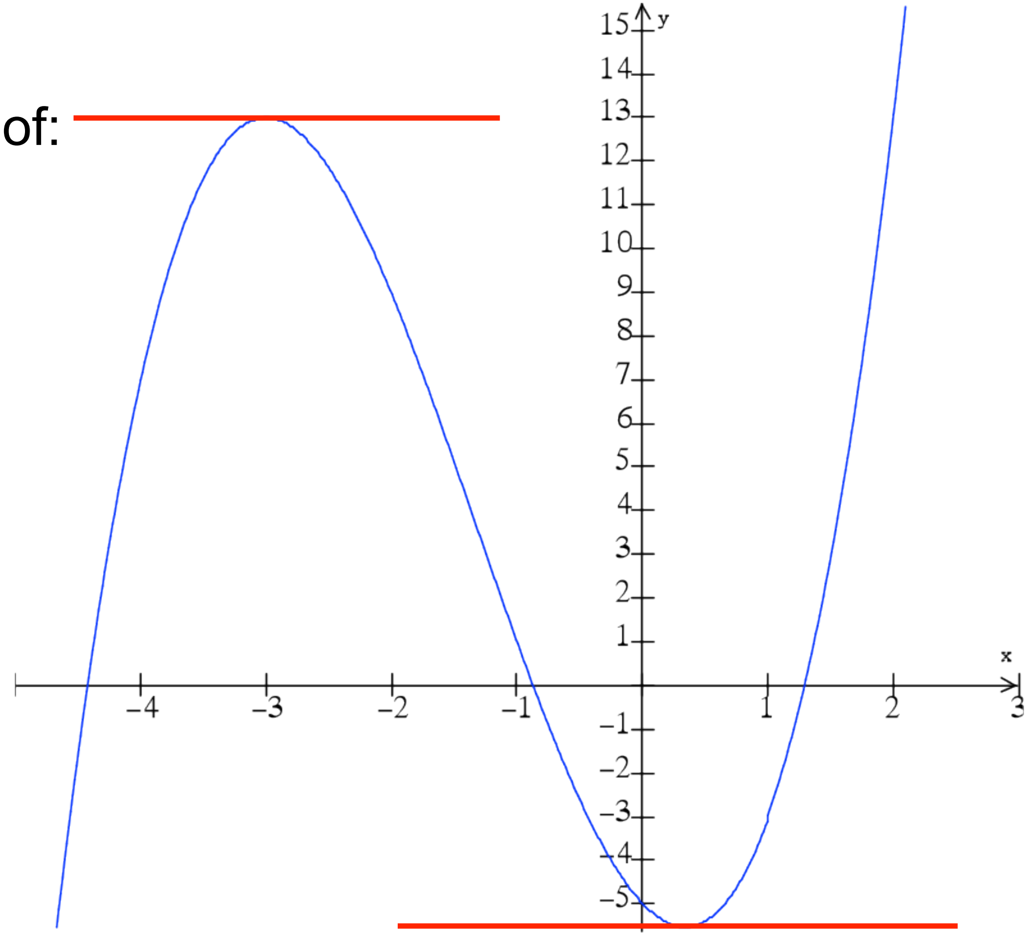
3-1
Extreme Values of Functions

Find any maximum and/or minimum points for the graph of:

$$y = x^3 + 4x^2 - 3x - 5$$

Notice that in two **critical** places, the tangent has a slope of zero.



In order to locate these points precisely, we need to find the values of x for which

$$y' = 3x^2 + 8x - 3 = 0$$

$$y' = (3x - 1)(x + 3) = 0$$

$$x = -3, \frac{1}{3}$$

Find any maximum and/or minimum points for the graph of:

$$y = x^3 + 4x^2 - 3x - 5$$

Without seeing the graph, how could we tell which of these two points is the maximum and which is the minimum?

