Probability Rules:

- The sum of the probabilities for all possible outcomes in a sample space is [redacted]
- The probability of an outcome is a number between 0 and 1 inclusive. An outcome that always happens has probability [redacted] An outcome that never happens has probability [redacted]
- The probability of an outcome occurring equals [redacted] probability that it doesn't occur.
- The probability that two mutually exclusive (disjoint) events occur is [redacted]

Variables

Numerical

Categorical

Numeric
- Discrete
- Continuous

Nonnumeric

We will now focus on ***Linear Transformations*** of these Random Variables

Strategies for Solving Probability Problems:

Draw a picture of the situation -

- Table/Charts

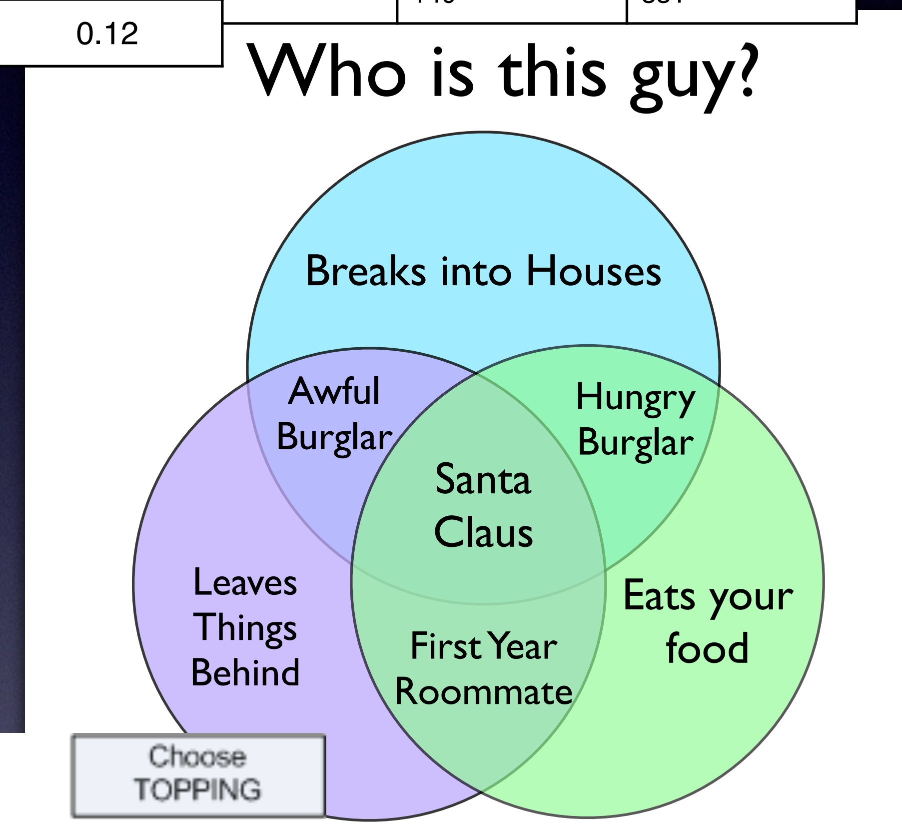
		Breakfast		
		Yes	No	Total
Sex	Male	66	66	132
	Female	125	74	199
			140	331

White	Black	Red	Silver	Gold
0.46	0.22	0.09	0.11	0.12

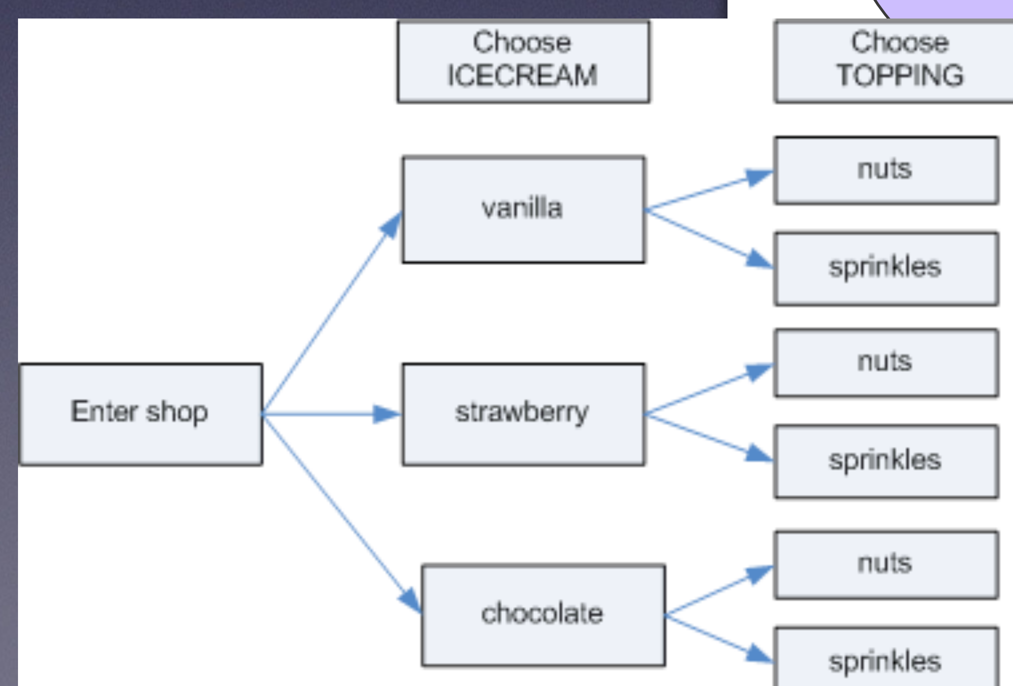
Some problems will require **YOU** to create the probability distribution for X, i.e. the chart.

