Paired Hypothesis Tests for Means

$$H_0 : \mu_d \ = \ \#$$

Note #1: Use colons

$$\neq$$

$$H_a : \mu_d \ < \ \#$$

$$>$$

Note #2: Use only PARAMETERS in your hypothesis...although there will be some problems where we'll use words/sentences

Note #4: The symbol used in the alternate will come from the context of the problem

$\neq$ - two-sided test, equivalent to a Confidence Interval (CI)

$<$
 } - one-sided test
$>$

Steps in Hypothesis Testing

1. Define the population characteristic (i.e. parameter) about which hypotheses are to be tested.

2. State the null hypothesis H_a.

3. State the alternative hypothesis H_0.

4. State the significance level for the test α.

5. Check all assumptions and state name of test.

6. State the name of the test.

7. State df if applicable (not applicable in proportion land).

8. Display the test statistic to be used without any computation at this point.

9. Compute the value of the test statistic, showing specific numbers used.

10. Calculate the P – value.

11. Sketch a picture of the situation.

12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0.
    2. Summarize in context discussing H_a.

Paired Hypothesis Tests for Mean

Steps in Paired Mean Hypothesis Testing

1. $\mu_d =$.......

2. $H_0 : \mu_d = \#$

   $\neq$

3. $H_a : \mu_d \quad < \quad \#$

   $>$

4. State the significance level for the test α.

Common Pairings:
1. Before and After
2. Twins
3. Two Treatments applied to different halves of the same object

5. Assumptions:

   1. Random **Paired** Sample
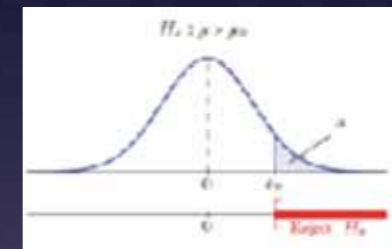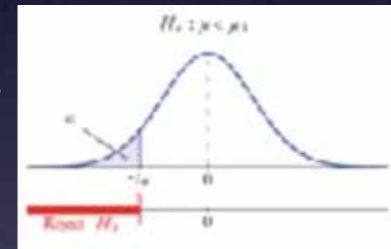
   2. Normality stated

      $n_d \geq 30$

      Boxplot of raw **difference** data shows roughly symmetric shape

   3. s_d known

6. Paired Mean t Test

7. $df = n - 1$

8/9. $t = \dfrac{\bar{x}_d - \mu_d}{s_d / \sqrt{n}} = \#$
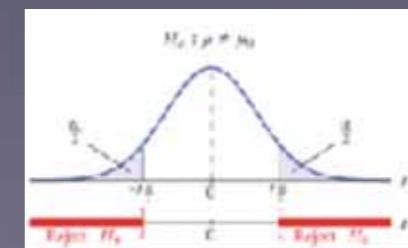
11.

10. $P - value =$

$$P(t > \#) = tcdf(\#, 1E99, df)$$
$$P(t < \#) = tcdf(-1E99, \#, df)$$

one-sided tests

$$2P(t > \#) = 2*tcdf(\#, 1E99, df)$$
$$2P(t < \#) = 2*tcdf(-1E99, \#, df)$$

two-sided tests

12. State the conclusion in two sentences -
   1. Summarize in theory discussing H_0.
   2. Summarize in context discussing H_a.

Confidence Intervals

General CI Formula

$$\text{Statistic} \;\pm\; (\text{Critical Value})(\text{Standard Deviation})$$

Paired t CI Formula

$$\overline{x}_d \pm t\,\frac{s_d}{\sqrt{n}}, \quad df = n-1$$

Use Table or Calculator
to get the t critical value

Interpretation for Paired Confidence Intervals for Means
We are __% confident that , the true mean difference of _____,
is between ___ and ___.

**Interpretation for the Confidence <u>Level</u> of a Paired
Confidence Intervals for Means**
We used a method to construct this estimate that in the long run
will successfully capture the true value of μ_d ___% of the time.

Chigs wonders if her naps after eating candy are longer or shorter depending on what she eats. Sometimes she eats Skittles (gross!) while other times she enjoys M & M's (much better choice). Mr. Murphy is convinced that M & M sugar crashes are much less severe. Chigs decides to do a study to see if there is a difference in the sugar crash caused by each candy. She randomly selects 36 SI students from a survey of healthy students who eat both candies. She has them eat Skittles one day and M & M's a few days later, then times their naps. Her results are in two TI-84 lists. Use a 2 sample t test.

