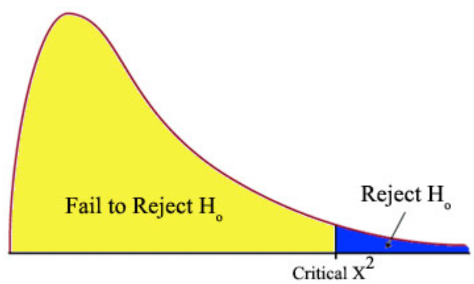
Unit 8: The Chi-Square Statistic

Introduction: In Units 6 and 7 we learned the fundamentals of both constructing a confidence interval and testing hypotheses. In both units we limited our work to single categories (Unit 6) or single variables (Unit 7). We are about to look at sampling more than one category and develop confidence intervals of our expected results from these samples. We will also be using hypothesis testing to check for relationships between two categories. To do these, we will be using a distribution called Chi-Square.

8-1 Chi Squared Test for Goodness of Fit

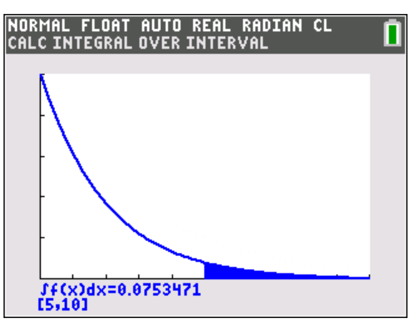
Goals: 1. Use a Chi Square Test for Goodness of Fit
2. Carry out a hypothesis test using the Chi Square distribution.

As we did in Unit 6, we will be conducting further testing with **categorical data.** In this unit we will be dealing with samples in which proportions are divided among multiple categories. For this we have what is called the Chi Squared χ^2 Distribution, a sample graph of which is shown below:

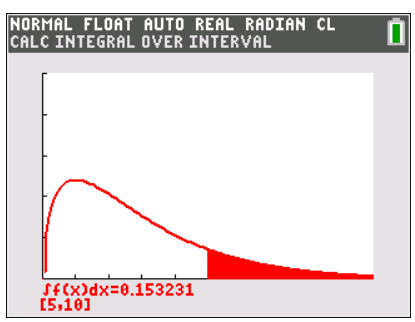


The actual shape of a Chi Square graph varies with what is called *degrees of freedom*. In Unit 7 you learned about this value being $n - 1$ in which n is the sample size. In the case of Chi Square, degrees of freedom will be determined differently which we will see shortly. Below are graphs of the Chi Square distribution for df = 2, 3, and 4

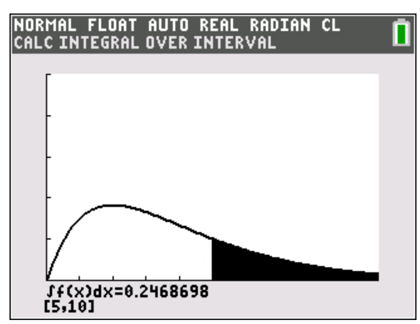
Degrees of freedom = 2



Degrees of freedom = 3



Degrees of freedom = 4



The above graphs use a sample Chi Squared value $\chi^2 = 5$. Notice how the tail area increases as the number of categories (and df) increases. So the more categories in your sample, the higher the value of χ^2 needs to be in order to reach a conventional significance level (level of rejection).

Example 1 For 1000 shoppers donating blood at a mall, the frequencies of blood types were as shown in the table below. Consider this an SRS of all mall shoppers.

In the general population, the blood type distribution is as follows:

Type O = 45%, Type A = 40%,
Type B = 10%, Type AB = 5%,.

Do these data provide evidence that the blood type proportions of mall shoppers differ from the blood type proportions of the general public? Test the appropriate hypotheses using $\alpha = 0.01$.

Blood Type	**Frequency**
O	465
A	294
B	196
AB	45
Total	**1000**

We will test the following hypotheses:

H_0: Mall shoppers have the same blood type proportions as the general public

H_a: Mall shoppers DO NOT have the same blood type proportions as the general public

Data was given for n = 1000 mall shoppers. If H_0 is true, what are the expected counts?

