

Sample Proportions & Point Estimates

The Dallas & Harlow Corporation has bought out Needoh so Eliza has now achieved her dream of endless free NiceCubes. But now that they own them, they have to make sure to maintain their quality for their customers. This task now falls to Dani since Eliza is now so stress free that she hardly comes to the office. They find that Needoh had a defect rate of no more than 5% (0.05). They recently implemented a new manufacturing process and want to assess if the defect rate will change as a result

(a) To investigate the defect rate of the new process, Dani takes a random sample of 200 stress balls and finds that 14 of them are defective. Calculate the sample proportion \hat{p} of defective stress balls and use it as a point estimate for the true proportion of defective stress balls produced by the new process.

(b) Eliza is concerned that the sample size of 200 may not be large enough to provide a reliable estimate. Explain why a larger sample size would generally lead to a more precise point estimate.

(c) Suppose the true proportion of defective stress balls produced by the new process is actually 7% (0.07).

(i) If Dani were to take many random samples of 200 stress balls and calculate the sample proportion of defective stress balls for each sample, describe the shape, center, and variability of the sampling distribution of the sample proportion.

(ii) Would a sample proportion of 4% (0.04) be considered unusual in this sampling distribution? Justify your answer.

(d) Dani and Eliza decide to take a larger random sample of 1000 stress balls. In this sample, 60 stress balls are found to be defective.

(i) Calculate the sample proportion of defective stress balls for this larger sample.

(ii) Explain why the point estimate from this larger sample is likely to be closer to the true proportion of 0.07 than the point estimate from the sample of 200 stress balls.

(e) Is the sample proportion (\hat{p}) an unbiased estimator of the true population proportion (p)? Explain your reasoning. In Dani's initial sample of 200 stress balls, the sample proportion was 0.07. Is this point estimate biased? Explain your answer.