L_1 represents the length of a nap of a student who ate M & M's. L_2 represents the length of a nap of the same student a few days later after eating Skittles's

The two lists are shown below (not all entries are visible but they are there)
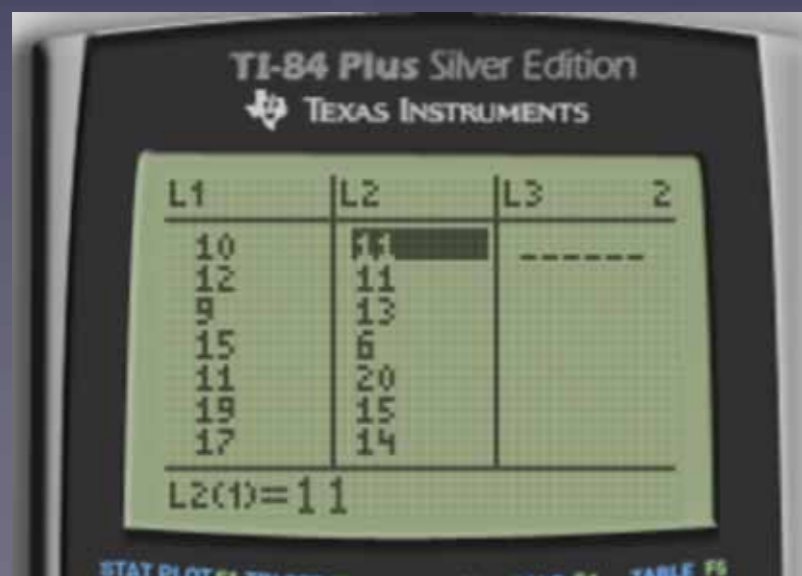
Since we are measuring the difference between the skittles nap and the m & m's nap, we need to find the mean and standard deviation of the two differences.
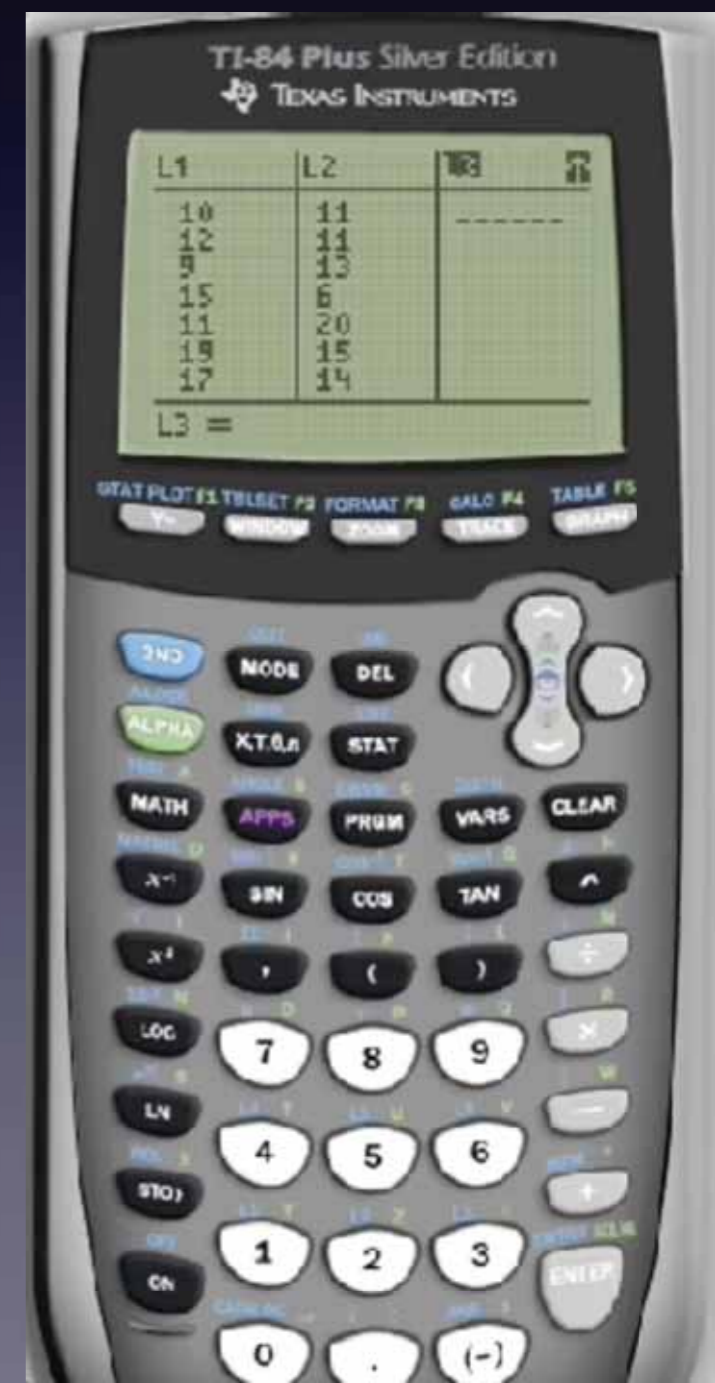
Paired Hypothesis Tests for Mean