In order to locate these points precisely, we need to find the values of x for which

$$y' = 3x^2 + 8x - 3 = 0$$

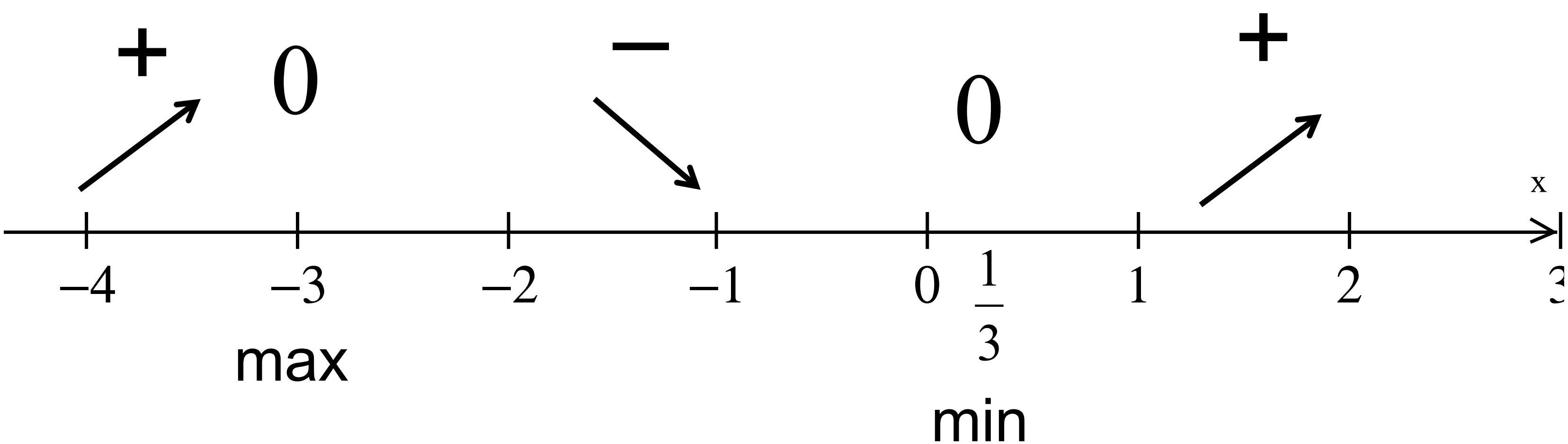
$$y' = (3x - 1)(x + 3) = 0$$

$$x = -3, \frac{1}{3}$$

Find any maximum and/or minimum points for the graph of:

$$x = -3, \frac{1}{3}$$

$$y = x^3 + 4x^2 - 3x - 5$$



So let's test the intervals:

