Jonah's Trophy Climb: When Linear Models Aren't Enough

After his reality check about gaming time vs. grades, Jonah decided to set some boundaries. He now limits himself to 1 hour per day of Clash Royale (Lucas and Sebastian are skeptical about this). To make the most of his limited gaming time, he's been tracking his progress in the game.

In Clash Royale, players earn "trophies" by winning battles. Jonah noticed something interesting: when he first started playing, his trophy count skyrocketed quickly. But now, even though he's playing the same amount each day, his trophy gains have slowed to a crawl. "It's like the game gets exponentially harder!" he complained.

Norah saw a teaching opportunity: "Jonah, not everything in life is linear. Let's analyze this!"

The Data

Jonah recorded his total trophy count at the end of each month since he started playing:

Months Playing	Total Trophies
1	1200
2	2100
3	2850
4	3400
5	3850
6	4200
7	4480
8	4700
9	4870
10	5010
11	5120
12	5210

1) Create a scatterplot of Months Playing (x) vs. Total Trophies (y). Describe what you observe. Does the relationship appear linear?

2) Calculate the correlation coefficient (r) for the original data. Based on this value alone, would you think a linear model is appropriate?

3) Create a linear regression model: $\hat{y} = a + bx$. Write the equation with the appropriate context.

4) Create a residual plot for your linear model. What does the residual plot reveal about the appropriateness of a linear model? Be specific about any patterns you observe.

Look at your scatterplot again. The pattern of Jonah's trophy progression shows:

- Rapid growth at first
- Gradual leveling off

5) What type of curve does this resemble? (Hint: Think about functions you've learned in previous math classes - exponential, logarithmic, power, etc.)

Calculate the differences in trophy gains between consecutive months:

- Month 1 to 2: _____ trophies gained
- Month 2 to 3: _____ trophies gained
- Month 3 to 4: _____ trophies gained
- Continue through Month 11 to 12

What pattern do you notice? Is Jonah gaining trophies at a constant rate?

Since the data shows diminishing returns (rapid growth that levels off), a logarithmic transformation might linearize the relationship.

6) Create a new variable: ln(*trophies*). Calculate the natural log of the trophy count for each month. Round to 3 decimal places.

Months Playing	**Total Trophies**	**ln(*trophies*)**
1	1200	
2	2100	
...	...	...

7) Create a new scatterplot with Months Playing on the *x*-axis and ln(*trophies*) on the y-axis. Does this relationship appear more linear? You can paste a TI or Desmos graph here if you like.

8) Calculate the correlation coefficient (r) for Months vs. ln(*trophies*). Compare this to your answer in Question 2. Which relationship is more linear?

9) Find the LSRL for the transformed data: ln(*trophies*) = $a + b$(*months*)**.** Write your equation with numerical values.

10) Create a residual plot for your transformed model. Compare this residual plot to the one from Question 4. Which shows a more random scatter? You may use Desmos or TI again.

11) Interpret the slope of your transformed model in context. What does the slope tell us about Jonah's trophy progression? (This is tricky - the units are now "ln(trophies) per month")

12) Use your transformed model to predict ln(Trophies) when Jonah has been playing for 15 months.

13) Convert your prediction back to the original scale by calculating $e^{\text{your prediction}}$ This gives you the predicted trophy count. What do you predict Jonah's trophy count will be after 15 months?
14) Based on your model, what is Jonah's predicted trophy count after 24 months (2 years)? Does this prediction seem reasonable? Why might the model break down for very large time values?
15) The transformed model is: ln(*trophies*) = $a + b$(*months*)

By using properties of logarithms, we can rewrite this in exponential form: Trophies = $e^a e^{b \cdot months}$

This is equivalent to: **Trophies =** $A \cdot B^{months}$ where $A = e^a$ and $B = e^b$

Calculate A and B for Jonah's model. This is called an exponential model.

16) Using your exponential form, what is the growth factor (B) for Jonah's trophy count? Express it as a percentage increase per month.

17) Complete this table comparing the two models:

Criterion	Linear Model	Transformed (Logarithmic) Model
Correlation (r)		
r^2		
Residual Plot Pattern		
Which is better?		

18) Jonah asks: "Why can't I just use the linear model? The *r* value was still pretty high!"

Write a response explaining:

- Why high correlation doesn't guarantee an appropriate model
- The importance of checking residual plots
- What could go wrong if he used the linear model to make predictions

19) Jonah's friend Tommy has been playing for 18 months and has 5800 trophies. Using your exponential model, is Tommy performing better or worse than expected? Calculate his residual.

20) Why does the logarithmic model make sense for this situation? Think about game design and player skill. Give at least two real-world reasons why trophy gains would slow down over time.

21) Your model predicts Jonah will eventually plateau around a certain trophy level as months approach infinity. Based on your exponential model form, does it predict infinite growth or a plateau? Is this realistic for a game? (Hint: Consider what happens as months gets very large in your model.)

22) Another possible transformation is a power transformation: log(*trophies*) vs. log(*months*).

Try this transformation:

- Create log(*trophies*) vs. log(*months*).
- Make a scatterplot and find the LSRL
- Calculate r and r^2

Compare the power model to your logarithmic model. Which provides a better fit? (In this case, the logarithmic model should be better, but it's good practice!)