	Blood Type			
	O	**A**	**B**	**AB**
Observed Count	465	294	196	45
Expected Count	0.45(1000) = 450	0.4(1000) = 400	0.1(1000) = 100	0.05(1000) = 50

Note: A different look for H_0 could be

$H_0 : p_O = 0.45, p_A = 0.40, p_B = 0.10, p_{AB} = 0.05$

We need a test statistic. We call it the χ^2 (Chi-square) test statistic. Here's how to calculate it:

$$\chi^2 = \sum_{\text{all cells}} \frac{(\text{observed cell count - expected cell count})^2}{\text{expected cell count}}$$

Let's calculate our χ^2 test statistic.

$$\chi^2 = \frac{(465-450)^2}{450} + \frac{(294-400)^2}{400} + \frac{(196-100)^2}{100} + \frac{(45-50)^2}{50} = 121.25$$

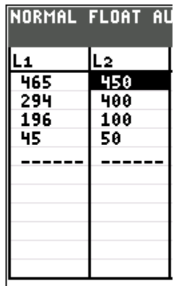
Now how do we get our P–value?

We will use the Chi Square distribution function on the calculator which can be found in the same menu as *normalcdf*

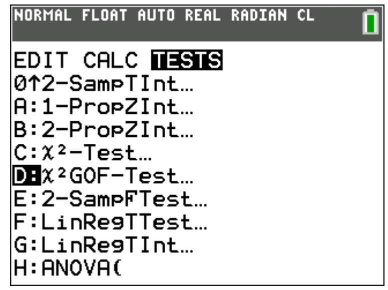
$P(\chi^2 > 121.25) = \chi^2 cdf(121.25, 1E99, 3) = 0$

You can also use the Chi Square test function on the TI calculator. See below:

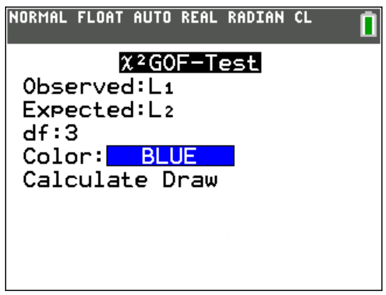
After entering both the observed and expected counts in separate lists,



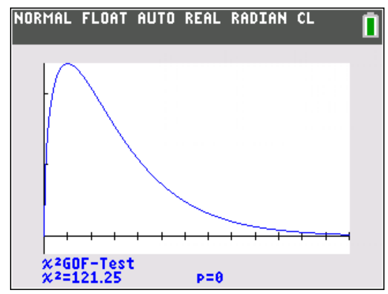
Find the Goodness of Fit test under the TESTS menu:



Choose the lists in which you entered your observed and expected counts and enter the degrees of freedom.



If you choose Draw, you will see the shaded graph along with the Chi Square and *P*-values



χ^2 Goodness-Of-Fit Procedure

1. Hypotheses:

$$
\begin{aligned}
H_0 : p_1 &= \text{hypothesized proportion for Category 1} \\
p_2 &= \text{hypothesized proportion for Category 2} \\
&. \\
&. \\
&. \\
p_k &= \text{hypothesized proportion for Category } k
\end{aligned}
$$

2. Test Statistic: $\chi^2 = \sum\limits_{\text{all cells}} \dfrac{(\text{observed cell count - expected cell count})^2}{\text{expected cell count}}$

3. *P* – values: The *P* – value associated with the computed test statistic value is the area to the **right** of χ^2 chi-square curve with $df = k - 1$.

4. <u>*Assumptions*</u>: 1. Observed cell counts are based on a *random sample.*
2. The *sample size is large*. The sample size is large enough for the chi-square test to be appropriate as long as every expected cell count is at least 5.

Contributions

This term refers to iterations of the Chi Square sum and is used to discuss the larger values and their effect on the result of the test. In example 1, the contributions are

$$\chi^2 = \frac{(465-450)^2}{450} + \frac{(294-400)^2}{400} + \frac{(196-100)^2}{100} + \frac{(45-50)^2}{50} = 121.25$$

$$= 0.5 + 28.09 + 92.16 + 0.5 = 121.25$$

Of these numbers the largest contribution is 92.16 from the observed count of 196 people with Type B blood. This tells us that those blood types had the largest effect on the outcome of our test. Though without it our *P*-value would still have been 0, the Chi Square statistic would have been $\chi^2 = 29.09$ which is still a big difference.

