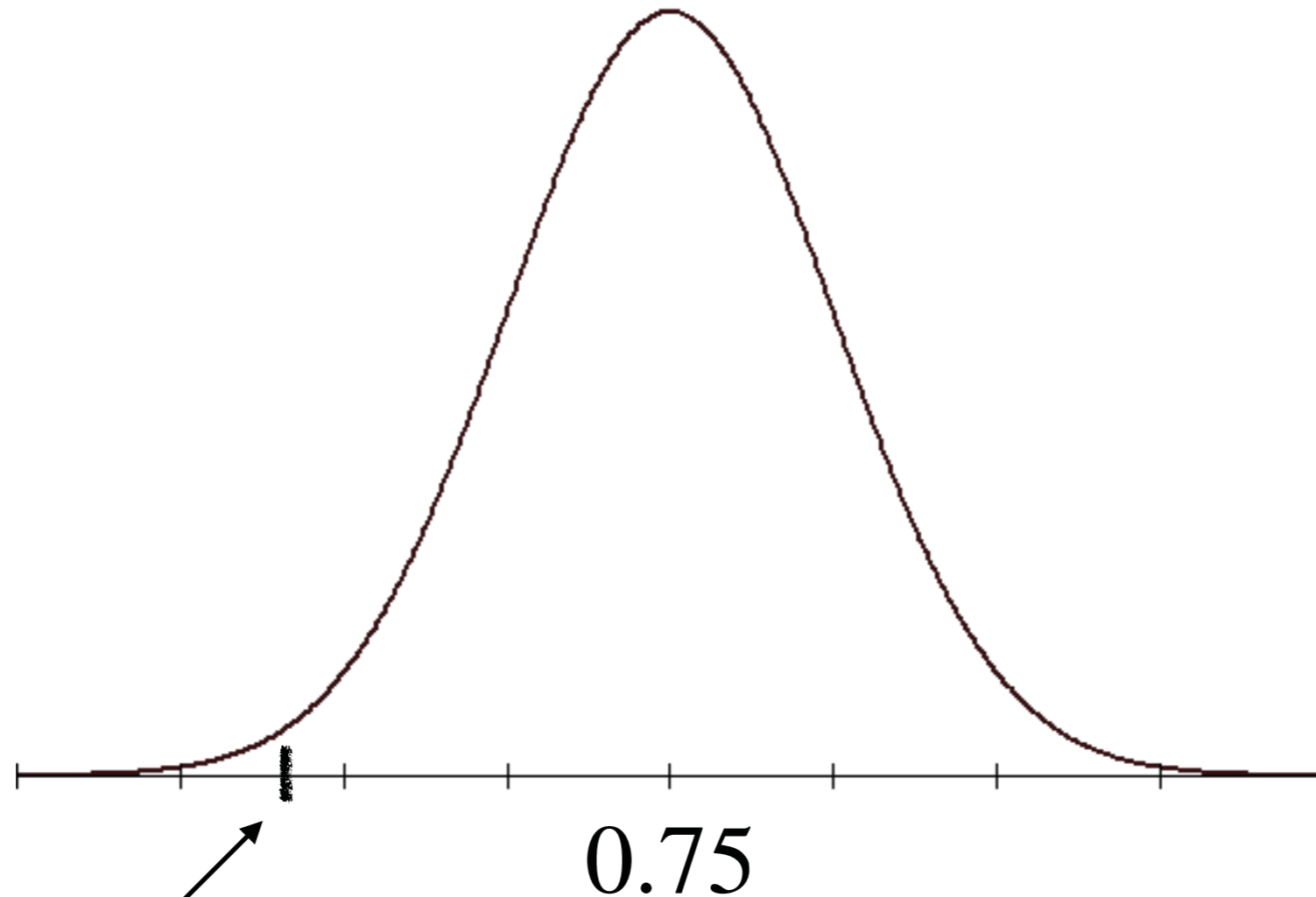


Reilly is claiming that 75% of prom shoppers prefer to get their prom dress (shoes and all) on Revolve.

Annabelle is convinced that the percentage is way less (she prefers Lulus). BTW, according to Reilly, Delaney plans to write an editorial in the next edition of *Girls in Journalism* about this debate. But I digress... Annabelle does a google form survey and is able to get a random sample of 156 students shopping for a prom dress to give their first choice. Her results are that of the 156 responses, 107 of them swear by Revolve.

If Annabelle is determined to reject Reilly's claim, what level of significance would allow her to do so?



0.75

If Reilly is right

0.686

Annabelle's result is about here

How far off can Annabelle's sample proportion be before we start to question Reilly's claim?

$\alpha$ 

Also called 'level of significance'  
or 'significance level'.

