confidence Interval (CI)

$<$ 
$>$ } - one-sided test

**Steps in Hypothesis Testing**

1. Define the population characteristic (i.e. parameter) about which hypotheses are to be tested.

2. State the null hypothesis $H_0$.

3. State the alternative hypothesis $H_a$.

4. State the significance level for the test $\alpha$.

5. Check all assumptions and state name of test.

6. State the name of the test.

7. State *df* if applicable (not applicable in proportion land).

Determining *df* will vary depending on the test.

8. Display the test statistic to be used without any computation at this point.

9. Compute the value of the test statistic, showing specific numbers used.

10. Calculate the $P$ – value.

11. Sketch a picture of the situation.

12. State the conclusion in two sentences -
    1. Summarize in theory discussing $H_0$.
    2. Summarize in context discussing $H_a$.

# Independent Samples Hypothesis Tests for Means

**Steps in 2 Sample Mean Hypothesis Testing**

1. $\mu_1 =$ .......
   $\mu_2 =$ .......
2. $H_0 : \mu_1 = \mu_2$
3. $H_a : \mu_1 \; \neq, <, > \; \mu_2$
4. State $\alpha$.

5. Assumptions:
   1. Random **Independent** Samples OR Treatments Randomly Assigned
   2. Normality stated

      $n_1 \geq 30, \;\; n_2 \geq 30$

      Boxplot of raw data for both set of data shows roughly symmetric shape
   3. $\sigma_1, \sigma_2$ known → 6. 2 Sample Mean $z$ Test; 7. $df = N/A$

      $\sigma_1, \sigma_2$ unknown → 6. 2 Sample Mean $t$ Test; 7. $df = \#$

8/9. $t = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \#$

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

$$df = \frac{\left(V_1 + V_2\right)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}}$$

$$V_1 = \frac{s_1^2}{n_1} \qquad V_2 = \frac{s_2^2}{n_2}$$

# Independent Samples Hypothesis Tests for Means

**Steps in 2 Sample Mean Hypothesis Testing**

1. $\mu_1 =$ .......
   $\mu_2 =$ .......
2. $H_0 : \mu_1 = \mu_2$
3. $H_a : \mu_1 \; \begin{matrix} \neq \\ < \\ > \end{matrix} \; \mu_2$
4. State $\alpha$.

5. <u>Assumptions:</u>
   1. Random **Independent** Samples
      OR Treatments Randomly Assigned
   2. Normality stated
      $n_1 \geq 30, \;\; n_2 \geq 30$
      Boxplot of raw data for both set of data shows roughly symmetric shape
   3. $\sigma_1, \sigma_2$ known → 6. 2 Sample Mean $z$ Test; 7. $df = N/A$
      $\sigma_1, \sigma_2$ unknown → 6. 2 Sample Mean $t$ Test; 7. $df = \#$

8/9. $$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \#$$

10. $P - value =$

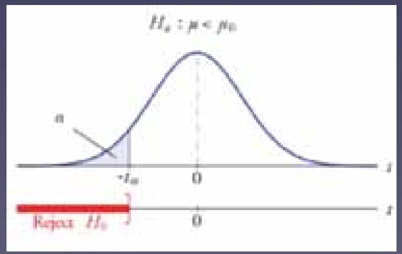
$$P(t > \#) = tcdf(\#, 1E99, df)$$
$$P(t < \#) = tcdf(-1E99, \#, df)$$

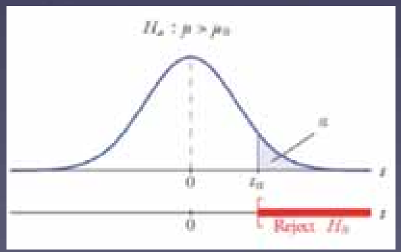
} one-sided tests

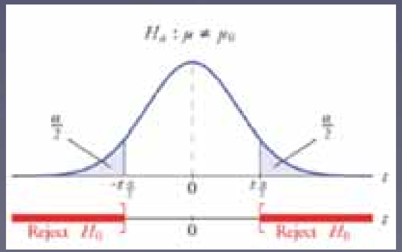
$$2P(t > \#) = 2 * tcdf(\#, 1E99, df)$$
$$2P(t < \#) = 2 * tcdf(-1E99, \#, df)$$

} two-sided tests

11.







12. State the conclusion in two sentences -
    1. Summarize in theory discussing $H_0$ .
    2. Summarize in context discussing $H_a$ .