<u>Example 2</u> Five partners in a law firm brought in the following numbers of new clients during the past year.

Partner	King	Wong	Patel	Allen	Pickens
Number of New Clients	35	42	22	41	30

Is there sufficient evidence at the 5% or 10% level of significance that the partners do not bring in equal numbers of new clients?

H_0: The partners bring in an equal proportion of new clients ($p_K = p_W = p_{Pa} = p_A = p_{Pi}$)

H_a: The partners bring in different proportions of new clients

Assumptions: The expected counts for all five observed counts here is 1/5 of 170

$$\chi^2 = \frac{(35-34)^2}{34} + \frac{(42-34)^2}{34} + \frac{(22-34)^2}{34} \frac{(41-34)^2}{34} \frac{(30-34)^2}{34} = 8.059$$

$P(\chi^2 > 8.059) = \chi^2 cdf(8.059, 1E99, 4) = 0.0894$

At significance level $\alpha = 0.1$ we will reject the null hypothesis but at $\alpha = 0.05$ we would fail to reject. We have evidence that each partner brings in new clients at different rates but the significance of this evidence will depend greatly on our choice of α.

Example 3 A genetic model for offspring of two Labrador retrievers states the ratios for dog colors to be 5:4:1 for black:yellow:chocolate. Two labrador retrievers are bred and a litter consisting of 3 black lab puppies, 5 yellow lab puppies, and 2 chocolate lab puppies is produced. For a goodness of fit test, the χ^2 statistic would be:

(a) 1.79
(b) 2.05
(c) 2.92
(d) 4.94
(e) The count number is too small for a goodness of fit test

Example 4 A die was rolled 24 times with the following results:

Number of Dots	1	2	3	4	5	6
Frequency	2	8	2	1	3	8

A goodness-of-fit chi-squared test is to be used to test the null hypothesis that the die is fair. At a significance level of $\alpha = 0.01$, the value of the chi-square test statistic and the decision reached is

(a) 12.5; fail to reject the null hypothesis
(b) 12.5; reject the null hypothesis
(c) 25.0; fail to reject the null hypothesis
(d) 25.0; reject the null hypothesis
(e) 75.5; reject the null hypothesis

Checkpoint:
Multiple Choice
1. A highway superintendent states that five bridges into a city are used in the ratio 2:3:3:4:6 during the morning rush hour. A highway study of a simple random sample of 6000 cars indicates that 720, 970, 1013, 1380, and 1917 cars use the five bridges, respectively. Can the superintendent's claim be rejected at the 2.5% or 5% level of significance?

(a) There is sufficient evidence to reject the claim at either of these two levels.
(b) There is sufficient evidence to reject the claim at the 2.5% but not at the 5% level.
(c) There is sufficient evidence to reject the claim at the 5% but not at the 2.5% level.
(d) There is not sufficient evidence to reject the claim at either of these two levels.
(e) There is not sufficient information to answer this question.

2. In a study to compare movie preferences among different age groups, a χ^2 statistic was used. If a small value of the test statistic is obtained, it suggests that

(a) the null hypothesis may not be rejected, since the differences between the observed and expected values are relatively large.
(b) the null hypothesis may be rejected, since the differences between the observed and expected values are relatively large.

(c) the null hypothesis may not be rejected, since the differences between the observed and expected values are relatively small.
(d) the null hypothesis may be rejected, since the differences between the observed and expected values are relatively small.
(e) the null hypothesis may not be rejected, since the differences between the observed and expected values are the same.

3. A reporter believed that police officers were required to write a specific quota of traffic tickets during a month. In order to meet the alleged quota, he believed officers would need to write more tickets during the last week of the month. To investigate the claim, the reporter collected the number of tickets written by the local police force in a month and organized them by weeks as shown in the table below.

Week	First Week	Second Week	Third Week	Fourth Week	Total
Tickets Written	133	124	154	145	556

A chi-square analysis was performed to test the claim that there is a relationship between the week of the month and the number of tickets written. What is the *P*-value of the test?