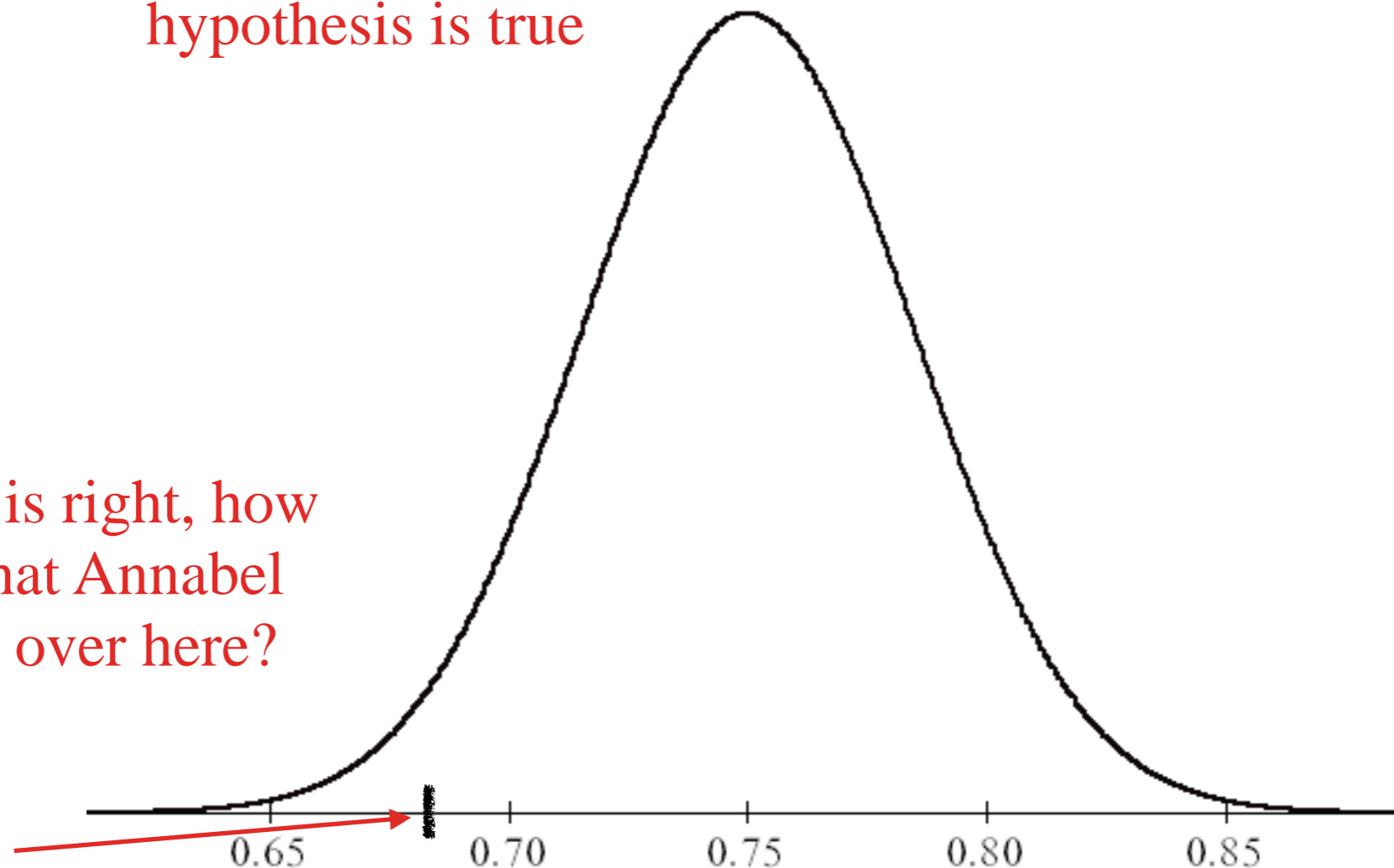
The level of significance is the smallest probability that we will allow to convince us to stick with Reilly's claim

We will calculate the probability ( $P$ -value) of getting Annabelle's result if Reilly's claim is true. The further away from the center Annabelle's results are, the more we question Reilly's claim.

**$P$ -Value** is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true

So if Reilly is right, how likely is it that Annabel gets a result over here?

0.686



$\alpha$ 

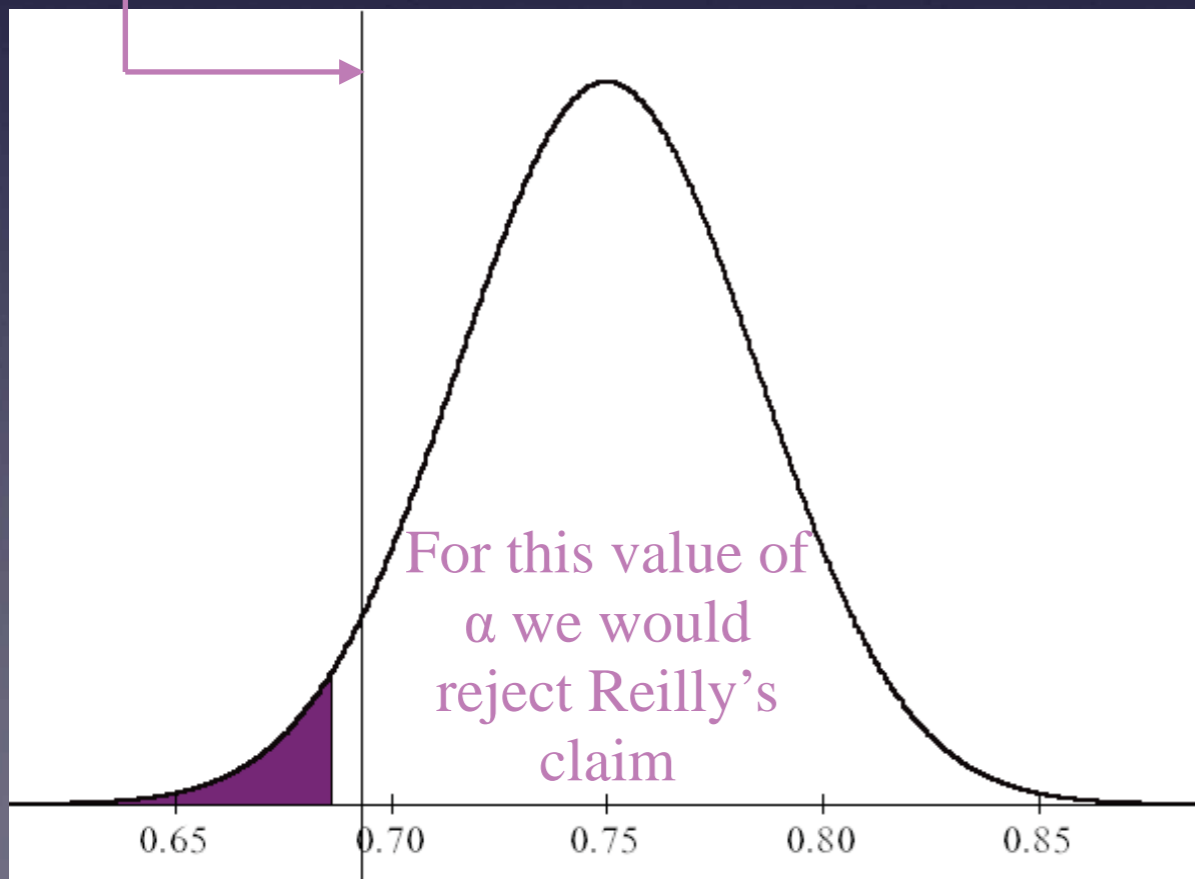
Also called 'level of significance'  
or 'significance level'.

