Dulce is debating with Dani whether with two dice it's more likely to

(a) take 10 rolls to get a 12 (roll nine times and get a 12 on the 10th roll) or

(b) to get exactly three 12's in exactly ten rolls. Which has a higher probability?

$$P(12) = \frac{1}{36} \qquad P(\text{not } 12) = \frac{35}{36}$$

Before anyone panics…

(a) $X = \#$ of rolls it takes to get a 12 $\qquad X = 10$

$$P(X = 10) = \left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^9 = 0.0216$$

**This is called a Geometric Probability**

(b) $Y = \#$ of 12's in ten rolls $\qquad Y = 3$

Ten rolls, three 12s

$$P(Y = 3) = \binom{10}{3}\left(\frac{1}{36}\right)^3\left(\frac{35}{36}\right)^7$$

What's this you ask?

How many different ways can we have three 12's and three non-12's?

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(b) $Y = \#$ of 12's in ten rolls $\quad Y = 3 \quad P(12) = \frac{1}{36} \quad P(\text{not } 12) = \frac{35}{36}$

| | Roll 1 | Roll 2 | Roll3 | Roll 4 | Roll 5 | Roll 6 | Roll 7 | Roll 8 | Roll 9 | Roll 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Combo 1 | 12 | 12 | 12 | Not 12 | Not 12 | Not 12 | Not 12 | Not 12 | Not 12 | Not 12 |
| Combo 2 | 12 | Not 12 | Not 12 | 12 | Not 12 | Not 12 | Not 12 | Not 12 | 12 | Not 12 |
| Combo 3 | 12 | Not 12 | Not 12 | Not 12 | Not 12 | 12 | Not 12 | Not 12 | 12 | Not 12 |
| Combo 4 | Not 12 | Not 12 | Not 12 | 12 | Not 12 | Not 12 | Not 12 | Not 12 | 12 | 12 |
| Combo 5 | Not 12 | Not 12 | 12 | Not 12 | 12 | Not 12 | 12 | Not 12 | Not 12 | Not 12 |

There are actually 120 different combinations of three 12's and seven non-12's

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Before anyone panics…

(a) $X = \#$ of rolls it takes to get a 12 $\qquad X = 10$

$$P(X = 10) = \left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^9 = 0.0216$$

**This is called a Geometric Probability**

(b) $Y = \#$ of 12's in ten rolls $\qquad Y = 3$

Ten rolls, three 12s

$$P(Y = 3) = \binom{10}{3}\left(\frac{1}{36}\right)^3\left(\frac{35}{36}\right)^7 = 120\left(\frac{1}{36}\right)^3\left(\frac{35}{36}\right)^7 = 0.0021$$

What's this you ask?

How many different ways can we have three 12's and three non-12's?

**This is called a Binomial Probability**

The debate changes to whether with two dice it's more likely to

(a) take ***up to*** 10 rolls to get a 12 (roll nine times and get a 12 on the 10th roll) or

(b) to get ***between one and three 12's (inclusive)*** in exactly ten rolls. Which has a higher probability?

How does this change the problem?

$$P(12) = \frac{1}{36} \qquad P(\text{not } 12) = \frac{35}{36}$$

(a) $X = \#$ of rolls it takes to get a 12 $\quad 1 \le X \le 10$

$$\sum_{n=1}^{10}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{n-1} = \underbrace{\left(\frac{1}{36}\right)}_{P(X=1)} + \underbrace{\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)}_{P(X=2)} + \underbrace{\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{2}}_{P(X=3)} \ldots + \underbrace{\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{9}}_{P(X=10)} = 0.2455$$

| 12 in just one roll | First roll not 12, second one a 12 | First two rolls not 12, third one a 12 | First nine rolls not 12, tenth one a 12 |
|---|---|---|---|

Remember that the task is to roll until you get a 12

**This is called a Geometric Distribution**

The debate changes to whether with two dice it's more likely to

(a) take ***up to*** 10 rolls to get a 12 (roll nine times and get a 12 on the 10th roll) or

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$$P(12) = \frac{1}{36} \qquad P(\text{not } 12) = \frac{35}{36}$$

(b) $Y = \#$ of 12's in ten rolls $\qquad 1 \le Y \le 3$

$P(Y = 3)$

Ten rolls,
three 12s

$$\binom{10}{3}\left(\frac{1}{36}\right)^3\left(\frac{35}{36}\right)^7 + \binom{10}{2}\left(\frac{1}{36}\right)^2\left(\frac{35}{36}\right)^8 + \binom{10}{1}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^9$$

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$$P(12) = \frac{1}{36} \qquad P(\text{not } 12) = \frac{35}{36}$$

(b) $Y = \#$ of 12's in ten rolls $\quad 1 \le Y \le 3$

$P(Y = 3)$ — Ten rolls, three 12s

$P(Y = 2)$ — Ten rolls, two 12s

$P(Y = 1)$ — Ten rolls, one 12

$$\binom{10}{3}\left(\frac{1}{36}\right)^3\left(\frac{35}{36}\right)^7 + \binom{10}{2}\left(\frac{1}{36}\right)^2\left(\frac{35}{36}\right)^8 + \binom{10}{1}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^9 = 0.2454$$

This would be 10 nCr 3 on your calculator

10 nCr 1 = 10 different combinations of one 12 and 9 non-12's

10 nCr 2 = 45 different combinations of two 12's and 8 non-12's

10 nCr 3 = 120 different combinations of three 12 and 9 non-12's

Because each 12 could happen anywhere in the order (could be the first roll, third, tenth, etc) they represent 10 nCr 3 possible combinations of three 12's in ten rolls