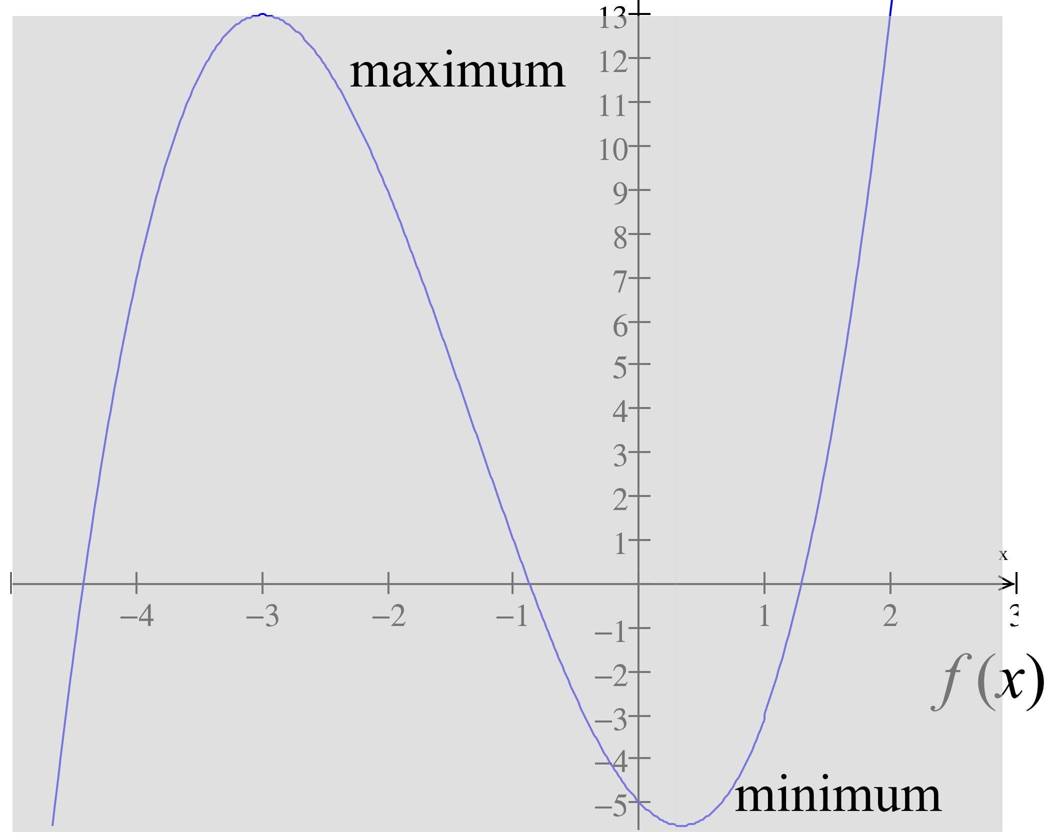
$$y' = 3x^2 + 8x - 3 = 0$$

$$y' = (3x - 1)(x + 3) = 0$$

Important note: The above number line without an explanation will *not* be considered sufficient justification on the AP Exam

Let's look again.