**P-Value** is the probability of getting Annabelle's results assuming that Reilly's claim is true.

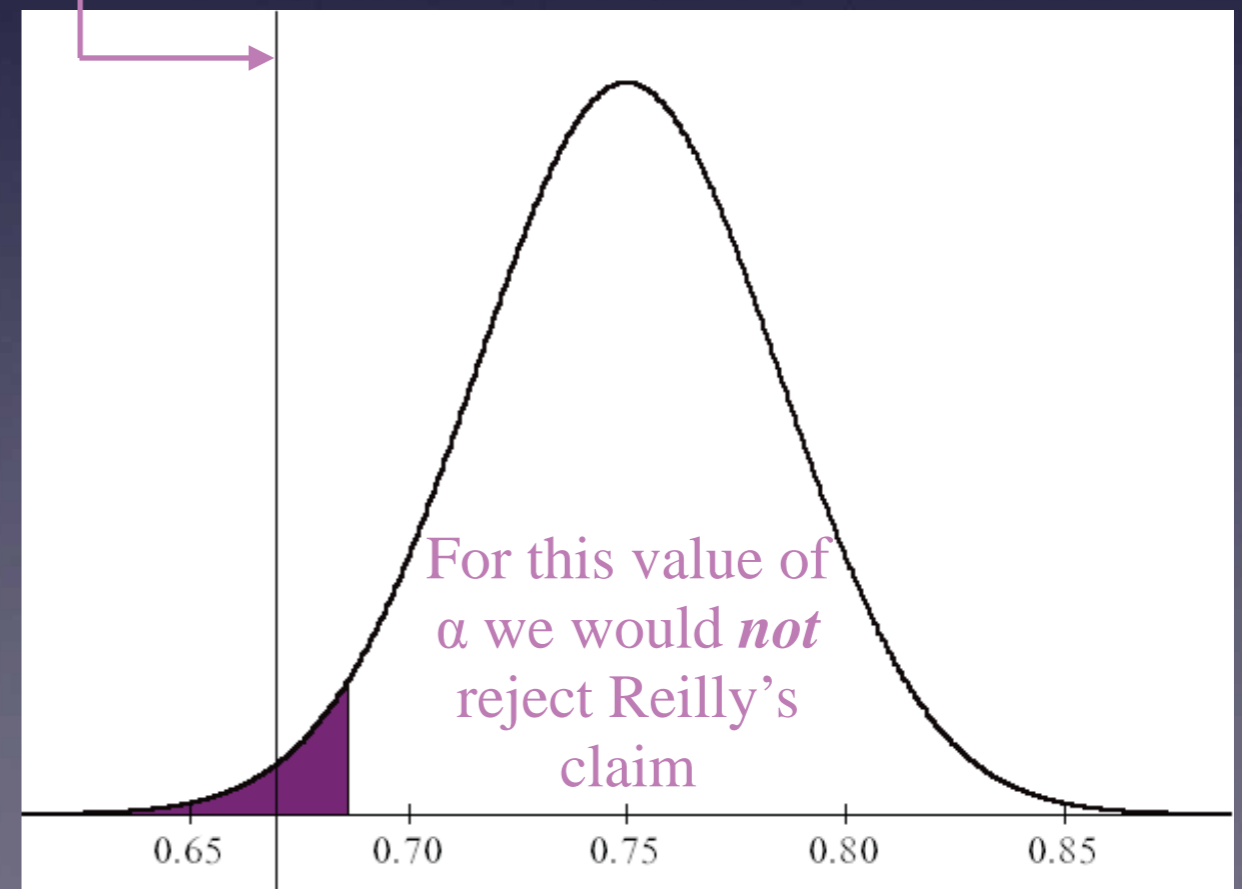
So if the  $p$  value is less than  $\alpha$  then either Annabelle's results are a fluke or Reilly is wrong/embellishing.

So how do we calculate this?

Critical Value where the  $P$ -value  
will reach  $\alpha$



Critical Value where the  $P$ -value  
will reach  $\alpha$



# Single Sample Hypothesis Tests for Proportions

Note #1: Use colons

$$H_0 : p = \#$$

$$H_a : p \neq \#$$

$$H_a : p < \#$$

$$H_a : p > \#$$

Note #3:  $H_0$  ALWAYS gets an = ...even if the wording in the problem sounds like it shouldn't

Note #2: Use only PARAMETERS in your hypothesis...although there will be some problems where we'll use words/sentences

Note #4: The symbol used in the alternate will come from the context of the problem

$\neq$  - two-sided test, equivalent to a Confidence Interval (CI)

$\{ \begin{matrix} < \\ > \end{matrix} \}$  - one-sided test

## Steps in Hypothesis Testing

1. Define the population characteristic (i.e. parameter) about which hypotheses are to be tested.
2. State the null hypothesis  $H_0$       Reilly's claim  $\rightarrow p = 0.75$       Proportion of Revolve users
3. State the alternative hypothesis  $H_a$       Annabelle's claim  $\rightarrow \hat{p} < 0.75$
4. State the significance level for the test  $\alpha$       What value of  $\alpha$  are we going with?
5. Check all assumptions.      We've done this before
6. State the name of the test.      Sample proportion  $z$
7. State  $df$  (degrees of freedom) if applicable (this will be in Unit 7).
8. Display the test statistic to be used without any computation at this point.      Which formula?
9. Compute the value of the test statistic, showing specific numbers used.      Show work on AP Exam
10. Calculate the  $P$  – value.      Is  $p$  - value greater than or less than significance level? This determines the outcome, reject or fail to reject.
11. Sketch a picture of the situation.
12. State the conclusion in two sentences -
  1. Summarize in theory discussing  $H_0$
  2. Summarize in context discussing  $H_a$

# Single Sample Hypothesis Tests for Proportions

## Steps in Proportion Hypothesis Testing

1.  $p = \dots\dots$  Proportion of Revolve users

2.  $H_0 : p = \#$  Reilly's claim  $\rightarrow p = 0.75$

$\neq$

3.  $H_a : p <$  Annabelle's claim  $\rightarrow \hat{p} < 0.75$

$>$

4. State the significance level  $\alpha$  for the test

8/9. 
$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \# \quad \longleftarrow \quad z = \frac{x - \mu}{\sigma}$$

Note the formula for  $z$  score using proportions

10.  $P - value =$

$$P(z > \#) = normalcdf(\#, 1E99, 0, 1)$$

$$P(z < \#) = normalcdf(-1E99, \#, 0, 1)$$

$$2P(z > \#) = 2 * normalcdf(\#, 1E99, 0, 1)$$

$$2P(z < \#) = 2 * normalcdf(-1E99, \#, 0, 1)$$

Is  $p$  - value greater than or less than significance level? This determines the outcome, reject or fail to reject.

12. State the conclusion in two sentences -

1. Summarize in theory discussing  $H_0$ .
2. Summarize in context discussing  $H_a$ .

5. Assumptions:

1. Random Sample

2.  $np \geq 10$

$n(1-p) \geq 10$

3. SSSRTP

6. 1 Sample Proportion  $z$  Test

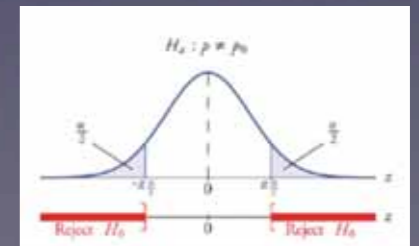
7.  $df = N / A$

11.