(a) 2.0989
(b) 0.5521
(c) 0.7176
(d) 0.8353
(e) Counts are too small for test to be valid

8-1 Homework

1. From the given information in each case below, state what you know about the *P*-value for a chi-square test and give the conclusion for a significance level of $\alpha = 0.01$.

   (a) $\chi^2 = 7.5$, $df = 2$
   (b) $\chi^2 = 13.0$, $df = 6$
   (c) $\chi^2 = 18.0$, $df = 9$
   (d) $\chi^2 = 21.3$, $df = 4$
   (e) $\chi^2 = 7.5$, $df = 2$

2. Are all M & M's colors distributions the same? Some students wanted to know if a typical 3.14 oz bag of M & M's has an equal distribution of colors. To test this, they bought four bags (around 60 each) and tallied each color entering their totals in the table below. Use a Chi Square GOF test with $\alpha = 0.01$ to make an inference about this study.

M & M's sorted by color					
Brown	**Red**	**Green**	**Orange**	**Yellow**	**Blue**
37	33	35	68	32	31

3. It's Girl Scout cookie season and Mr. Maychrowitz is again claiming that Samoas are axiomatically the superior of all the GS cookies. Mr. Murphy (who low key knows that the truly great cookie is the Tagalong) believes that of the top four kinds of GS cookies (Samoas, Tagalongs, Trefoils, and Thin Mints) all are equally popular and he is willing to put all claims to the test by randomly sampling boxes sold by SI girl scouts. His results are tabulated below.

Girl Scout Cookie Boxes Sold			
Samoas	**Tagalongs**	**Thin Mints**	**Trefoils**
47	29	27	27

(a) Conduct a Chi Squared GOF test with significance level $\alpha = 0.05$ to determine who is likely correct

(b) By the way, if the null is rejected then Mr. Murphy will be sad and not speak to his AP Stats students for a week but if the null is not rejected then Mr. Maychrowitz will refuse to share his cookies with students even the ones who prefer Samoas. What are the consequences of a Type I and Type II error in this scenario? Based on the results from part (a) which outcome is now a possibility?

4. The report "Fatality Facts 2004: Bicycles" (Insurance Institute, 2004) included the following table classifying 715 fatal bicycle accidents according to time of day the accident occurred.

Time of Day	Number of Accidents
Midnight to 3 am	38
3 am to 6 am	29
6 am to 9 am	66
9 am to Noon	77
Noon to 3 pm	99
3 pm to 6 pm	127
6 pm to 9 pm	166
9 pm to Midnight	113

(a) Assume it is reasonable to regard the 715 bicycle accidents summarized in the table as a random sample of fatal bicycle accidents in 2004. Do these data support the hypothesis that fatal bicycle accidents are not equally likely to occur in each of the 3-hour time periods used to construct the table? Test the relevant hypotheses using a significance level of 0.05.

(b) Suppose a safety office proposes that bicycle fatalities are twice as likely to occur between noon and midnight as during midnight to noon and suggests the following hypothesis: H_0: $p_1 = 1/3$, $p_2 = 2/3$, where p_1 is the proportion of accidents occurring between midnight and noon and p_2 is the proportion occurring between noon and midnight. Do the given data provide evidence against this hypothesis, or are the data consistent with it? Justify your answer with an appropriate test. (Hint: Use the data to construct a one-way table with just two time categories.)

8-2 Test for Homogeneity and Independence in a Two-Way Table

GOALS:
1. Run a chi-square test for homogeneity.
2. Run a chi-square test for independence.
3. Understand what the next four weeks of class look like!

There are two more types of tests we can use with the χ^2(Chi-square) test statistic. We can test:

- two or more populations for homogeneity (same proportions), or we can investigate
- whether two variables from the same population are independent

Both tests have identical mechanics and only differ in conclusion.

Comparing Two or More Populations for Homogeneity

1. State your target population, null and alternative hypothesis (in symbols and words).

   H_0 : The true category proportions are the same for all populations (homogeneity of the populations)
   H_a : H_0 is not true. The true category proportions are not the same for all of the populations.