$$f(x) = x^3 + 4x^2 - 3x - 5$$

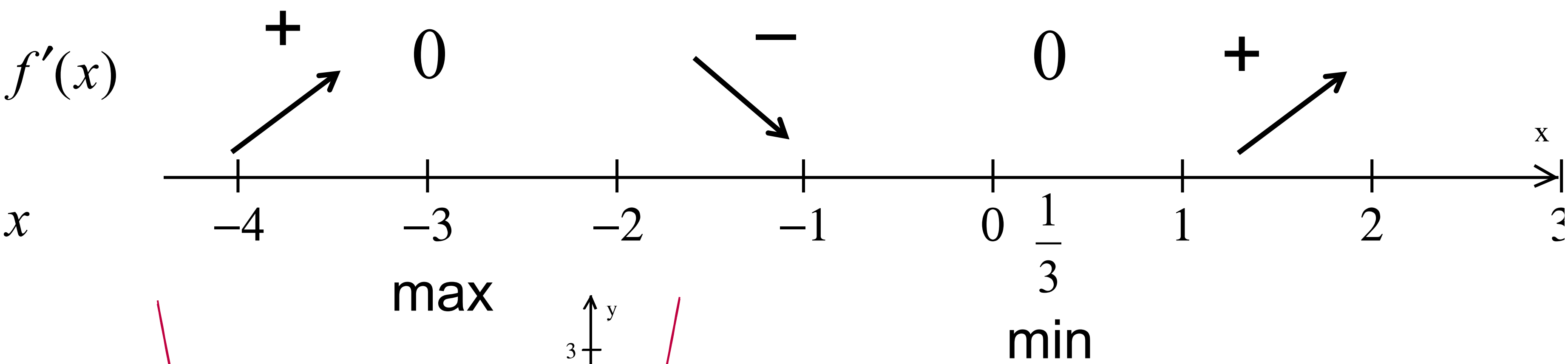


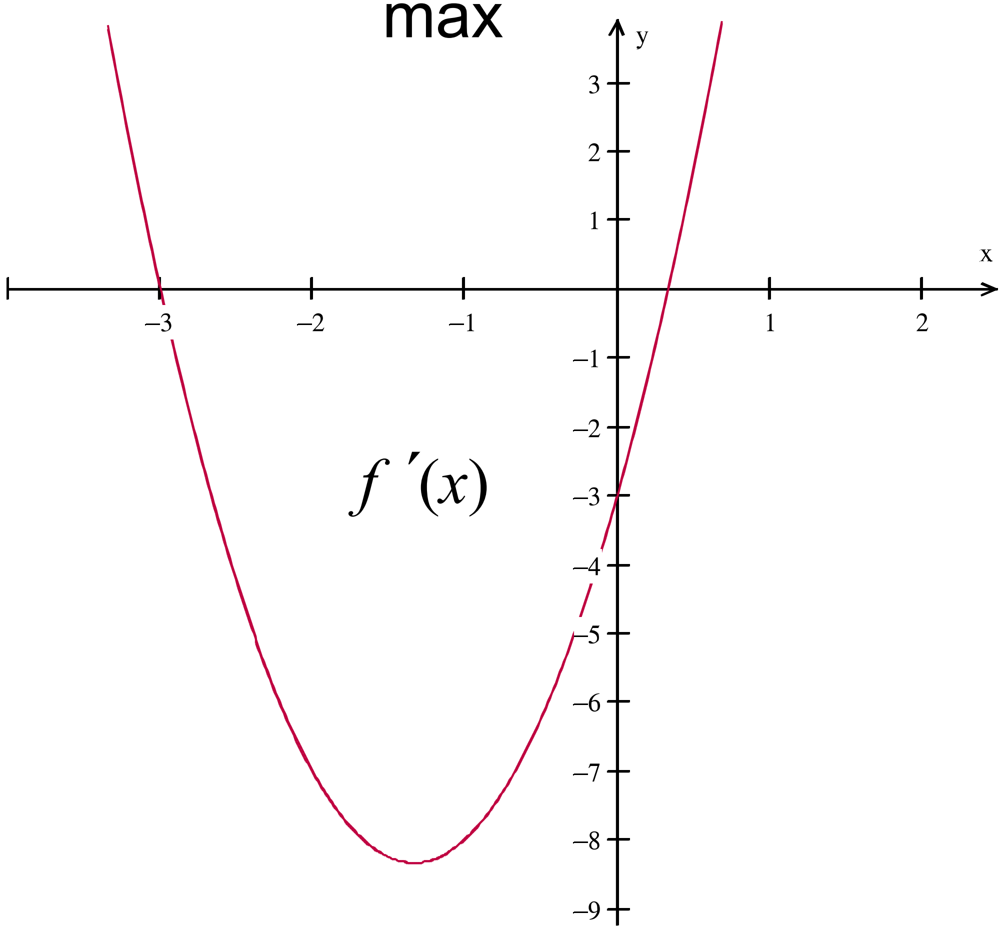
$f'(x) > 0$ $f'(x) < 0$ $f'(x) > 0$

$f(x)$ is increasing $f(x)$ is decreasing $f(x)$ is increasing

Since we also know that $f'(x) = 3x^2 + 8x - 3$ we can sketch a graph of it based on the info we have

