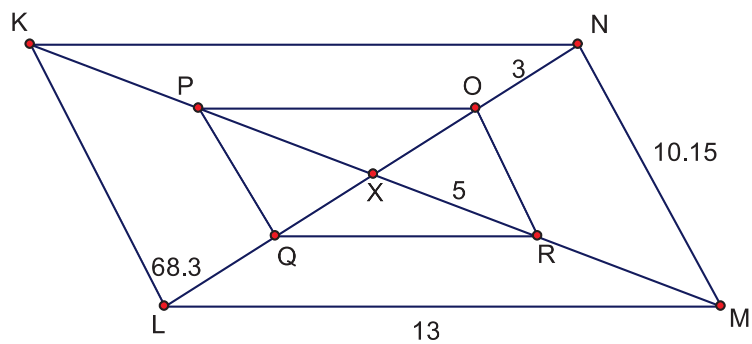
7 ~~6~~-2: Properties of Parallelograms

A ***parallelogram*** is a quadrilateral whose opposite sides are parallel.

The following theorems apply to all parallelograms:

- If a quadrilateral is a parallelogram, then its **opposite sides** are **congruent**.
- If a quadrilateral is a parallelogram, then its **opposite angles** are **congruent**.
- If a quadrilateral is a parallelogram, then its **consecutive (or same-side interior) angles** are supplementary.
- If a quadrilateral is a parallelogram, then its **diagonals bisect each other**.

EX 1) In parallelogram *KLMN* below, points *O, P, Q, R* are midpoints of $\overline{XN}$, $\overline{XK}$, $\overline{XL}$, and $\overline{XM}$, $\angle NKL = 61°$ and $\angle NLK = 68.3°$. Find the indicated measures.



a) $KN = 13$

b) $PX = 5$

c) $KL = 10.15$

d) $XN = 6$

e) $LN = 12$

f) $KP = 5$

g) $KR = 15$

h) $m\angle MNL = 68.3°$

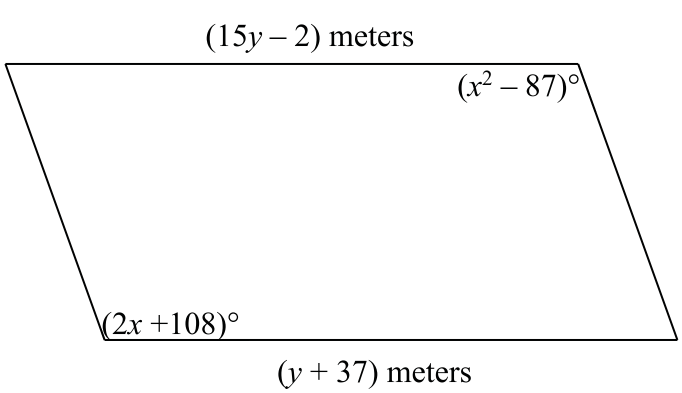
i) $m\angle NLM = 50.7°$

j) $m\angle NML = 61°$

k) $m\angle XQP = 68.3°$

l) Perimeter of *KLMN* $= 46.3$

EX 2) Solve for *x* and *y* in the parallelogram below.



Solution:

$15y - 2 = y + 37$

$y = \dfrac{39}{14}$ meters

$x^2 - 87 = 2x + 108$

$x^2 - 2x - 195 = 0$

$x = 15$ or -13

7 ~~6~~-3: Properties of Parallelograms

In order to prove that a quadrilateral is a parallelogram, you can show that *both* pairs of opposite sides are parallel (since this is the definition of a parallelogram).

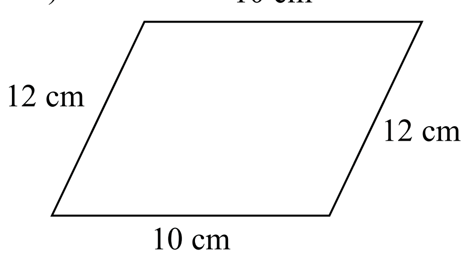
In addition, you can prove a quadrilateral is a parallelogram any of the following ways:

- If the **opposite sides** of a quadrilateral are **congruent**, then it is a parallelogram.
- If the **opposite angles** of a quadrilateral are **congruent**, then it is a parallelogram.
- If the **consecutive angles** of a quadrilateral are **supplementary**, then it is a parallelogram.
- If the **diagonals** of a quadrilateral **bisect each other**, then it is a parallelogram.
- If *one* pair of **opposite sides** are **parallel** and **congruent**, then the quadrilateral is a parallelogram.