# Confidence Intervals

## General CI Formula

$$\text{Statistic} \pm \left(\text{Critical Value}\right)\left(\text{Standard Deviation}\right)$$

## 2 Sample Mean *z* CI Formula

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Use Table or Calculator
to get the *z* critical value

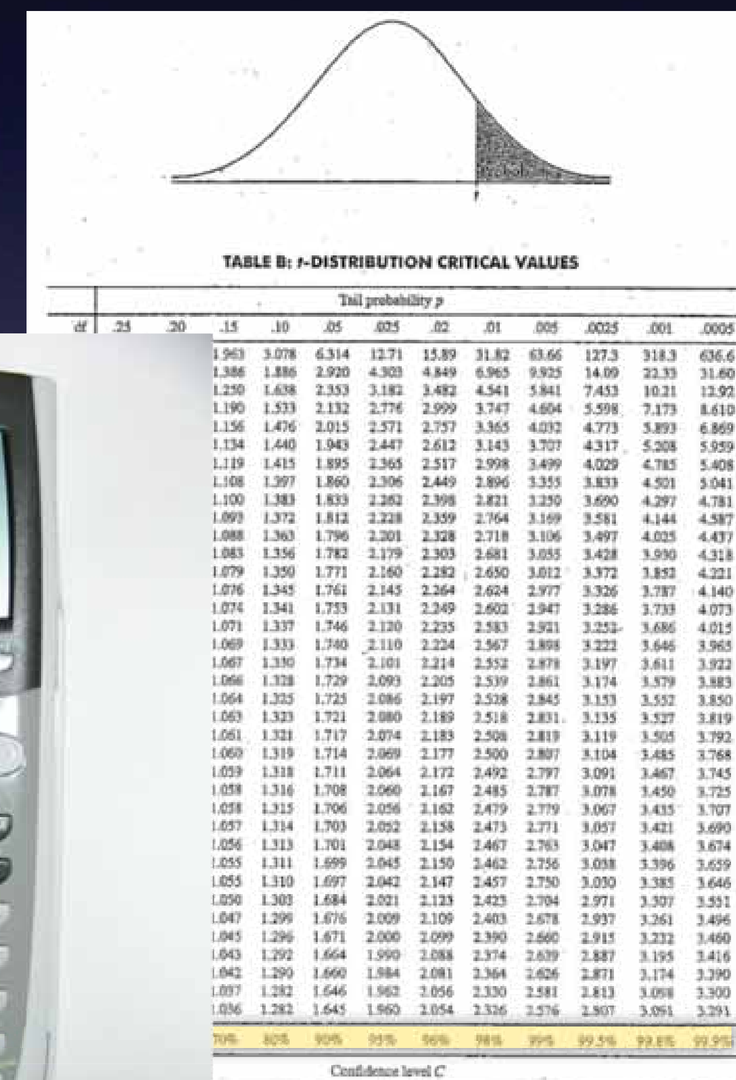
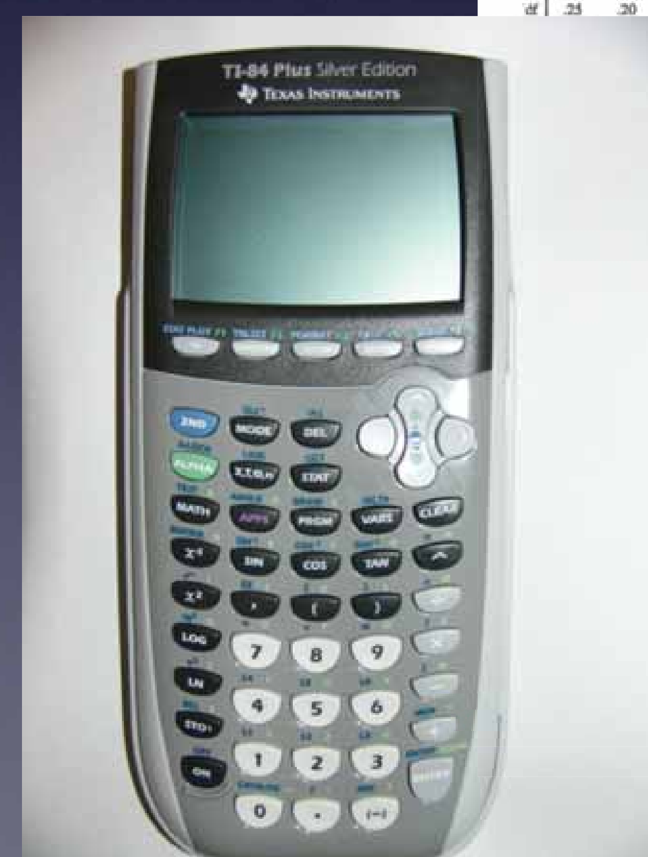


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

| Tail probability *p* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |

Confidence level *C*



# Confidence Intervals

## General CI Formula

$$\text{Statistic} \pm \left(\text{Critical Value}\right)\left(\text{Standard Deviation}\right)$$

## 2 Sample Mean *t* CI Formula

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$df = \#$ ← get from calculator

Use Table
to get the *t* critical value

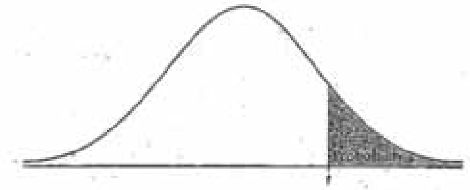


**TABLE B: *t*-DISTRIBUTION CRITICAL VALUES**

| | Tail probability *p* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *df* | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | .679 | .849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | .679 | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | .678 | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | .677 | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675 | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| ∞ | .674 | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |
| | Confidence level *C* | | | | | | | | | | | |

**Interpretation for Independent Samples Confidence Intervals for Means**

We are __% confident that $\mu_1 - \mu_2$, the true mean difference of _____, is between ___ and ___.