$$f'(x) = 3x^2 + 8x - 3$$

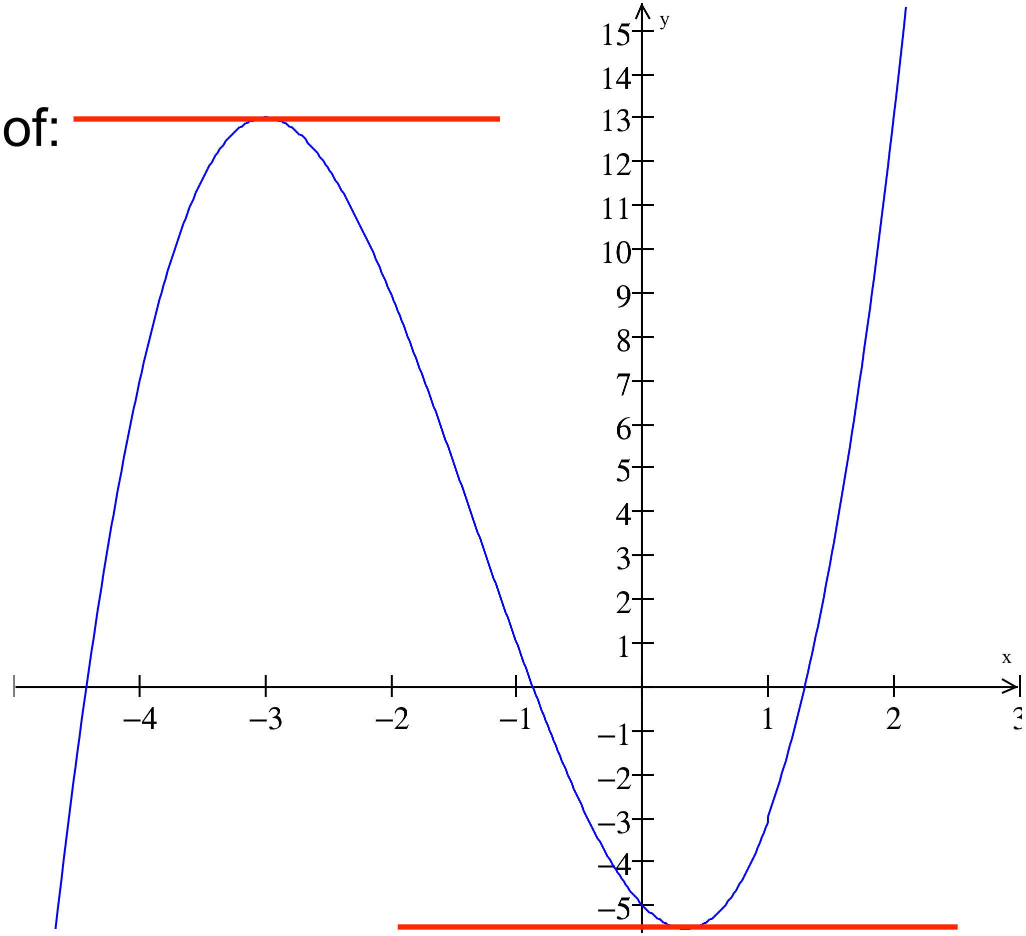




Note that we now also have a graph of $f'(x)$ This is <u>important!</u>

Find any maximum and/or minimum points for the graph of:

$$y = x^3 + 4x^2 - 3x - 5$$



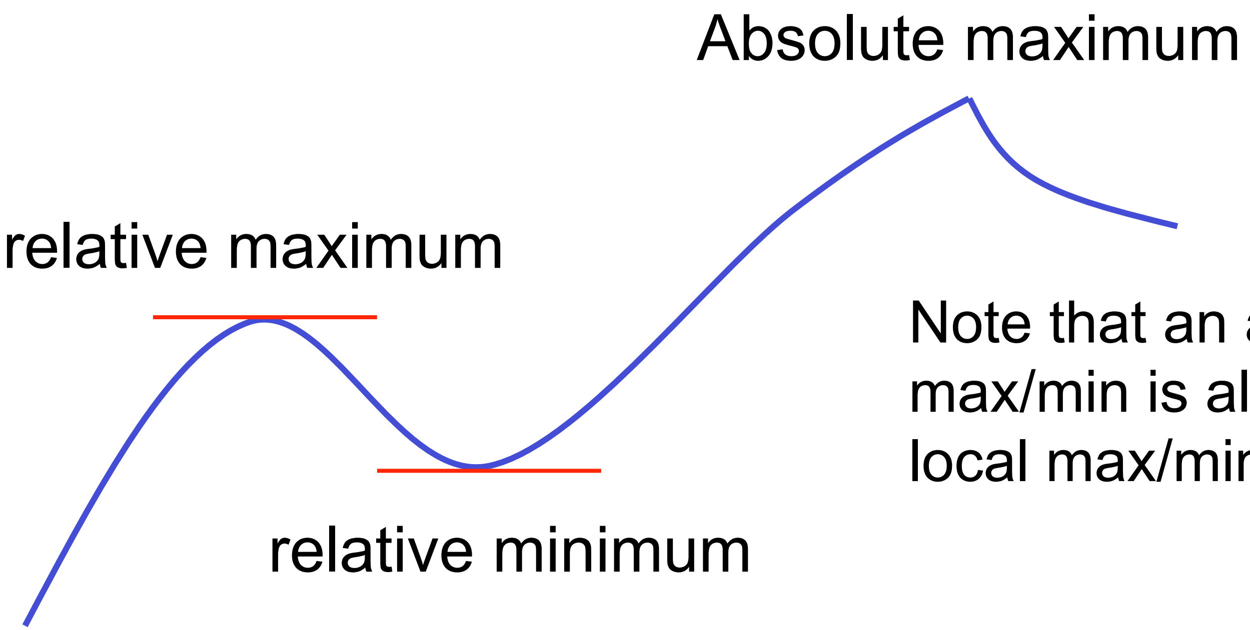
Now we have a way of finding maxima and minima. But we still need to better classify these points…

$$y' = 3x^2 + 8x - 3 = 0$$

$$y' = (3x-1)(x+3) = 0$$

$$x = -3, \frac{1}{3}$$

Terms to remember for you note-takers:



Note that an absolute max/min is already a local max/min

Notice that local extremes in the interior of the function occur where f' is zero or f' is undefined.