} one-sided tests

} two-sided tests



**Alternate Hypothesis:**

- A**  $H_a: p >$  hypothesized value
- B**  $H_a: p <$  hypothesized value
- C**  $H_a: p \neq$  hypothesized value

**P – value:**

Area under z curve to right of calculated z

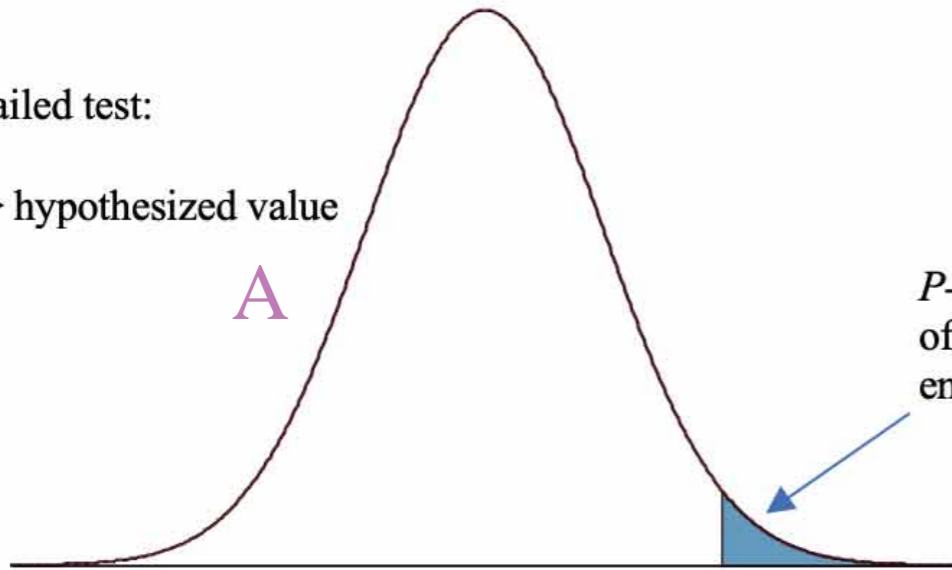
Area under z curve to left of calculated z

- (1) 2(area to right of z) if z is positive, or
- (2) 2(area to left of z) if z is negative

Right tailed test:

$H_a: p >$  hypothesized value

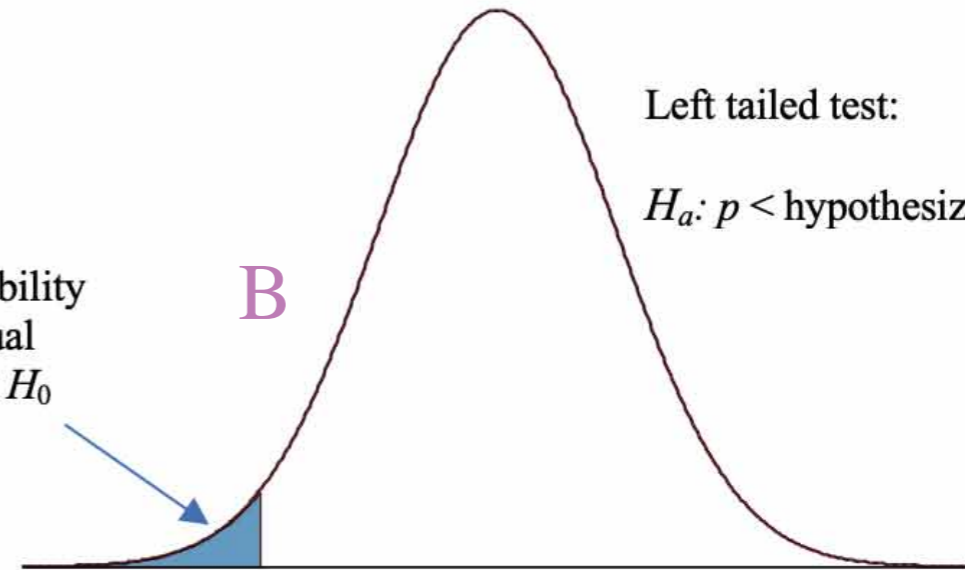
**A**



Left tailed test:

$H_a: p <$  hypothesized value

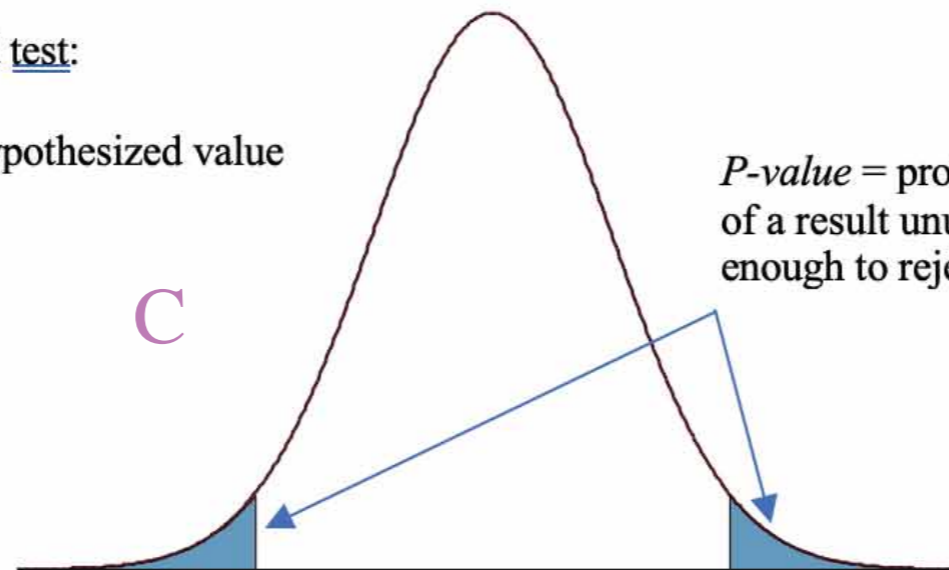
**B**



Two tailed test:

