**Calculus Chapter 10 Review 2** Name Solutions

Show all your work. Do not use your calculators to evaluate any integrals

A particle is moving along a curve defined by the parametric equations $x = t^3 - 3t$ and $y = t - t^2$ over the interval $0 \leq t \leq 3$. Use this to do problems 1 through 5.

1) At what time $t$ is the direction of the particle's motion vertical? $\frac{dx}{dt} = 0$ $\frac{dy}{dt} \neq 0$

$\frac{dx}{dt} = 3t^2 - 3 = 0 \Rightarrow t = 1$

$\frac{dy}{dt} = 1 - 2t \Rightarrow t = 1$ $\frac{dy}{dt} = -1$ $t = 1$

2) Find the speed and position of the particle at this time.

$s(1) = \langle -2, 0 \rangle$

Speed $= \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = \sqrt{1 + 0} = 1$

3) Write the expression to find the distance the particle has traveled by this time. Use your calculator to evaluate the final answer.

$$\int_0^1 \sqrt{(3t^2-3)^2 + (1-2t)^2}\, dt \approx 2.134$$

4) Find the equation of the line tangent to the path of the particle at $t = 2$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-2t}{3t^2-3}$ $x = 2$ $y = -2$

at $t = 2$ $\frac{dy}{dx} = \frac{-3}{9} = -\frac{1}{3}$

$$\boxed{y + 2 = -\frac{1}{3}(x - 2)}$$

5) Find $d^2y/dx^2$. $= \dfrac{\frac{d}{dx}\frac{dy}{dx}}{\frac{dx}{dt}} = \dfrac{\frac{-6t^2+3-6t(1-2t)}{(3t^2-3)^2}}{(3t^2-3)} = \dfrac{-6t^2+3-6t+12t^2}{(3t^2-3)^3}$

$$= \frac{6t^2-6t+3}{(3t^2-3)^3}$$

6) Reliving memories of their junior year, Delanie, Sophia, and Richard drag Mr. Murphy up to the top of another high diving board and throw him off. His acceleration vector is ⟨0, – 32⟩ measured in feet/second squared and the bottom of the diving board ladder has the position vector ⟨0, 0⟩. Since he travels upward and outward after being released, his initial velocity vector is ⟨4, 16⟩.

a) Find the velocity vector $\langle x'(t), y'(t)\rangle$ = $\langle C_1, -32t + C_2 \rangle$

$-32(0) + C = 16$ $\quad\quad\langle 4, -32t+16\rangle$

$C = 16$

acceleration is 0 in the x direction so 4 is a constant horizontal velocity

b) Use the answer to part a) to determine the height of the diving board if Mr Murphy's fall lasted 4 seconds.

height of diving board = displacement (vertical)

$$\int_0^4 -32t+16\, dt = -16t^2+16t\Big]_0^4 = -16(16)+16(4)$$

$$= -256+64 = -192 \text{ ft}$$

c) Find the position vector $\langle x(t), y(t)\rangle$ for Mr. Murphy's fall

$x(t) = 4t + C_1 \quad x(0) = 0$

$y(t) = -16t^2 + 16t + C_2 \quad y(0) = 192$

$y(t) = -16t^2 + 16t + 192$

$$\boxed{\langle 4t, -16t^2+16t+192\rangle}$$

7) Find the length of the polar curve $r = \theta^2$ on the interval $0 \le \theta \le \sqrt{5}$.

$r' = 2\theta \qquad L = \int_0^{\sqrt{5}} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{\sqrt{5}} \sqrt{\theta^2(\theta^2+4)}\, d\theta$

$= \int_0^{\sqrt{5}} \theta\sqrt{\theta^2+4}\, d\theta \qquad u = \theta^2+4 \quad du = 2\theta\, d\theta$

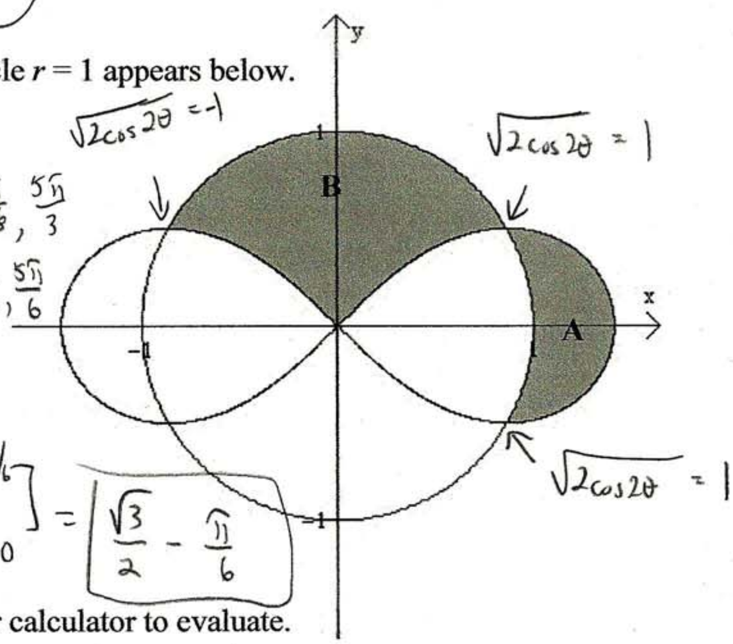
$\theta = 0 \quad \theta = \sqrt{5}$

$u = 4 \quad u = 9$

$\frac{1}{2}\int_4^9 u^{1/2}\, du = \frac{1}{3}\left[u^{3/2}\right]_4^9 = \frac{1}{3}(27-8) = \frac{19}{3}$

8) The graph of the polar curve $r = \sqrt{2\cos 2\theta}$ and the circle $r = 1$ appears below.

(a) Find the area of shaded region **A**.
Do not use your calculator to evaluate.




$\sqrt{2\cos 2\theta} = 1 \Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

$\theta = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

$A_{\text{Region A}} = \frac{1}{2}\int_{-\pi/6}^{\pi/6} 2\cos 2\theta - 1\, d\theta$

$= \int_0^{\pi/6} 2\cos 2\theta - 1\, d\theta = \sin 2\theta - \theta \Big]_0^{\pi/6} = \boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{6}}$

(b) Find the area of shaded region **B**. Do not use your calculator to evaluate.

$A_{\text{Region B}} = \frac{1}{2}\int_{\pi/6}^{5\pi/6} 1 - 2\cos 2\theta\, d\theta = \frac{1}{2}\left[\theta - \sin 2\theta\right]_{\pi/6}^{5\pi/6} = \frac{1}{2}\left[\frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right)\right] - \frac{1}{2}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{2}\left[\frac{2\pi}{3} + \sqrt{3}\right] = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$

9) Show that the slope of the line tangent to the graph of $r = \cos\theta$ is 0 at $\theta = \frac{\pi}{4}$ $\qquad r' = -\sin\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{-\sin^2\theta + \cos^2\theta}{-\sin\theta\cos\theta - \sin\theta\cos\theta}$

$= \frac{-\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4}}{-2\sin\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{-\frac{1}{2} + \frac{1}{2}}{-1} = 0$