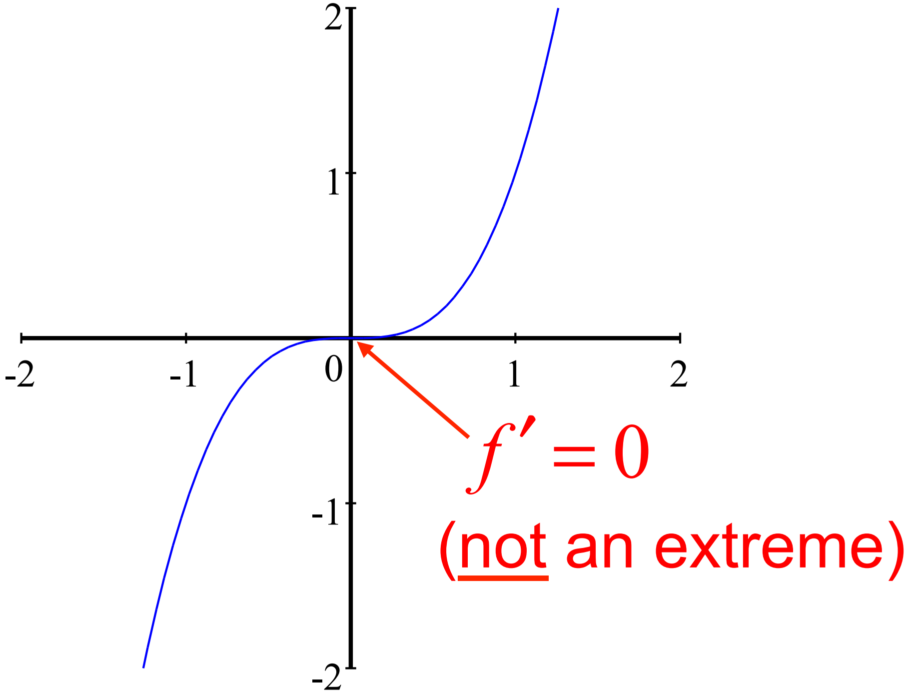
Vocabulary:

1. ***Extreme Points*** – the collective word for maximum and minimum points
2. ***Critical Value*** – the *x*-coordinate of an extreme point
3. ***Maximum Value*** – the *y*-coordinate of a high point
4. ***Minimum Value*** – the *y*-coordinate of a low point
5. ***Relative Extreme Values*** – the highest or lowest *y*-values in any section of the curve
6. ***Absolute Extreme Values*** – the highest or lowest *y*-values of the entire function

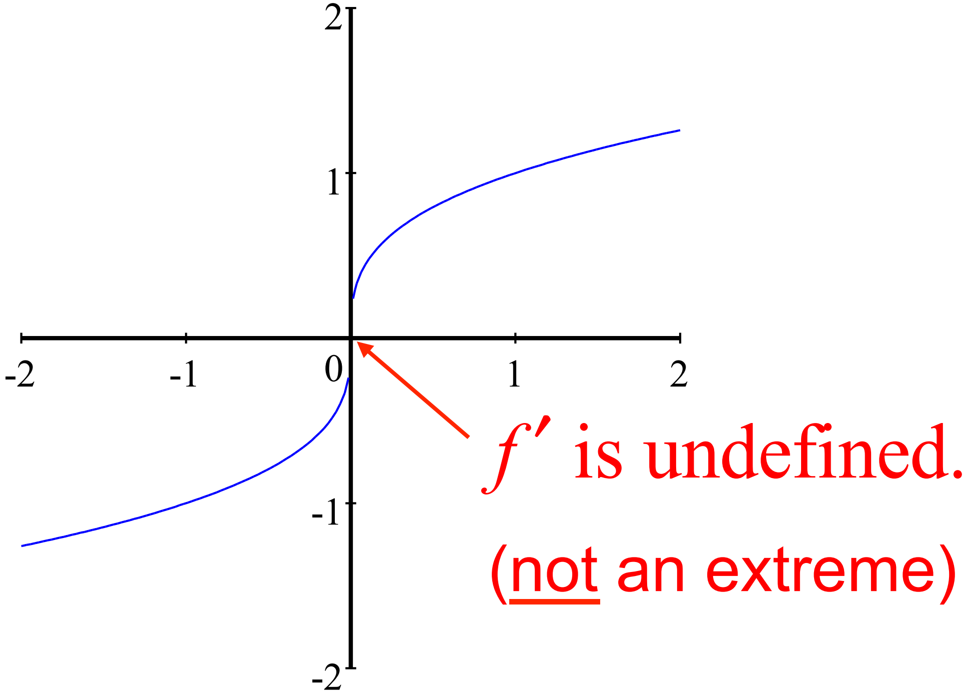
Note: The word "value" indicates an *x*-value OR a *y*-value, not both. An extreme POINT, on the other hand, indicates a coordinate point, and must be written as such.