2. Assumptions:
   1. The data consist of *independently chosen random samples.*
   2. The *sample size is large*. The sample size is large enough for the chi-square test to be appropriate as long as every expected cell count is at least 5.

3. Test Statistic: $\chi^2 = \sum\limits_{\text{all cells}} \dfrac{(\text{observed cell count - expected cell count})^2}{\text{expected cell count}}$

   The expected cell counts are estimated from the sample data (assuming H_0 is true) using the formula:
   $$\text{expected cell count} = \frac{(\text{row marginal total})(\text{column marginal total})}{\text{grand total}}$$

   df = (number of rows – 1) (number of columns – 1)

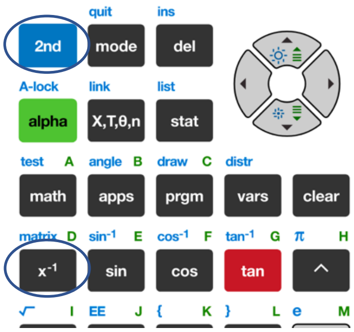
4. Calculation of P – value

5. Conclusion, in theory and context.

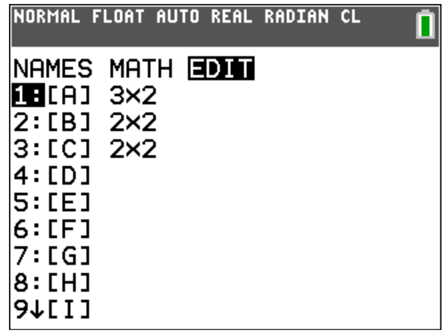
<u>Notes About Using Matrices on the TI Calculator</u>

To use the Chi Square tests for homogeneity or independence on the graphing calculator, we will need

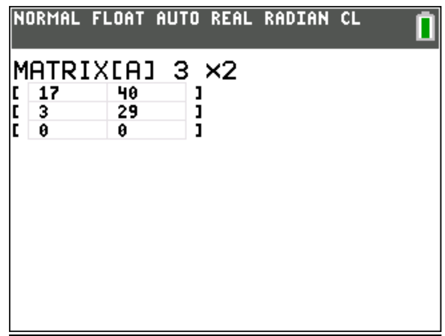
to use the matrix features. The matrix menu can be found by pressing and then



These two buttons will lead you to the matrix menu on the right. Choose EDIT and then one of the matrices to begin entering values



In the matrix edit window, you can define the number of rows and columns before entering your values.



<u>Example 1</u> Several AP Stats students noticed a difference in air circulation and temperature between Cowell and McCullough Gymnasiums. They wondered if the difference in environments affect AP test results so they randomly sample 100 AP results from SI students who took their exam in Cowell and 100 results from students who took their exam in McCullough. The results are tabulated below:

Test Grade	**5**	**4**	**3**	**2**	**1**
Cowell	10	14	40	24	12
McCullough	15	21	39	15	10

To answer this, we will need to conduct a Chi Square Test for Homogeneity because we are drawing students from two different populations. First let's state our hypotheses:

H_0: Students will test at the same success rate for both gyms

H_a: Students will test at different success rates for each gym

Significance level: $\alpha = 0.05$

Let's check our assumptions first:

1. The data consist of *independently chosen random samples.*

2. The *sample size is large*. The sample size is large enough for the chi-square test to be appropriate as long as every expected cell count is at least 5.

   In order to confirm that each expected count is at least 5, we have to divide the product of the row total and the column total by the total number of students. For example, in the AP score of 5 column, both expected counts will be (70*25)/140 = 12.5

Expected Counts	**5**	**4**	**3**	**2**	**1**
Cowell	12.5	17.5	39.5	19.5	11
McCullough	12.5	17.5	39.5	19.5	11

   Each count is well above 5 so our assumptions are met. Notice that when the populations are equally divided (70 in Cowell, 70 in McCullough) our expected count rows are identical.

