2-4: Tangent Line Review & Linear Approximations

How to find the tangent line at $x = a$:

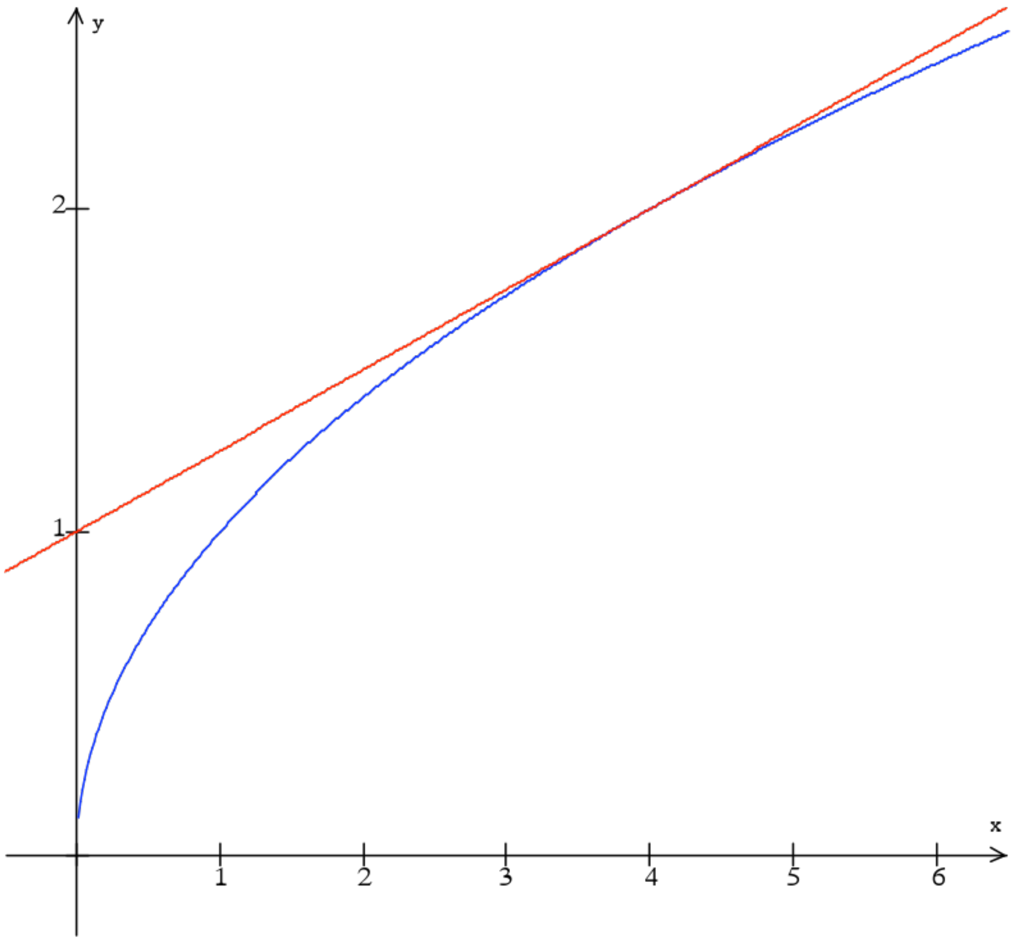
- Find $f(a)$ unless you already know it
(Plug a into the equation for y to find the point)

- Find $f'(a)$
(Plug a into y' to find the slope)