Critical points are not always extremes!

$$y = x^3$$



$y = x^{1/3}$



If a value is a critical value, then either

i) $\frac{dy}{dx} = 0$ at that value;

ii) $\frac{dy}{dx}$ does not exist at that value;

or iii) a value at an endpoint of an arbitrarily stated domain.

FINDING ABSOLUTE EXTREMA

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

$$f(x) = x^{2/3}$$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$$

$$f'(x) = \frac{2}{3\left(\sqrt[3]{x}\right)}$$

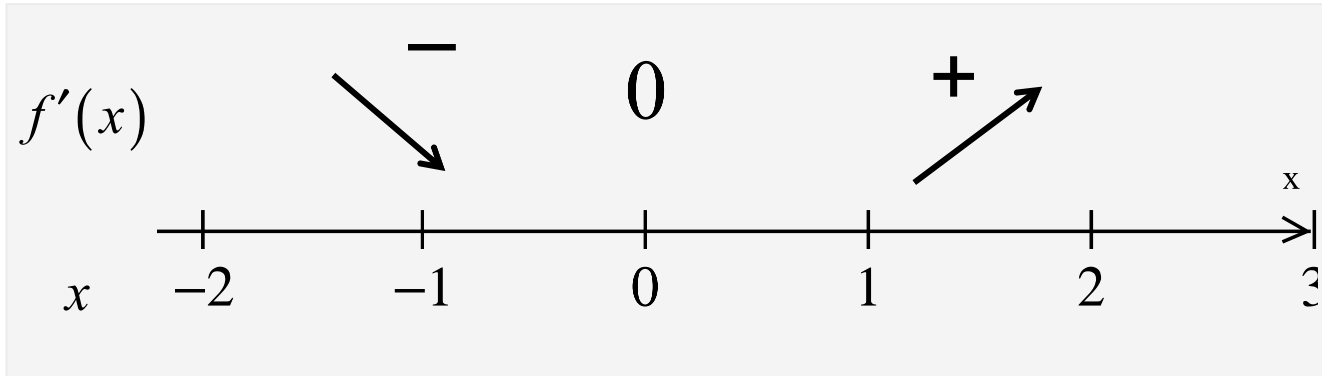
There are no values of x that will make the first derivative equal to zero.

The first derivative is undefined at $x = 0$, so $(0,0)$ is a critical point.

Because the function is defined over a closed interval, we also must check the endpoints.