Now we can find our test statistic:

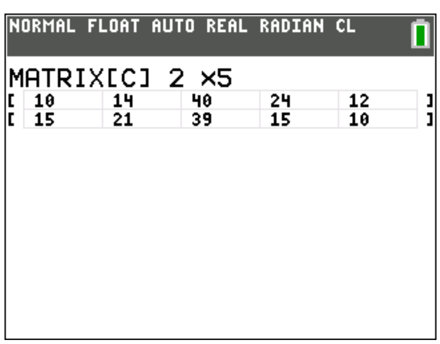
$$\chi^2 = \frac{(10-12.5)^2}{12.5} + \frac{(14-17.5)^2}{17.5} + \frac{(40-39.5)^2}{39.5} + \frac{(24-19.5)^2}{19.5} \ldots = 4.6714$$

The number of rows is 2 and columns is 5 so our *df* = (5 – 1)(2 – 1) = 4

$P(\chi^2 > 121.25) = \chi^2 cdf(4.6714, 1E99, 4) = 0.3227$ so we don't even come close to rejecting the null hypothesis. This data is not sufficient evidence of any effect on student test achievement by the gym in which they take the AP exam.

Before we move on, let's see how this can all be done on the calculator using matrices:

In the matrix edit window, enter your observed counts then select the Chi Square test from the TESTS menu.



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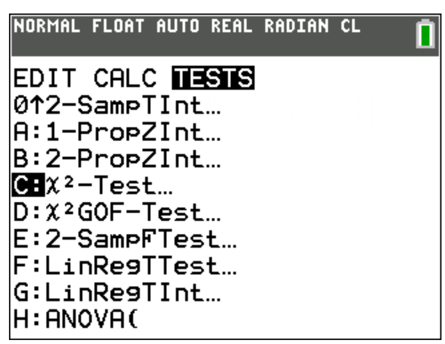
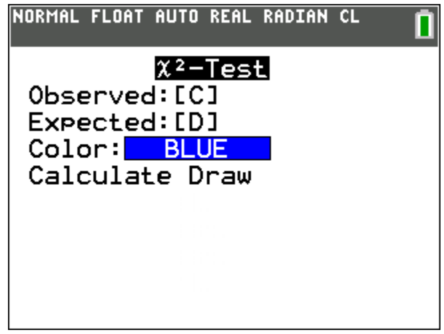
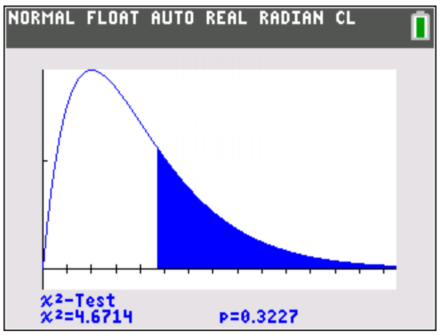
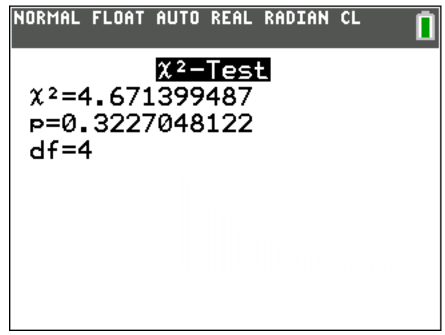


For Observed, select the matrix you just entered. For Expected, choose any other matrix from the matrix list. The function will calculate the expected counts and store them in this other matrix. Notice here that I shoes D which will be explained below

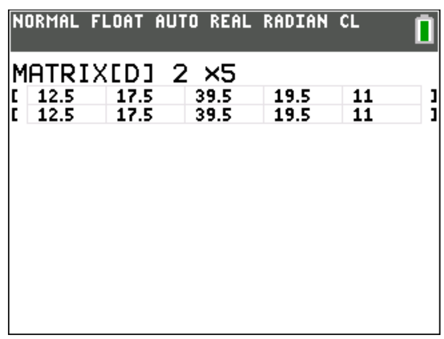




If you choose Draw you will get the graph shown to the left. If you choose Calculate you will get the screen shown to the right



Notice that when I look at matrix D, the function stored the expected counts there. If I want to see the



Comparing Two or More Populations for Independence

- State your target population, null and alternative hypothesis (in symbols and words).

  H_0 : The two variables are independent.

  $H_a : H_0$ is not true. The two variables are not independent.