Note that these are the converses of the theorem in 6-2 (with the exception of the last statement).
(page 413 has a good summary for recognizing what is a parallelogram)

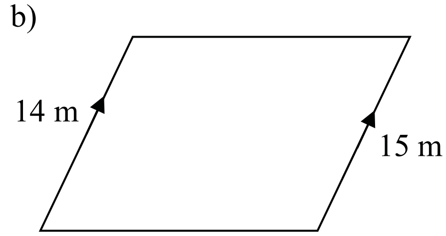
EX 3) For each of the figures below, which **MUST** be parallelograms. If it is a parallelogram, write the reason why it is. If it is not, explain why not.

a)



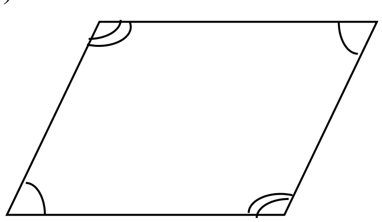
Yes – opposite sides are congruent

b)



No – one pair of sides is not congruent and parallel

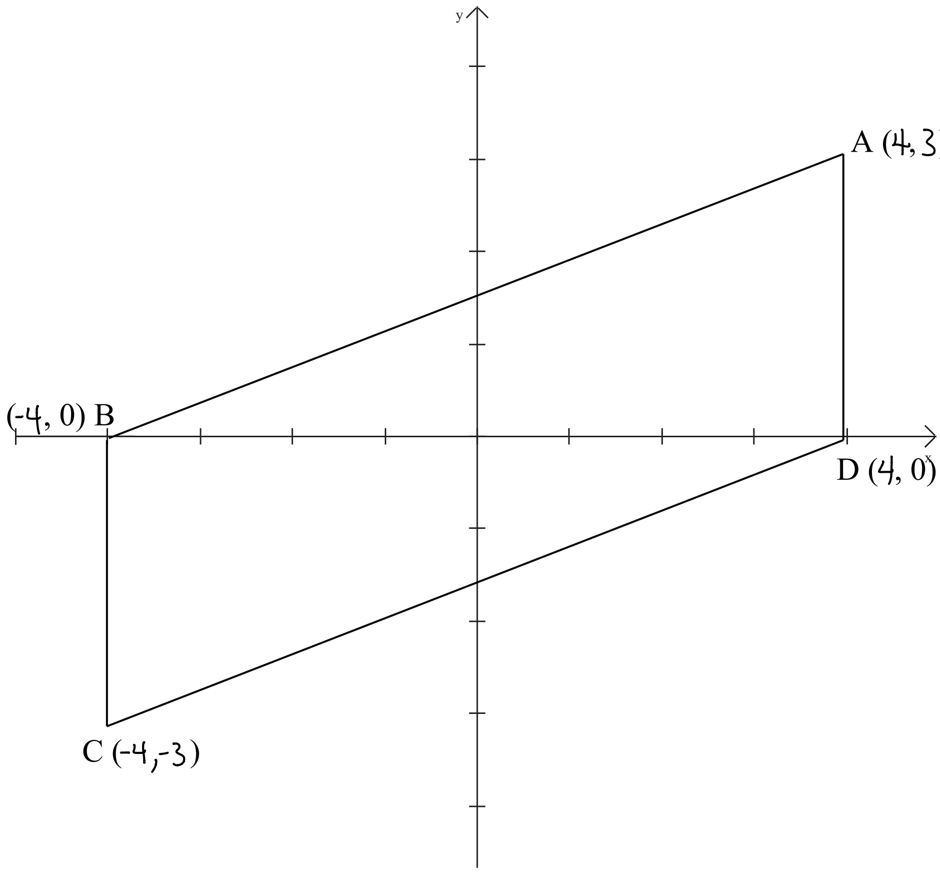
c)



Yes – opposite angles are congruent

EX 4) Prove that quadrilateral *ABCD* below is a parallelogram. There are 3 different ways to attack this coordinate proof, so find one that works best for you.

- Method 1: Show that opposite sides are parallel (have the same slope).
- Method 2: Show that opposite sides are congruent (have the same length).
- Method 3: Show that *one* pair of opposite sides is congruent and parallel.



- Method 1: Show that opposite sides are parallel (have the same slope).

Vertical lines are parallel

$\overline{BC} \parallel \overline{AD}$

$m_{\overline{AB}} = \frac{3-0}{4-(-4)} = \frac{3}{8}$

$m_{\overline{DC}} = \frac{0-(-3)}{4-(-4)} = \frac{3}{8}$

$\overline{AB} \parallel \overline{DC}$

- Method 2: Show that opposite sides are congruent (have the same length).

$BC = 3$

$AD = 3$

$AB = \sqrt{3^2 + 8^2} = \sqrt{73}$

$DC = \sqrt{3^2 + 8^2} = \sqrt{73}$

Method 3

One pair of opposite sides are $\parallel$ and $\cong$

Choose the two vertical lines because they are already $\parallel$

Their lengths are both 3 so they are both $\parallel$ and $\cong$