$H_a: p \neq$  hypothesized value

**C**

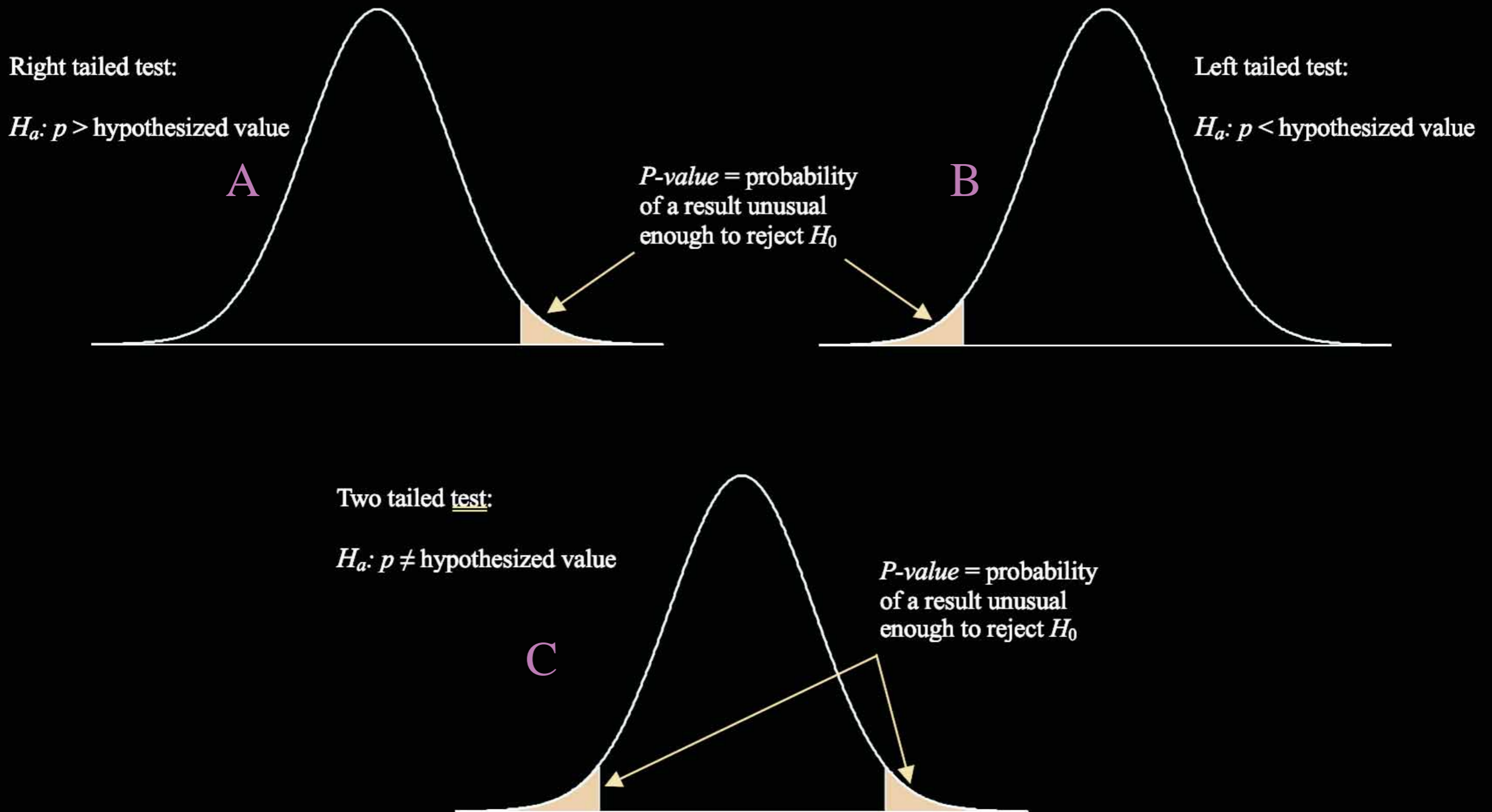


*P-value* = probability of a result unusual enough to reject  $H_0$



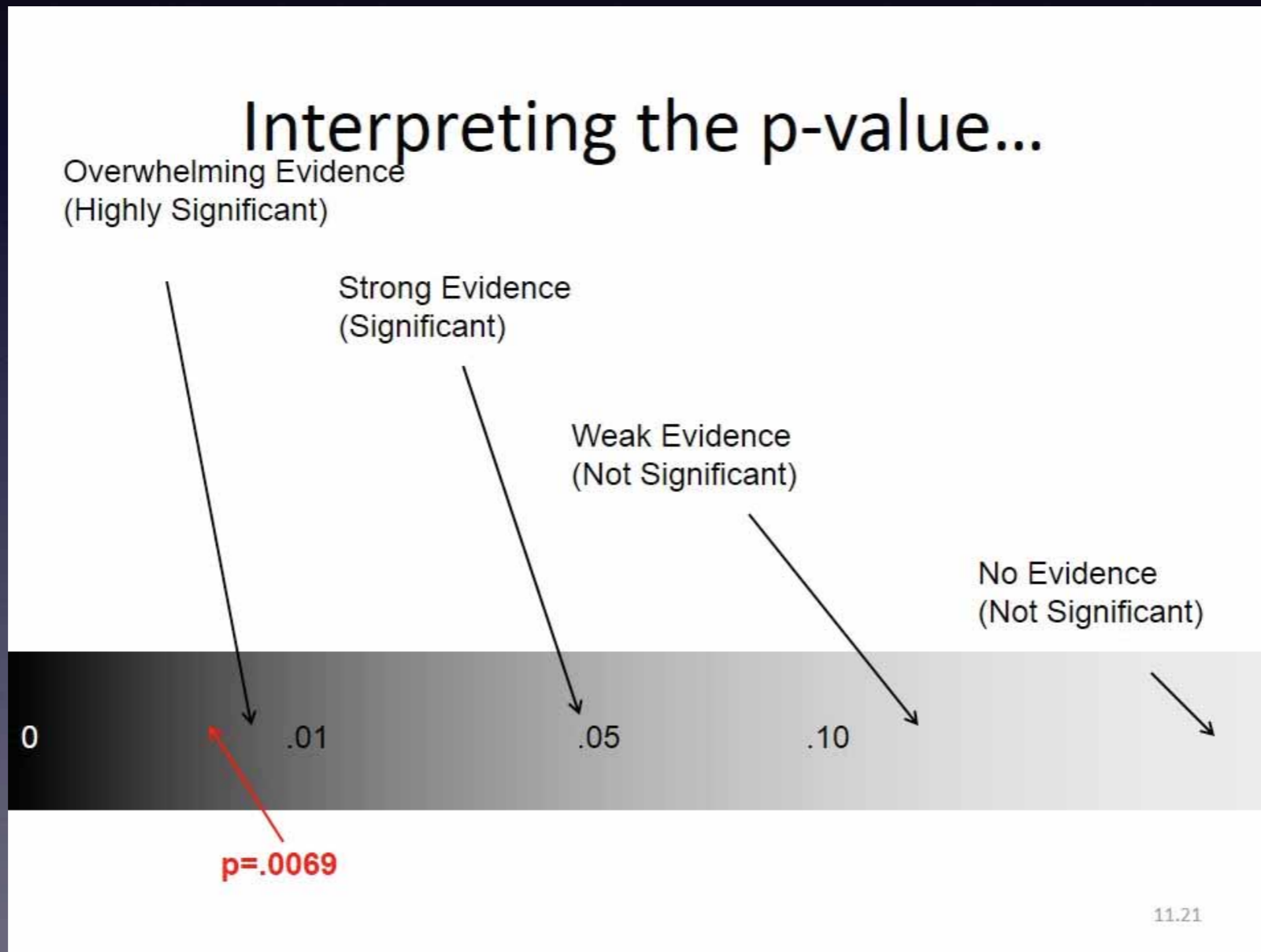
10.  $P\text{-value} =$

<b>A</b>	$P(z > \#) = \text{normalcdf}(\#, 1E99, 0, 1)$	} one-sided tests
<b>B</b>	$P(z < \#) = \text{normalcdf}(-1E99, \#, 0, 1)$	
<b>C</b>	$2P(z > \#) = 2 * \text{normalcdf}(\#, 1E99, 0, 1)$	} two-sided tests
	$2P(z < \#) = 2 * \text{normalcdf}(-1E99, \#, 0, 1)$	



$P\text{-Value} < \alpha \Rightarrow \text{Reject } H_0 ; \text{Evidence for } H_a$

$P\text{-Value} > \alpha \Rightarrow \text{Fail to Reject } H_0 ; \text{No Evidence for } H_a$



The Public Policy Institute of California reported that 71% of people nationwide prefer to live in a single-family home. To determine whether the preferences of Californians are consistent with this nationwide figure, a random sample of 2002 Californians were interviewed. Of those, 1682 said they consider a single-family home the ideal. Can we reasonably conclude that the proportion of Californians who prefer a single-family home is different from the national figure? We will answer this question by carrying out a hypothesis test with  $\alpha = 0.01$ .