- Assumptions:
  1. The observed counts are from a *random sample.*
  2. The *sample size is large*. The sample size is large enough for the chi-square test to be appropriate as long as every expected cell count is at least 5.

3. Test Statistic: $\chi^2 = \sum_{\text{all cells}} \frac{(\text{observed cell count - expected cell count})^2}{\text{expected cell count}}$

The expected cell counts are estimated from the sample data (assuming H_0 is true) using the formula:

$$\text{expected cell count} = \frac{(\text{row marginal total})(\text{column marginal total})}{\text{grand total}}$$

Degrees of Freedom, df = (number of rows – 1) (number of columns – 1)

Calculation of P – value

4. Conclusion, in and out of context.

Example 3 Here is a table showing who survived the sinking of the Titanic based on whether they were crew members, or passengers booked in first-, second-, or third-class staterooms:

	Crew	**First**	**Second**	**Third**	**Total**
Survived	212	202	118	178	710
Died	673	123	167	528	1491
Total	885	325	285	706	2201

(a) If an individual is drawn at random, what is the probability that we will draw a member of the crew?

$$P(\text{crew member}) = \frac{212}{2201}$$

(b) What's the probability of randomly selecting a third-class passenger who survived?

$$\text{P(Third \& Survived)} = \frac{178}{2201}$$

(c) If someone's chances of surviving were the same regardless of their status on the ship, how many members of the crew would you have expected to survive?

Here we use the expected counts equation: $\text{E(Crew \& Survive)} = \frac{710 \cdot 885}{2201} = 285.48$

(d) Is there a relationship between a person's status on the Titanic and whether or not they survived the sinking? Perform the appropriate test.

To answer this, we will need to conduct a Chi Square Test for Independence because we are drawing from one population and testing if a particular factor affects survival rate. First, let's state our hypotheses:

H_0: A passenger's status and survival rate are independent

H_a: A passenger's status and survival rate are **not** independent

Significance level: $\alpha = 0.05$

Let's check our assumptions first:

1. The data consist of *independently chosen random samples.*

2. The *sample size is large*. The sample size is large enough for the chi-square test to be appropriate as long as every expected cell count is at least 5.

   In order to confirm that each expected count is at least 5, we have to divide the product of the row total and the column total by the total number of passengers. For example, in the Crew column and Survived row, the expected count will be (885*710)/2201 = 285.48

	Crew	**First**	**Second**	**Third**	**Total**
Survived	212 (285.48)	202 (104.84)	118 (90.935)	178 (227.74)	710
Died	673 (599.52)	123 (220.16)	167 (193.06)	528 (478.26)	1491
Total	885	325	285	706	2201

   Each count is well above 5 so our assumptions are met.

Now we can find our test statistic:

$$\chi^2 = \frac{(212-285.48)^2}{285.48} + \frac{(202-104.84)^2}{104.84} + \ldots \frac{(528-478.26)^2}{478.26} = 187.79$$

The number of rows is 2 and columns is 4 so our $df = (4 - 1)(2 - 1) = 3$

$P(\chi^2 > 187.79) = \chi^2 cdf(187.79, 1E99, 3) = 0$

Because our P-value = 0 < 0.05 we reject the null hypothesis. We have significant evidence of a relationship between the status of the passenger and their likelihood of survival.

Checkpoint
Multiple Choice

1. Find the expected value of the cell marked with the "***" in the following 3 × 2 table (the bold face values are the marginal totals):

observation	observation	**19**
observation	***	**31**
observation	observation	**27**
45	**32**	**77**

(a) 74.60 (b) 18.12 (c) 12.88 (d) 19.65 (e) 18.70

2. A study is to be conducted to help determine if race is related to blood type. Race groups are identified as White, African-American, Asian, Latino, or Other. Blood types are A, B, O, and AB. How many degrees of freedom are there for a chi-square test of independence between Race and Blood Type?

(a) 5 × 4 = 20
(b) 5 × 3 = 15
(c) 4 × 4 = 16
(d) 5 + 4 – 2 = 7
(e) 4 × 3 = 12

3. A group separated into men and women are asked their preference toward certain types of television shows. The following table gives the results.