**Interpretation for the Confidence <u>Level</u> of Independent Samples Confidence Interval for Means**

We used a method to construct this estimate that in the long run will successfully capture the true value of $\mu_1 - \mu_2$ ___% of the time.

**<u>ALWAYS</u>** check your assumptions and interpret your interval, even you are not specifically asked to in the problem.  Just do it.  Seriously.

General Work Flow -

1. Assumptions
2. Construction of Interval
3. Interpretation(s)

1. Suppose we compare the data on two samples, *A* and *B*, and come up with the following data: $n_A = n_B = 10, \bar{x}_A = 25, s_A = 3.21, \bar{x}_B = 22.2, s_B = 3.09$. Find a 99% confidence interval for the difference between the means. Assume the samples are independent.

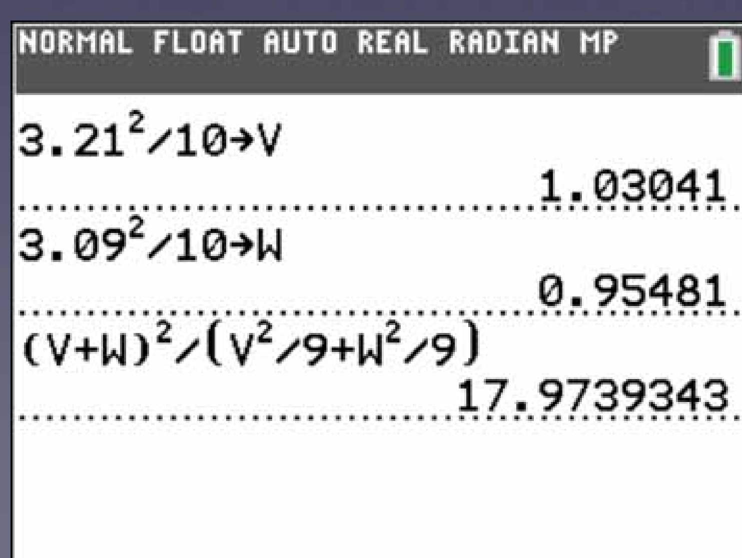
(a) (-1.26, 6.86) (b) (-1.78, 7.38) (c) (-1.67, 7.27) (d) (-0.83, 6.43) (e) None of the above

$$(\bar{x}_1 - \bar{x}_2) \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$df = \#$ $\leftarrow$ get from calculator

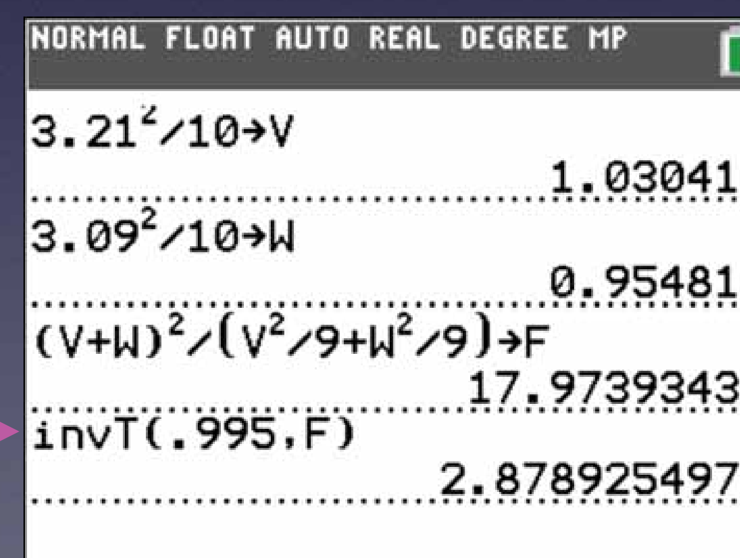
$$(25 - 22.2) \pm 2.898\sqrt{\frac{3.21^2}{10} + \frac{3.09^2}{10}}$$

$df = 17?$




$$(25 - 22.2) \pm 2.878\sqrt{\frac{3.21^2}{10} + \frac{3.09^2}{10}}$$

While we truncated the stated *df* to 17, for the purposes of calculating your *t*-value, you use the raw result from the calculator.




Normality assumption was unable to be verified. Sometimes the CB is inconsistent with this. 😠