- Formulas

- Venn Diagram



- Tree Diagram



Formulas

Is there a formula on the AP formula sheet that applies?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Intersection in both equations - sometimes you will need to use both equations to solve one problem

Expected Value

$$E(X) = \mu_X = \sum x_i p_i$$

Variance

$$Var(X) = \sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$$

Remember that you can crunch these stats on your calculator, using your lists.

Remember also that Standard Deviation = $\sqrt{\text{Variance}}$

$$\sigma_X = \sqrt{\sigma_X{}^2}$$

Note: These formulas are not on the AP formula sheet

For such linear transformations on X, i.e. $Y = aX + b$,

$$\mu_Y = a\mu_X + b$$

$$\sigma_Y = a\sigma_X$$

Notice that the constant (y-intercept) does not affect the standard deviation

To raise money for the cause of youth volleyball, Bryn and JJ decide to sell passes to their St Cecilia Volleyball Extravaganza. Bryn thinks that the tickets should be \$5 for entry and each food/snack ticket is \$0.50 each. JJ insists that given that the average sale of food tickets per person is 10 with a standard deviation of 2, they should sell both admission and food tickets for \$1.

Which fund raising model will yield a higher average dollar amount raised per person? What is the standard deviation for both fund raising models?

$$X = \text{\# of food tickets} \qquad\qquad Y = \text{\$ earned}$$

Recall linear modeling from algebra

$$Y = a\mathrm{X} + b$$

$a = ?$ Price per food ticket $\qquad$ $b = ?$ The price of admission

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Which fund raising model will yield a higher average dollar amount raised per person? What is the standard deviation for both fund raising models?

$$\mu_X = 10 \qquad \sigma_X = 2$$

$$\mu_{Bryn_Y} = 0.5\mu_X + 5 \quad = 0.5(10) + 5 = \$10 \qquad \mu_{JJ_Y} = \mu_X + 1 \quad = 10 + 1 = \$11$$

$$\sigma_{Bryn_Y} = 0.5\sigma_X \quad = 0.5(2) = \$1 \qquad \sigma_{JJ_Y} = \sigma_X = \$2$$

Answer: JJ

Furthermore, one SD to the left for both models gives you \$9 but one SD right with JJ's model gives you \$13

For such linear transformations on X, i.e. $Y = aX + b$,

Means are more intuitive.

$$\mu_Y = a\mu_X + b$$

$$\sigma_Y = a\sigma_X$$

For any 2 random variables X and Y, $\mu_{X \pm Y} = \mu_X \pm \mu_Y$

Examples of 2 random variable means and variances to come

For any 2 random **independent** variables X and Y,

$$\sigma^2_{X \pm Y} = \sigma^2_X + \sigma^2_Y$$

$$\sigma_{aX \pm bY} = \sqrt{(a\mu_X)^2 + (b\mu_Y)^2}$$

ALWAYS, and I mean ALWAYS, add variances.

Do not add the standard deviations. Square root the variance after you've added the two.

Think Pythagorean theorem or the Distance Formula

Remember: These formulas are not on the AP formula sheet

Mr Maychrowitz can play endless Tetris for an average of 44 minutes with a standard deviation of 3 minutes while Mr. Murphy can manage 27 with a standard deviation of 4 minutes. Find the mean and standard deviation of the difference between Mr. Maychrowitz and Mr Murphy's endless Tetris times. Since they never play in the same room nor compare their scores, we can say that their scores are independent.

The following random variables represent each player's scores

X = Mr Maychrowitz' score

Y = Mr Murphy's score

$\mu_X = 44$ $\sigma_X = 3$

$\mu_Y = 27$ $\sigma_Y = 4$

What do these two numbers represent?

The mean difference between their scores μ_{X-Y}

σ_{X-Y} The standard deviation of the difference between their scores

$$\mu_{X-Y} = 44 - 27 = 17 \text{ minutes}$$

$$\sigma_{X-Y} = \sqrt{3^2 + 4^2} = 5 \text{ minutes}$$

Now suppose that in order to give Mr. Murphy a sporting chance at matching his average, Mr Maychrowitz is willing to multiply all of Mr. Murphy's results by 1.5 and even add a 2 minute head start to the result (because Mr. Murphy whined that 1.5 wasn't enough) when comparing their outcomes. Find the mean and variance of their transformed results.

X = Mr Maychrowitz' score

$1.5Y + 2$ = Mr Murphy's adjusted score

$$\mu_X = 44$$

$$\sigma_X = 3$$

$$\mu_{1.5Y+2} = 1.5 * 27 + 2 = 42.5$$

$$\sigma_{1.5Y} = 1.5 * 4 = 6$$

$$\mu_{X-(1.5Y+2)} = 44 - 42.5 = 1.5 \text{ minutes}$$

$$\sigma_{X-1.5Y} = \sqrt{3^2 + (1.5 * 4)^2} = \sqrt{3^2 + 6^2} = \sqrt{45} \approx 6.708$$

For such linear transformations on X, i.e. $Y = aX + b$,

$$\mu_Y = a\mu_X + b$$

Means are more intuitive.

$$\sigma_Y = a\sigma_X$$

For any 2 random variables X and Y, $\mu_{X\pm Y} = \mu_X \pm \mu_Y$

For any 2 random **independent** variables X and Y,

$$\sigma^2_{X\pm Y} = \sigma^2_X + \sigma^2_Y$$

$$\sigma_{aX\pm bY} = \sqrt{(a\mu_X)^2 + (b\mu_Y)^2}$$

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