- Write the equation of the line

Find the equation of the line tangent to the function $y = \sqrt{x}$

at $x = 4$



Remember that the slope of the tangent line is just the derivative

$$y = \sqrt{x} = x^{1/2}$$

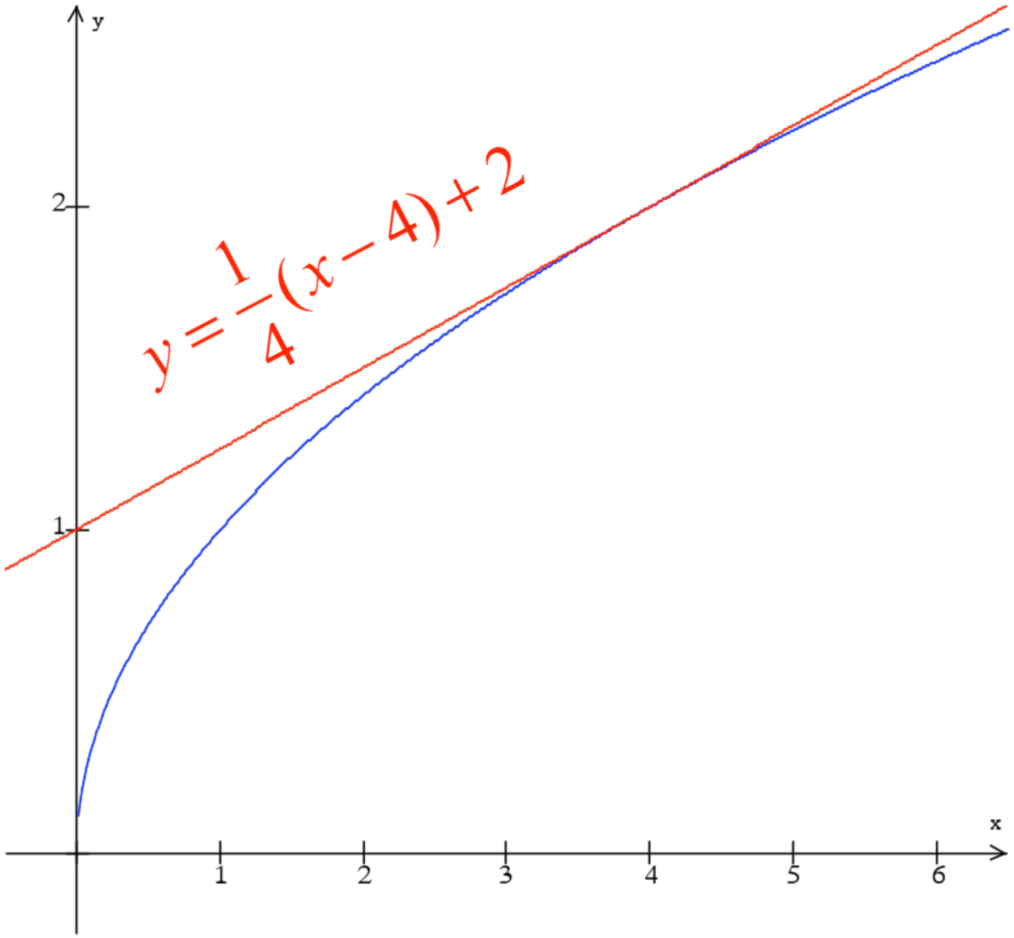
$$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

at $x = 4$

$$m_{\tan} = y' = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

Find the equation of the line tangent to the function $y = \sqrt{x}$

at $x = 4$



But we want the whole equation, not just the slope

$$m_{\tan} = y' = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

But at $x = 4$, $y = 2$ so now we have the point (4, 2)

Using point-slope, we get a final equation of

$$y = \frac{1}{4}(x-4)+2$$

Approximate the square root of 5 without your calculator (remember, they haven't always been around). But how?

Let's start with the function $y = \sqrt{x}$ at $x = 4$ because we know what $\sqrt{4}$ is

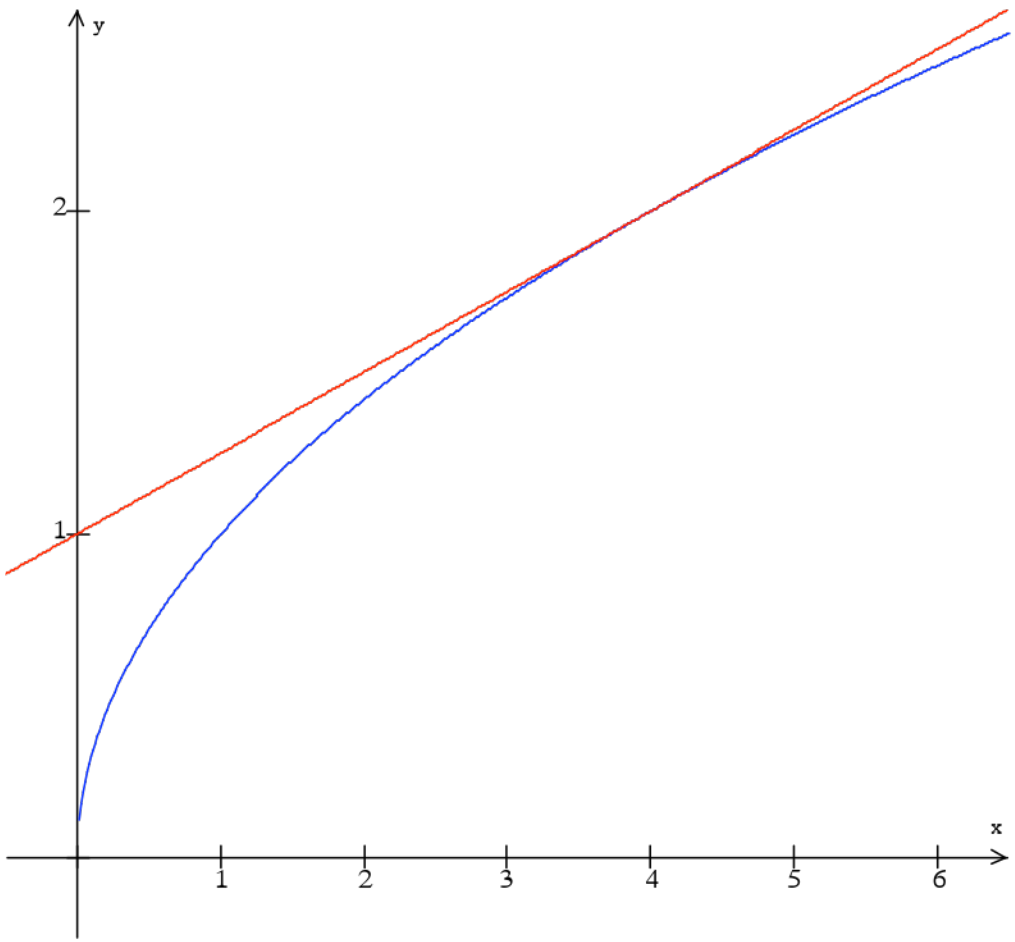
First of all, what is the tangent line at $x = 4$?