1.  $p$  = true proportion of Californians who prefer a single-family home

2.  $H_0 : p = 0.71$

3.  $H_a : p \neq 0.71$  which means that it will be a two sided test

4.  $\alpha = 0.01$

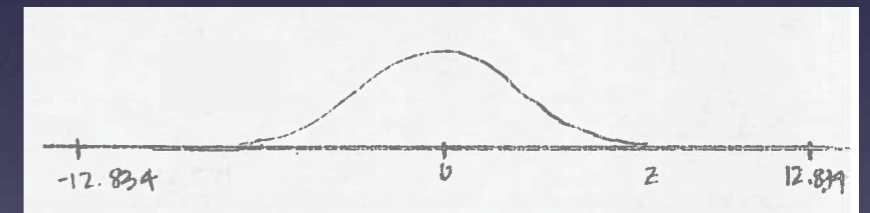
5. Assumptions: 1. Random Sample ✓ 2.  $np = 2002(0.71) = 1421.42 \geq 10$  ✓ 3. SSSRTP ✓

$$n(1 - p) = 2002(1 - 0.71) = 580.58 \geq 10 \checkmark$$

6. 1 Sample Proportion  $z$  Test

$$8/9. \quad z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.84 - 0.71}{\sqrt{\frac{0.71(1-0.71)}{2002}}} = 12.834$$

11.

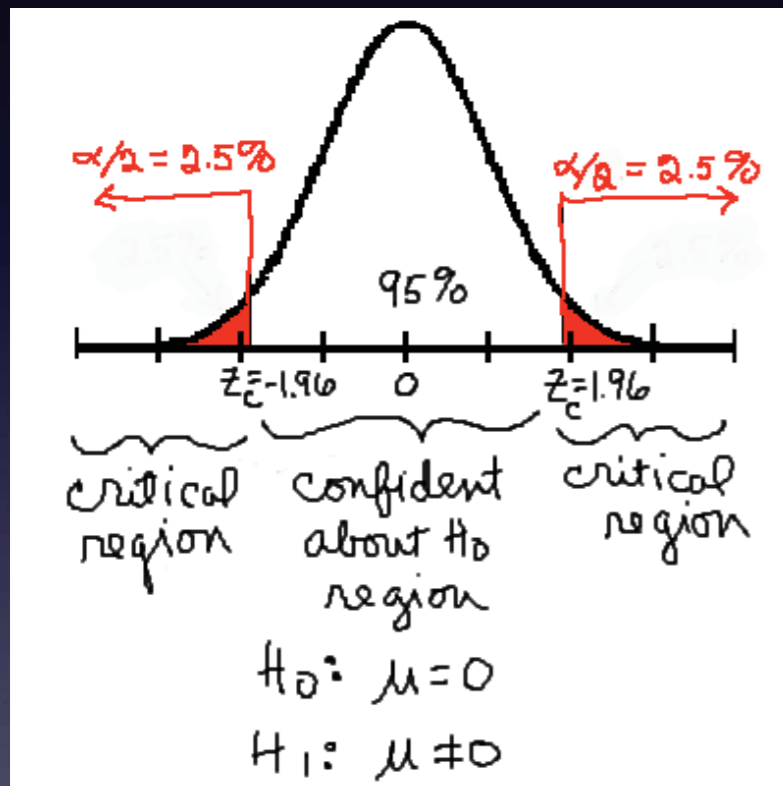


10.  $P - value = 2P(z > 12.834) = 2 * normalcdf(12.834, 1E99, 0, 1) = 0$  ← two sided test

12. Because our  $P - value = 0 < 0.01 = \alpha$ , we reject  $H_0$  at the 0.01 level of significance. We have evidence that the true proportion of Californians who prefer a single-family home differs from the national figure.

# Confidence Intervals are Related to Two-Sided Tests

In general, for every two-sided test of hypothesis there is an equivalent statement about whether the hypothesized parameter value is included in a confidence interval.