$$f(x) = x^{2/3} \qquad D = [-2, 3]$$

$$f'(x) = \frac{2}{3\left(\sqrt[3]{x}\right)}$$



At: $x = 0$ $\quad f(0) = 0$

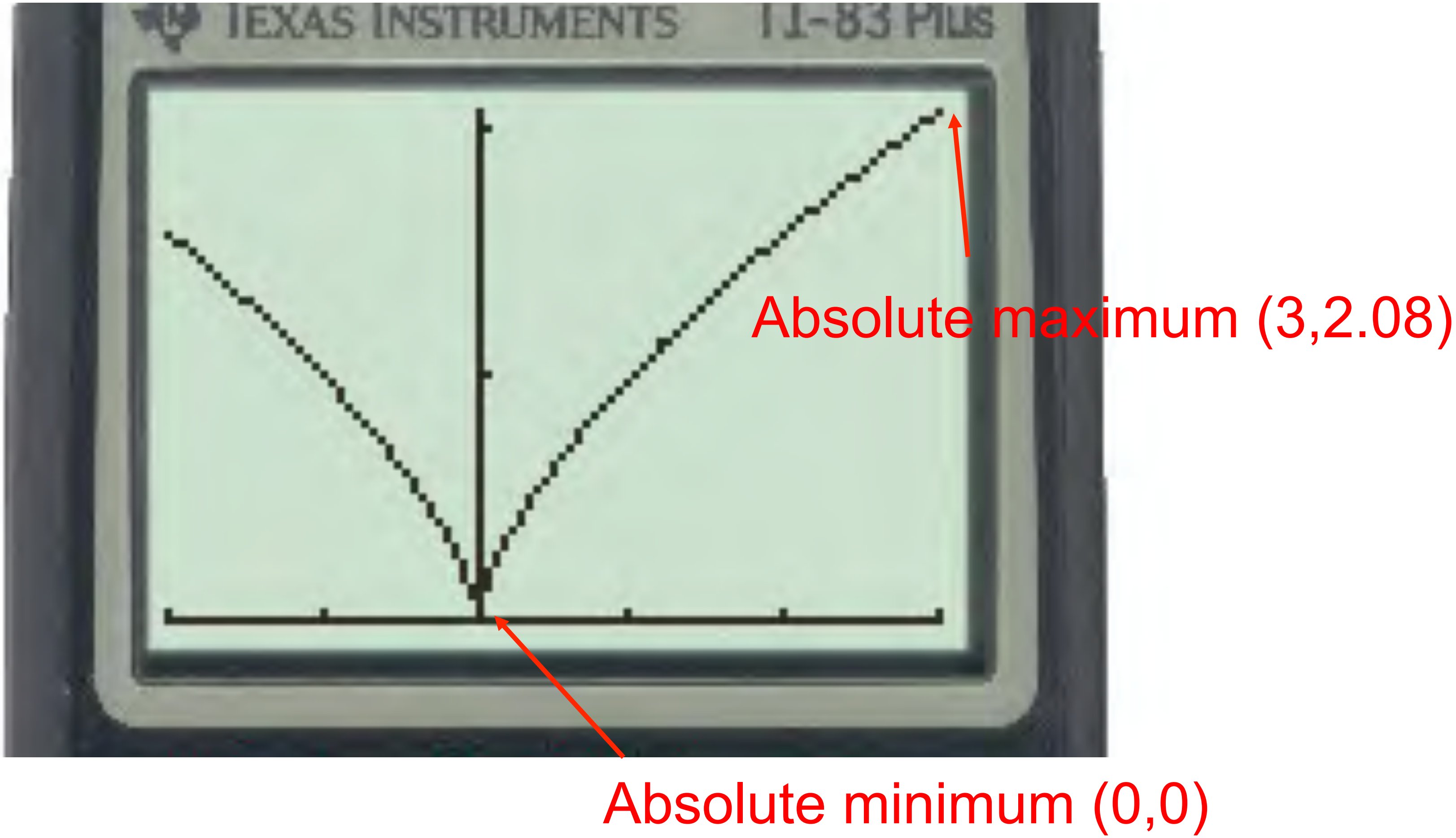
At: $x = -2$

$$f(-2) = (-2)^{\frac{2}{3}} \approx 1.5874$$

At: $x = 3$ $\quad f(3) = (3)^{\frac{2}{3}} \approx 2.08008$

Absolute minimum: $(0, 0)$

Absolute maximum: $(3, 2.08)$


TEXAS INSTRUMENTS
TI-83 Plus
Absolute maximum (3,2.08)
Absolute minimum (0,0)

$$f(x) = x^{2/3}$$

Finding Maximums and Minimums Analytically:

1. Find the derivative of the function, and determine where the derivative is zero or undefined. These are the critical points.

2. Find the value of the function at each critical point.

3. Find values or slopes for points between the critical points to determine if the critical points are maximums or minimums.

4. For closed intervals, check the end points as well.

Now you are ready to handle Assignment 3-1