$$y = \frac{1}{4}(x-4)+2$$

Now graph both the function and its tangent line…

Use the tangent line to the function $y=\sqrt{x}$ at $x=4$ to approximate $\sqrt{5}$

$$y=\frac{1}{4}(x-4)+2$$



Notice that for numbers close to 4, the tangent line is very close to the curve itself…

So lets try plugging 5 into the tangent line equation and see what we get…

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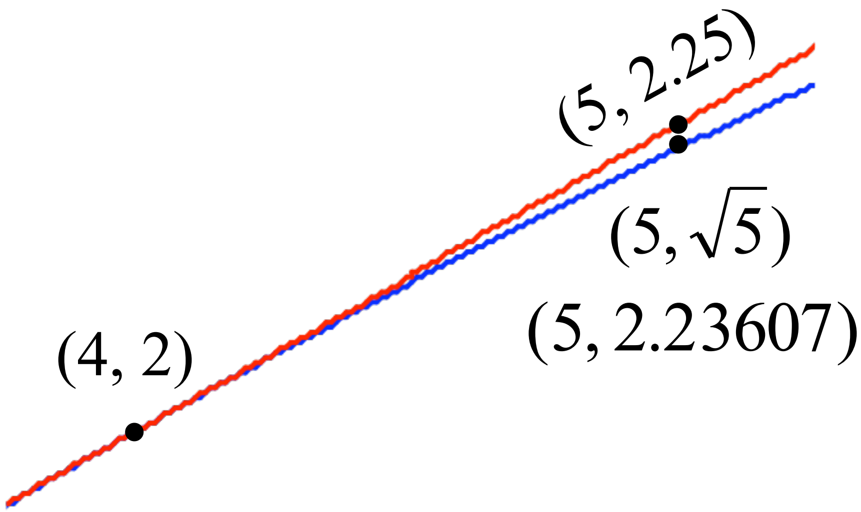
Use the tangent line to the function $y = \sqrt{x}$ at $x = 4$ to approximate $\sqrt{5}$

$$y = \frac{1}{4}(x-4)+2$$

Notice how close the point on the line is to the curve…



So let's plug 5 into the tangent line to get…

$$y = \frac{1}{4}(5-4)+2 = 2.25$$

As it turns out…

$$\sqrt{5} = 2.23607$$

So in this case, our approximation will be a very good one as long as we use a number close to 4.

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How to approximate values of f using the tangent line at $x = a$:

- Find the equation of the tangent line at $x = a$

- Plug the value of x for which you are trying to approximate f into the *tangent line* (not the original function)