	Program Type A	Program Type B
Men	5	20
Women	3	12

Which of the following statements is/are true?

(a) The variables gender and program preference are independent.
(b) For these data, $\chi^2 = 0$
(c) The variables gender and program preference are related.

(a) I only
(b) I and II only
(c) II only
(d) III only
(e) II and III only

4. Is there a relationship between education level and sports interest? A study cross-classified 1500 randomly selected adults in three categories of education level (not a high school graduate, high school graduate, and college graduate) and five categories of major sports interest (baseball, basketball, football, hockey, and tennis). The value of χ^2 is 13.95. Is there evidence of a relationship between education level and sports interest?

(a) The data prove there is a relationship between education level and sports interest.
(b) The evidence points to a cause-and-effect relationship between education level and sports interest.
(c) There is evidence at the 5% significance level of a relationship between education level and sports interest.
(d) There is evidence at the 10% significance level, but not at the 5% significance level, of a relationship between education level and sports interest.
(e) The *P*-value is greater than 0.10, so there is no evidence of a relationship between education level and sports interest.

5. A disc jockey wants to determine whether middle school students and high school students have similar music tastes. Independent random samples are taken from each group, and each person is asked whether he/she prefers hip-hop, pop, or alternative. A chi-square test of homogeneity of proportions is performed, and the resulting *P*-value is below 0.05. Which of the following is a proper conclusion?

(a) There is evidence that for all three music choices the proportion of middle school students who prefer each choice is equal to the corresponding proportion of high school students.
(b) There is evidence that the proportion of middle school students who prefer hip-hop is different from the proportion of high school students who prefer hip-hop.
(c) There is is evidence that for all three music choices the proportion of middle school students who prefer each choice is different from the corresponding proportion of high school students.
(d) There is evidence that for at least one of the three music choices the proportion of middle school students who prefer that choice is equal to the corresponding proportion of high school students.
(e) There is evidence that for at least one of the three music choices the proportion of middle school students who prefer that choice is different from the corresponding proportion of high school students.

8-2 Homework

1. Mr. Murphy is now concerned that Mr. Maychrowitz is exerting undue influence on his students about GS cookie preferences so he ups his random sampling game. He conducts a more rigorous random sample that now divides students who bought GS cookies into three categories: Those who have Mr. Murphy for any math subject this year, those who have Mr. Maychrowitz, and those who have neither teacher. (Samoas, Tagalongs, Trefoils, and Thin Mints) all are equally popular and he

is willing to put all claims to the test by randomly sampling from boxes bought by students from each category of math experience (Murphy, Maychrowitz, neither). The results are tabulated below.

	Girl Scout Cookie Boxes Sold			
Teacher	**Samoas**	**Tagalongs**	**Thin Mints**	**Trefoils**
Mr. Maychrowitz	22	26	31	18
Mr. Murphy	7	41	28	19
Neither	23	22	30	23

(a) Would a Chi Squared test for independence or homogeneity be the approach here? Explain your answer.

(b) Conduct your Chi Squared test with significance level $\alpha = 0.05$ to determine who is likely correct

(c) What are the two largest contributions to the result and what inference about your test results might you make from them? Explain your answer.

2. More students have entered the M & M's study with some wondering if the distribution of colors within a bag differs by the size of the bag. They randomly bought several bags of different sizes from different stores and tabulated their results below. Is there sufficient evidence to conclude that the distribution of M & M's colors differs by the size of the bag?

M & M's sorted by color						
	Brown	**Red**	**Green**	**Orange**	**Yellow**	**Blue**
Four 60 piece bags	37	33	35	68	32	31
Three 20 piece bags	9	9	12	11	8	16
Three 100 piece bags	61	40	86	58	66	64

3. A random sample of 1000 registered voters in a certain county is selected, and each voter is categorized with respect to both educational level (four categories) and preferred candidate in an upcoming election for county supervisor (five possibilities). The hypothesis of interest is that educational level and preferred candidate are independent.

(a) If $\chi^2 = 7.2$, what would you conclude at significance level 0.10?

(b) If there were only four candidates running for election, what would you conclude if $\chi^2 = 14.5$ and $\alpha = .05$?