The 95% confidence interval for the mean weight of all the Dole Pineapples grown in the field this year is 31.255 to 32.616 ounces.

$$95\% \text{ CI: } \mu \in (31.255, 32.616)$$

$$H_0: \mu = 31$$

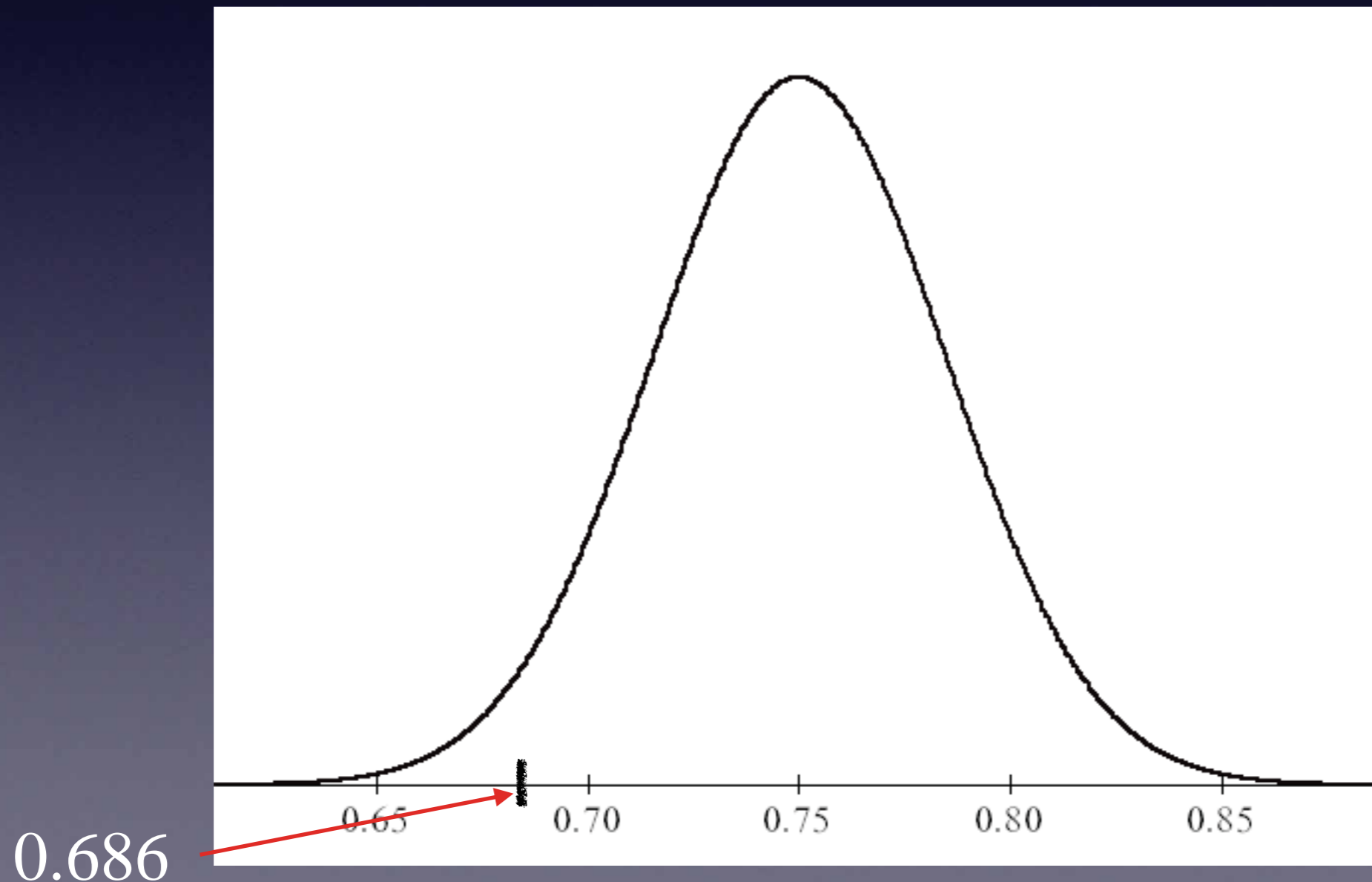
$$H_a: \mu \neq 31$$

When the two-sided significance test at level  $\alpha$  rejects  $H_0: \mu = \mu_0$ , the  $100(1 - \alpha)\%$  confidence interval for  $\mu$  will not contain the hypothesized value  $\mu_0$ .

When the two-sided significance test at level  $\alpha$  fails to reject the null hypothesis, the confidence interval for  $\mu$  will contain  $\mu_0$ .

P-Value is the probability of getting Annabelle's results assuming that Reilly's claim is true.

So if the  $p$  value is less than  $\alpha$  then either Annabelle's results are a fluke or Reilly is wrong/embellishing.



If Annabelle is determined to reject Reilly's claim, what level of significance would allow her to do so?

Reilly's claim:  $p = 0.75$

Annabelle's results: 107/156 prefer Revolve

The test statistic is the  $z$  score from a presumed normal distribution with 0.75 the mean

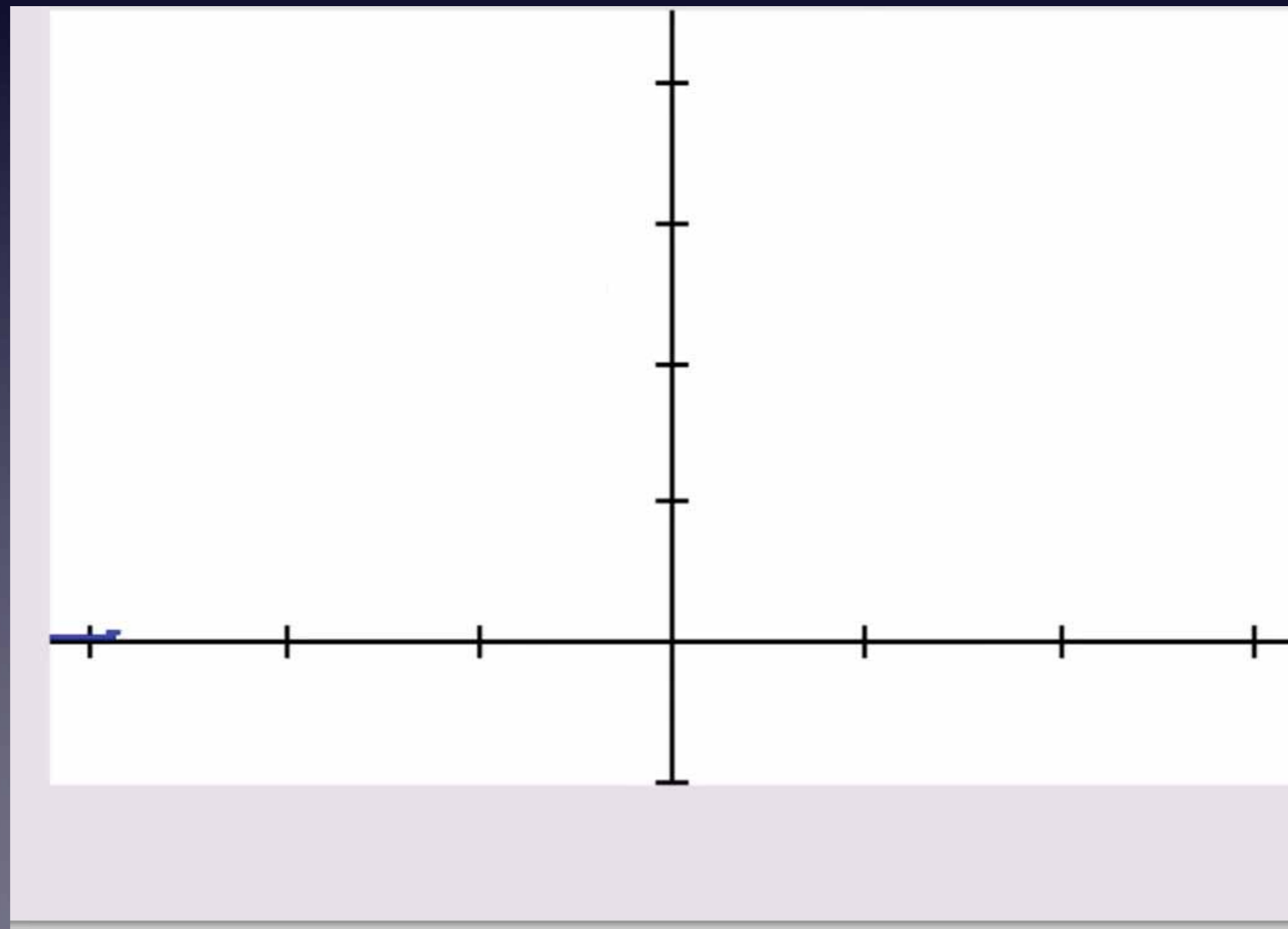
$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{(107/156) - 0.75}{\sqrt{\frac{0.75(0.25)}{156}}} = -1.849$$