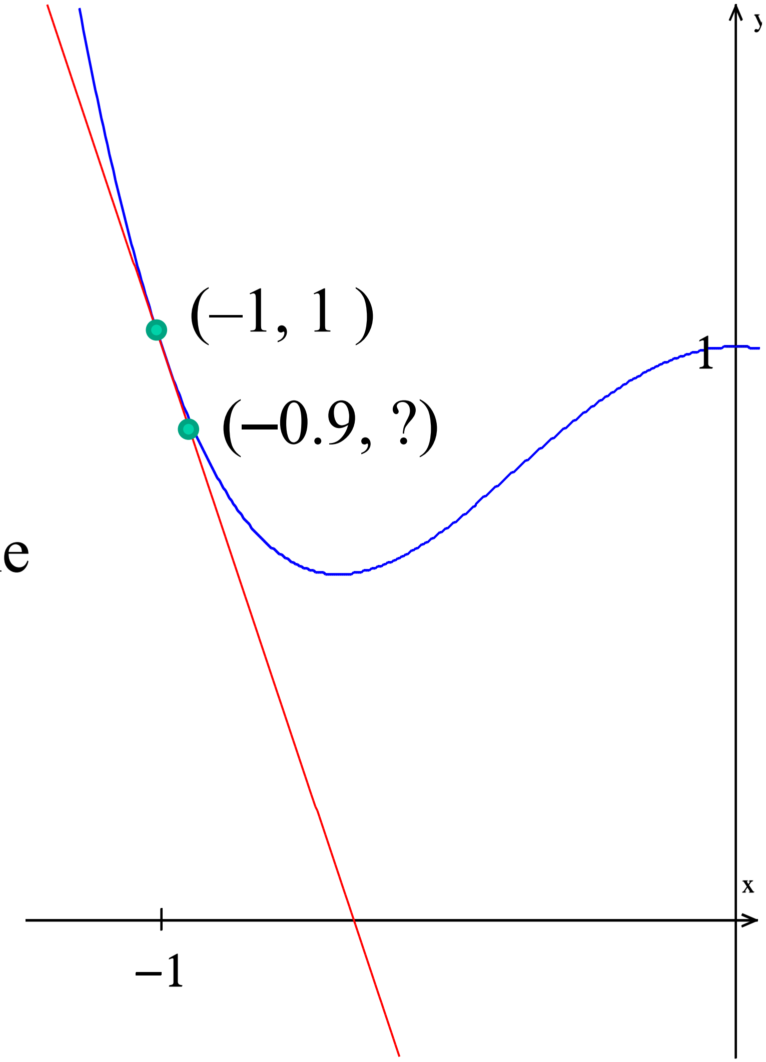
Example 1 on page 95

$$f(x) = x^4 - x^3 - 2x^2 + 1$$

Use the tangent line of f at $x = -1$ to approximate $f(-0.9)$

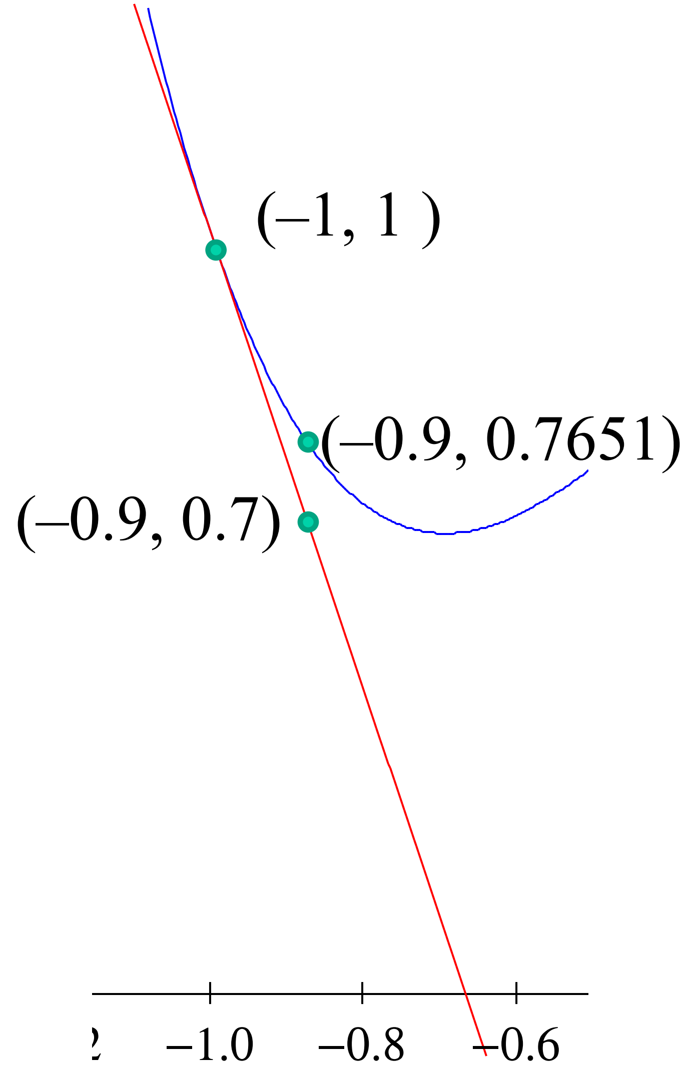
So we are going to find the tangent line

Then plug in -0.9 for x to the tangent line equation because that point on the line is so close to $f(-0.9)$



$$f'(x) = 4x^3 - 3x^2 - 4x$$

$$f'(-1) = -3$$



And since we have the point (–1, 1) , the equation of the tangent line is

$$y = -3(x+1)+1$$ or

$$y = -3x - 2$$

Now plug in –0.9 to approximate

$$y = -3(-0.9) - 2$$

$y = 0.7$ which is not far from the actual value:

$$f(-0.9) = 0.7651$$

So just to recap…

How to approximate values of f using the tangent line at $x = a$:

- Find the equation of the tangent line at $x = a$
- Plug the value of x for which you are trying to approximate f into the *tangent line* (not the original function)