Steps in Paired Mean Hypothesis Testing

1. $\mu_d = 0$

Null Hypothesis is that there is no difference in nap length and therefore sugar crashes (Chig's claim)

2. $H_0 : \mu_d = 0$

3. $H_a : \mu_m < \mu_s$

Alternate Hypothesis is that Skittles crashes are worse hence longer naps (Mr. Murphy's claim)

4. Significance level $\alpha = 0.05$

8/9. $t = \dfrac{-1-0}{6.74 \big/ \sqrt{36}} = -0.8902$

5. Assumptions:

1. Random **Paired** Sample

2. Normality stated

$n_d \geq 30$

Boxplot of raw **difference** data shows roughly symmetric shape (see prev page for calculator display)

3. s_d known

6. Paired Mean t Test

7. $df = 35$

11.

$H_a : \mu < \mu_0$

α

$-t_\alpha$ 0 t

Reject H_0 0 t

mean(L₃)→M
                -1
stdDev(L₃)→S
        6.740072064

M/(6.74/6)→T
        -.8902077151
tcdf(-1ε99,T,35)
        .1897164271
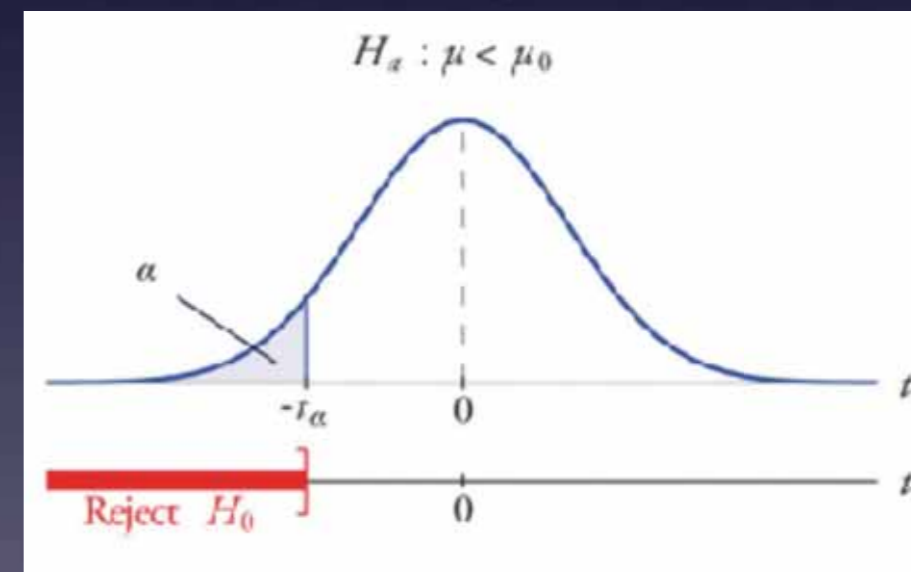
10. $P-value = P(t < -0.8902)$

$= tcdf(-1E99, -0.8902, 35)$

$= 0.1897 > 0.05$

12. State the conclusion in two sentences -
    1. Summarize in theory discussing H_0.
    2. Summarize in context discussing H_a.