$normalcdf(1E99, -1.849) = 0.0322$       So now what?

If Annabelle is determined to reject Reilly's claim, what level of significance would allow her to do so?

$$p\text{-value} = 0.0322$$

What is the meaning of this number?

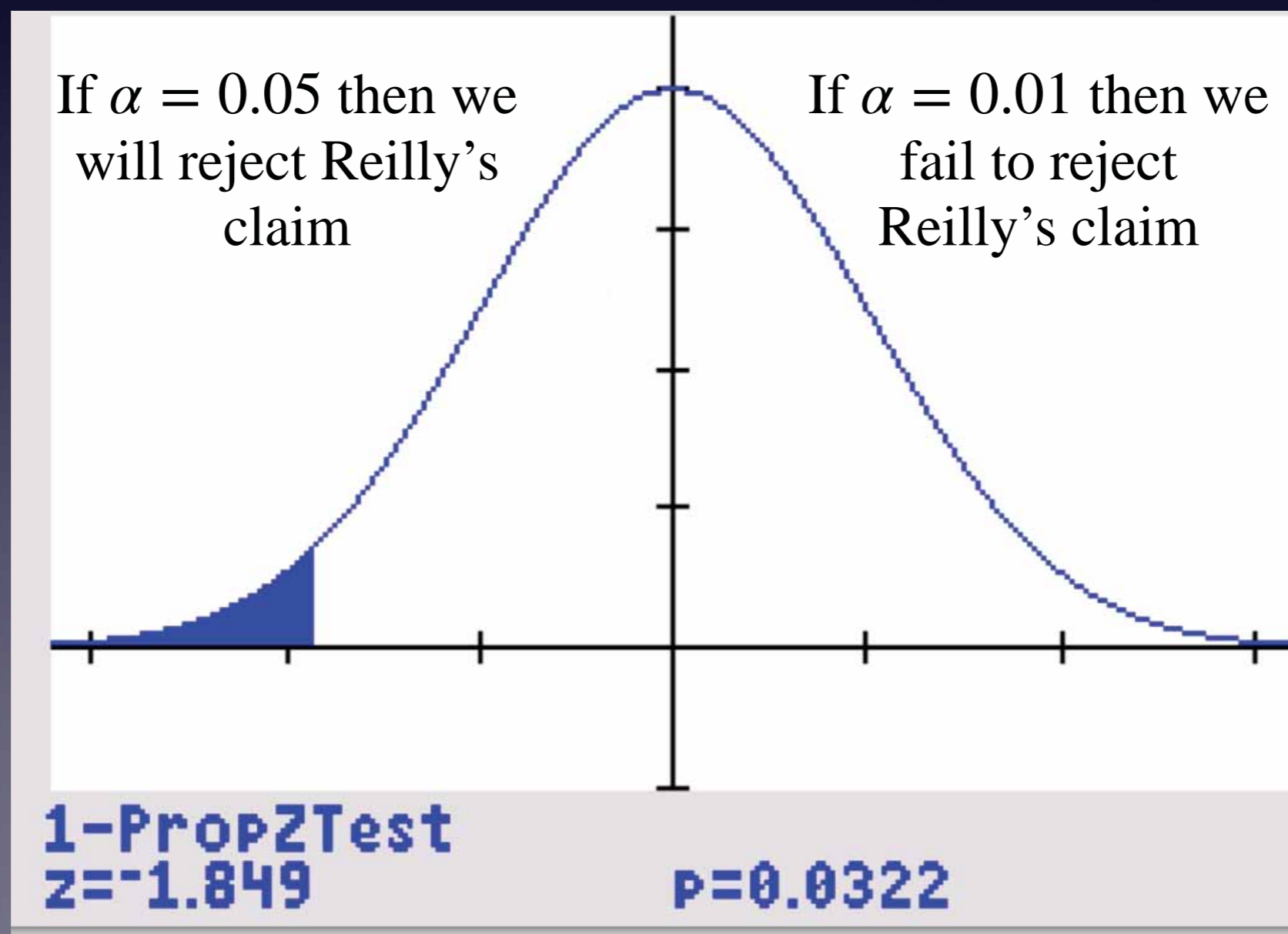


If Annabelle is determined to reject Reilly's claim, what level of significance would allow her to do so?

$$p\text{-value} = 0.0322$$

Depending on our choice of  $\alpha$  (our level of significance)

Because  
 $p < \alpha$



Because  
 $p > \alpha$



Justify = hypothesis test

estimate = CI

Statistically significant = reject null

Next time, Kevin challenges Raven's claim about Patrick Mahomes...and we find out what can go wrong in a hypothesis test