**This is called a Binomial Distribution**

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$$P(12) = \frac{1}{36} \qquad P(\text{not } 12) = \frac{35}{36}$$

(a) $X = \#$ of rolls it takes to get a 12 $\quad 1 \le X \le 10$

$$\underbrace{}_{P(X=1)} \quad \underbrace{}_{P(X=2)} \quad \underbrace{}_{P(X=3)}$$

$$\sum_{n=1}^{10}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{n-1} = \left(\frac{1}{36}\right)+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{2}\ldots+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{9} = 0.2455$$

**This is called a Geometric Distribution**

(b) $Y = \#$ of 12's in ten rolls $\quad 1 \le Y \le 3$

$$P(Y=3) \qquad P(Y=2) \qquad P(Y=1)$$

$$\binom{10}{3}\left(\frac{1}{36}\right)^{3}\left(\frac{35}{36}\right)^{7}+\binom{10}{2}\left(\frac{1}{36}\right)^{2}\left(\frac{35}{36}\right)^{8}+\binom{10}{1}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{9} = 0.2454$$

**This is called a Binomial Distribution**

These can both be done on the calculator but you have to know how the formulas work for the AP Exam. Stay tuned…

# When is a binomial distribution appropriate?

- If there are exactly 2 outcomes, usually designated success and failure, for each trial.
- If the number of trials is fixed.
- If the trials are independent.
- If the probability of success is the same for each trial.

# Formulas on AP sheet

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

$n$ trials
$k$ successes

We can use binompdf/cdf instead of the probability formula, but you need to recognize the probability formula for MC problems

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

This by the way can be calculated with the nCr function on the calculator

# Calculator

$binompdf(n, p, k)$ i.e. particular value of $x$, $P(k=7)$

$binomcdf(n, p, k)$ i.e. cumulative values of $x$, $P(k \leq 7)$ Important note: this includes $k=0$

## When is a binomial distribution appropriate?

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$$\mu_x = np$$

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The Mean and Standard Deviation of a Binomial Distribution

So the expected number of 12's we would get in 10 rolls is $\mu_x = 10\frac{1}{36} = \frac{5}{18}$

## Calculator

And the standard deviation is

$$\sigma_x = \sqrt{10\frac{1}{36}\frac{35}{36}} = 0.520$$

$binompdf(n,\ p,\ k)$ i.e. particular value of $x$, $P(k=7)$

$binomcdf(n,\ p,\ k)$ i.e. cumulative values of $x$, $P(k \le 7)$

Important note: this includes $k=0$

## When is a geometric distribution appropriate?

- If there are exactly 2 outcomes, usually designated success and failure, for each trial.
- If the trials are independent.
- If the probability of success is the same for each trial.
- If there is **NOT** a fixed number of trials. The trials continue **until** a success/failure is achieved.

## Formulas

$$P(X = k) = (1 - p)^{k-1}\, p$$

Note #1: This formula is NOT on your formula sheet

Note #2: We use geometpdf/cdf instead of the probability formula, but you need to recognize the probability formula for MC problems

## Calculator

$geomet\text{pdf}(p,\ k)$ i.e. particular value of $x$, $P(k = 7)$

$geomet\text{cdf}(p,\ k)$ i.e. cumulative values of $x$, $P(k \leq 7)$

# Formulas

Is there a formula on the AP formula sheet that applies?

$$E(X) = \mu_X = \sum x_i p_i$$

$$Var(X) = \sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$$

Might be helpful to crunch these stats on your calculator, using your lists.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

We use our calculators to get the probabilities, binompdf/cdf

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{geom} = \frac{1}{p} \qquad \sigma_{geom} = \frac{\sqrt{1-p}}{p}$$

The debate changes to whether with two dice it's more likely to

(a) take ***up to*** 10 rolls to get a 12 (roll nine times and get a 12 on the 10th roll) or

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Geometric Distribution $P(X=k)=(1-p)^{k-1}p$

$$\sum_{n=1}^{10}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{n-1}=\left(\frac{1}{36}\right)+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{2}\ldots+\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{9}=0.2455$$

```
geometcdf(1/36,1
0)
      .2455066161
```

Binomial Distribution $P(X=k)=\binom{n}{k}p^{k}(1-p)^{n-k}$

$$\binom{10}{3}\left(\frac{1}{36}\right)^{3}\left(\frac{35}{36}\right)^{7}+\binom{10}{2}\left(\frac{1}{36}\right)^{2}\left(\frac{35}{36}\right)^{8}+\binom{10}{1}\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{9}=0.2454$$

Why the subtraction?

Recall that $P(k\le 3)$ includes $k=0$

```
binomcdf(10,1/36
,3)-binomcdf(10,
1/36,0)
      .2453973233
```

We only want to find $1\le k\le 3$

Is there a formula/idea that is not on the AP formula sheet that applies?

If events are disjoint, then $P(A \cap B) = 0$

If events are independent, then $P(A|B) = P(A)$ or

$P(A \cap B) = P(A)P(B)$

Use these formulas when appropriate, i.e. based on what information is given

If $Y = aX + b$, then $\mu_Y = a\mu_X + b$, $\sigma_Y = a\sigma_X$

If $X$ and $Y$ are independent, then $\mu_{X \pm Y} = \mu_X \pm \mu_Y$, $\sigma^2_{X \pm Y} = \sigma^2_X + \sigma^2_Y$

$$\sigma^2_{aX \pm bY} = a^2\sigma^2_X + b^2\sigma^2_Y$$

If $X$ has a geometric distribution, then $P(X = k) = (1 - p)^{k-1} p$

If $X$ has a binomial distribution, then $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$