But first…
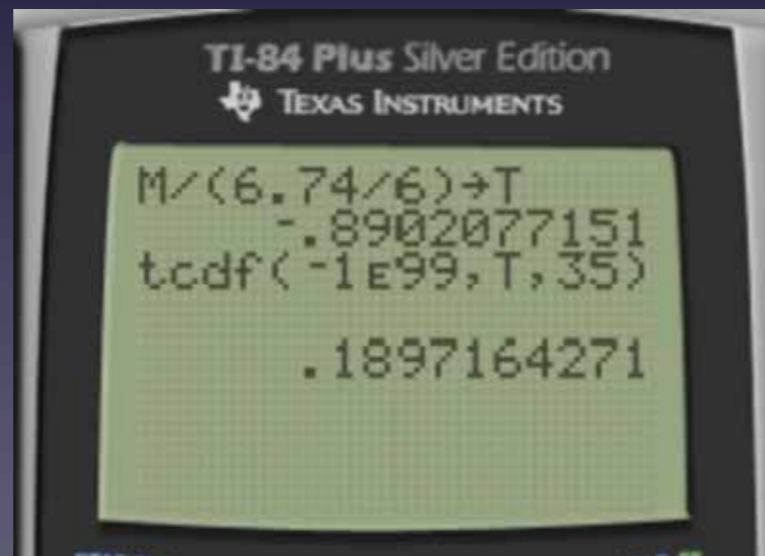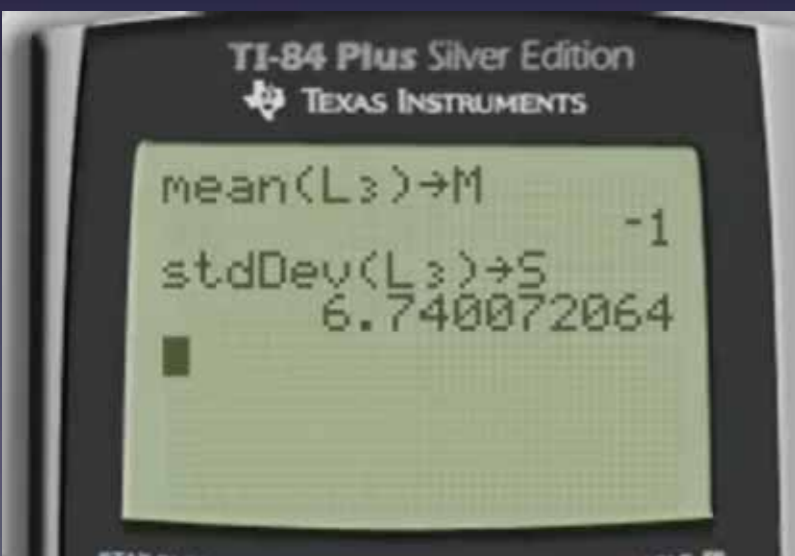
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A faster way to do it on the calculator… remember that you still need to show work on the AP Exam

We can find all the numbers we need here:

We can also graph the Type I error probability (shaded area) to see why we should not reject the null hypothesis
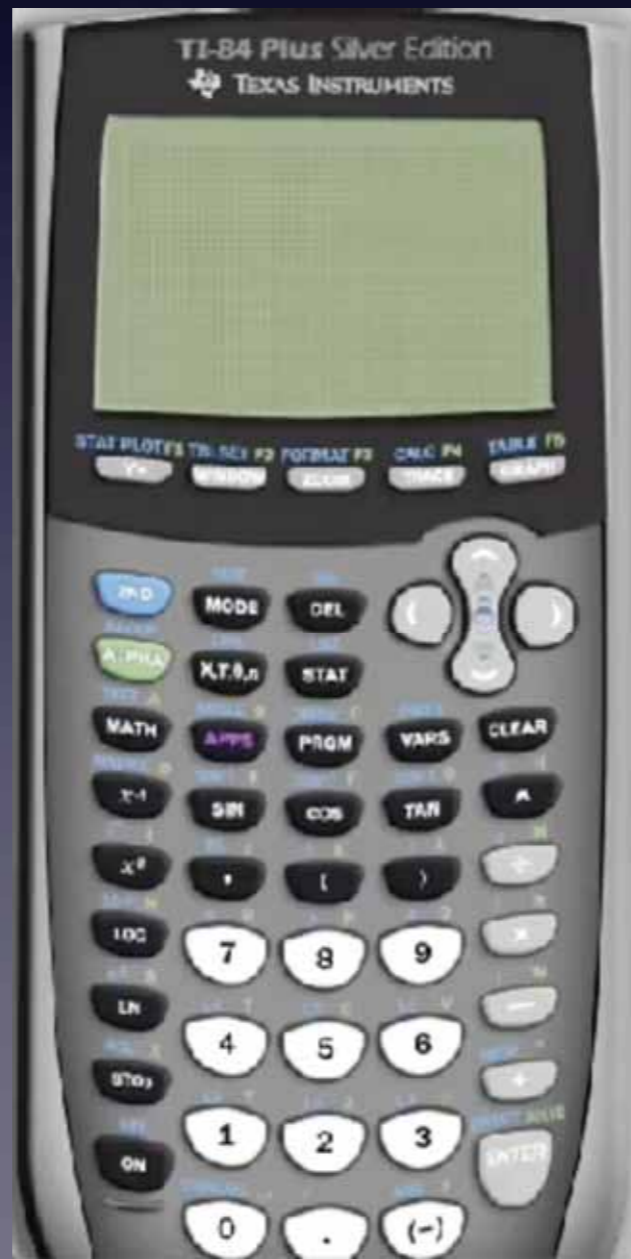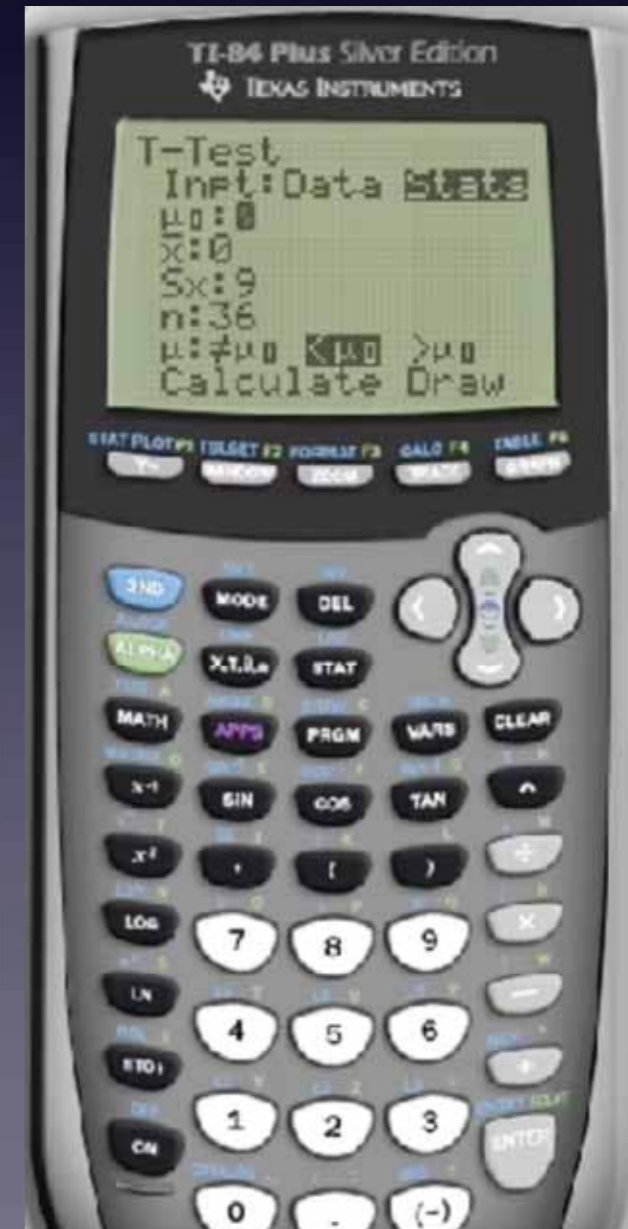
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Because our p-value = 0.18 > 0.05 we fail to reject the null hypothesis at the 0.05 level of significance

We do not have evidence that M & M sugar crashes lead to shorter naps than Skittles sugar crashes.

Chigs appears to be correct and Mr. Murphy just needs to get over it and accept that some people just like Skittles

ALWAYS check your assumptions and interpret your interval, even you are not specifically asked to in the problem. Just do it. Seriously.

General Work Flow -
1. Assumptions
2. Construction of Interval
3. Interpretation(s)

Check for screencast examples involving CI's